EXERCISES IN ALGEBRA

BY

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PREFACE.

THE desirable results to be attained in the study of Algebra are the thought of letters as general symbols of number, familiarity with the laws of the algebraic notation, facility and accuracy in performing algebraic processes, skill in the use of the equation as a means of mathematical investigation, and the development of the reasoning powers. In the attainment of these results, the teacher is the important factor, but the character of the book used determines in a greater or less degree the character of the work done and the effects upon the student.

These considerations have been the motives for the preparation of this work, and it is confidently believed that the use of it will lead to broader views of numbers and their properties, and produce increasing mental power. It is designed for use in high schools and academies, and advanced classes in grammar schools. A distinguishing feature of the book is its arrangement. The definitions, demonstration of principles, derivation of rules, model solutions, and illustrations occupy the last half of the book, and the exercises and problems the first half. The reasons for this arrangement are evident, and it is believed that the separation of the text from the exercises and problems can be no inconvenience in the legitimate use of either. The text is complete in the clearness and conciseness of definitions, thorough demonstration of principles, careful derivation of rules, and the abundance of illustrations and model

solutions. Frequent notes to the teacher will be found in the first half of the book. The numbers in these notes refer to articles in the text, and the notes indicate that the following exercise contains new work, for which students must be prepared.

The other prominent features of the work are the frequent exercises in algebraic expression, the unusual number and variety of examples and problems and the careful grading of the same, the early introduction of the equation and its use in the solution of problems, the thorough treatment of factoring, the completeness of the work on involution and evolution, the discussions and exercises on the signification of exponents, the character and amount of work in radicals, and the emphasis on the solution of complete quadratics by the method of factoring.

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The work is submitted to the profession with the hope that teachers and students may use it with the greatest pleasure and profit. This is the highest reward the author can desire.

GEORGE E. ATWOOD.

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CONTENTS.

ALGEBRA.

INTRODUCTION.

1. Algebra is not altogether unlike arithmetic. Both treat of numbers, but the method of representing numbers is not the same in both. In arithmetic, numbers are represented by ten characters, which are called *figures*. Each figure, when standing alone, represents a definite and fixed value, or number of things. The figure 4 standing alone always represents four things; the figure 7, seven things; the figure 9, nine things.

2. In algebra, numbers are represented by letters, by figures, or by a combination of figures and letters. The figures used in algebra always represent definite numbers, as in arithmetic, but the letters represent any numbers. The letter x, for example, may represent 5 in one problem, 15 in another, 36 in another, $7\frac{3}{5}$ in another. When figures are written with letters, without any sign between them, the figure indicates how many times the quantity, or number represented by the letter, is taken; and when one letter is written with another, either one of the letters indicates how many times the quantity, or number represented by the other letter, is taken. For example,

Whatever number x represents,

$3x$ means 3 times x,
$5x$ means 5 times x,
ax means a times x.

3. The signs of addition, subtraction, multiplication, and division are used in algebra with the same signification that they have in arithmetic.

EXERCISES IN ALGEBRAIC EXPRESSION.

1. If x represents a certain number, what does $4x$ represent? $2x$? $9x$? ax? bx?

2. If m represents a certain number, what represents 6 times the number? a times the number?

3. Indicate the sum of 8 and 7. Of x and 5. Of a and b. Of a, b, and c.

4. Indicate the difference between x and y, when x is greater than y. When y is greater than x.

5. Indicate the sum of a and b diminished by c. The sum of x and $3x$ diminished by y.

6. Indicate in two ways the product of 5 and a. Of a and x. Of 3, x, and y. Of a, b, and x.

7. A man paid x dollars for a harness and $4x$ dollars for a horse. How much did both cost?

8. If x represents the price of a yard of silk, what does $7x$ represent? $3x$? ax?

9. If a man works for x dollars a day, how much will he earn in nine days? In a days?

10. A man is x years old to-day. How old was he 7 years ago? Sixteen years ago?

11. If a yard of ribbon is worth a cents, how much is a foot worth? Two feet?

12. If you are x years old to-day, how old will you be in 8 years? In twelve years?

13. A man bought a carriage for x dollars and sold it at a loss of y dollars. How much did he get for it?

14. A lady paid a dollars for silk at b dollars a yard. How many yards did she buy?

15. A man is 3 times as old as his son. If the son is x years old, how old is the father?

16. A boy had a dollars. He earned b dollars more, and then spent c dollars. How much had he left?

17. A man sold a horse for x dollars, thereby gaining y dollars. Find the cost of the horse.

18. A boy bought a apples at n cents apiece. How much did he pay for them?

19. I have x dollars. If I pay two debts of a dollars and b dollars, how much shall I have left?

20. A horse cost $5x$ dollars, a harness x dollars, and a carriage $4x$ dollars. Express the cost of all in two ways.

21. A boy has x dimes. How many cents has he? How many dollars has he?

22. A grocer bought a barrels of flour for x dollars. What was the price of the flour per barrel?

23. If the difference between two numbers is 12 and the smaller one is x, what is the larger number?

24. A boy bought x apples at m cents apiece and sold them at n cents. If he gained, what was his gain?

25. If one number is x, and another number is 4 times as great, what is the sum of the numbers?

26. How long will it take a man to walk x miles at the rate of 5 miles an hour?

27. A man was x years old a years ago. How old will he be in b years? How old was he c years ago?

28. If one part of 7 is x, what is the other part? If one part of x is a, what is the other part?

29. A man worked x hours a day for eight days at y cents an hour. With the money earned he bought a coat for a dollars. How much money did he have left?

30. A has x sheep, B has twice as many as A, and C has twice as many as A and B together. What is the value of all their sheep at a dollars a head?

31. A man paid a dollars for b yards of silk. At the same price, how much will c yards cost?

32. A boy bought x oranges at a cents each and sold them at b cents. If he lost, what was his loss?

33. In b years a man will be x years old. How old was he c years ago?

34. If thirty dollars is divided equally among x boys, how much will each boy receive?

35. A has x cows, B has 5 more than A, and C has as many as A and B together. How many have all?

36. How far can a man walk in x hours at the rate of a miles in b hours?

37. A grocer bought a pounds of coffee at x cents a pound and sold it at y cents a pound. If y is greater than x, did he gain or lose, and how much? If x is greater than y, did he gain or lose, and how much?

38. A man bought a pounds of tea at x cents a pound, and paid for it in butter at b cents a pound. How many pounds of butter did he give for the tea?

39. A dealer bought a crates of berries at n cents a quart, and b crates of another kind at m cents a quart. If each crate contained x quarts, how much did they cost?

40. A merchant bought a yards of silk at x dollars a yard. If he sold it all at a profit of y dollars a yard, how much did he receive for it?

41. A speculator bought x acres of land at a dollars an acre. If he sold it all at a loss of b dollars an acre, how much did he receive for it?

4. The *sign of equality* is used in arithmetic to indicate that the numbers between which it is placed are equal.

The expression $9 + 7 = 8 \times 2$ denotes that the sum of 9 and 7 is equal to the product of 8 and 2.

The expression $8 - 5 = 9 \div 3$ denotes that the difference between 8 and 5 is equal to the quotient of 9 divided by 3.

5. In like manner, the sign of equality is used in algebra to indicate that numbers represented wholly or partly by letters are equal.

The expression $a + x = by$ denotes that the sum of the numbers represented by a and x is equal to the product of the numbers represented by b and y.

The expression $ax - 5 = bx + 9$ denotes that the product of a and x diminished by 5 is equal to the product of b and x increased by 9.

6. An Equation is the expression of the equality of two numbers.

$3x = 12.$	$4x = 20.$	$2x - 7 = 9.$
$3x = 15.$	$4x = 32.$	$2x + 2 = 8.$
$3x = 10.$	$4x = 30.$	$2x - 5 = 7.$

Since $3x$, in the above equations, means 3 times x;

$3x$ and 12 are equal only when x represents 4,
$3x$ and 15 are equal only when x represents 5,
$3x$ and 10 are equal only when x represents $3\frac{1}{3}$.

Since $4x$, in the above equations, means 4 times x;

$4x$ and 20 are equal only when x represents 5,
$4x$ and 32 are equal only when x represents 8,
$4x$ and 30 are equal only when x represents $7\frac{1}{2}$.

Since $2x$, in the above equations, means 2 times x;

$2x - 7$ and 9 are equal only when x represents 8,
$2x + 2$ and 8 are equal only when x represents 3,
$2x - 5$ and 7 are equal only when x represents 6.

It is evident from the above that when an equation contains only one letter, as x, the letter used represents some *particular* number.

The number represented by the letter in such an equation is called the *unknown number.*

EXERCISES.

1. If $7x = 28$, what does x represent?
2. If $5x = 40$, what does x represent?
3. If $3x = 29$, what does x represent?
4. If $3x + x = 24$, what does x represent?
5. If $2x - x = 17$, what does x represent?
6. If $2x + 3x = 27$, what does x represent?
7. If $7x - 3x = 36$, what does x represent?
8. If $5x - 4x = 15$, what does x represent?
9. If $4x + 3x + x = 32$, what does x represent?
10. If $7x - 3x - x = 36$, what does x represent?
11. If $9x + 4x - x = 60$, what does x represent?
12. If $4x + 2x + 3x = 54$, what does x represent?
13. If $5x + 3x - 7x = 13$, what does x represent?
14. If $9x - 4x - 2x = 30$, what does x represent?
15. If $8x - 3x - 4x = 17$, what does x represent?
16. If $7x - 4x + 5x = 23$, what does x represent?

7. The process of finding the number represented by the letter in an equation is called the *solution of the equation.*

8. Problems in arithmetic or algebra involve an unknown number or numbers, and the process by which the unknown number is determined is called the *solution of the problem.*

9. The unknown number or numbers are found in arithmetic by one or more of the processes of addition, subtraction, multiplication, and division.

10. In the solution of problems in algebra, the equation is employed. In fact, the practical utility of algebra consists in the application of the equation to the solution of problems.

ALGEBRAIC SOLUTION OF PROBLEMS.

1. The sum of two numbers is 84, and the greater number is 6 times the less. What are the numbers?

Let x = the less number;
then $6x$ = the greater number.
Since the *sum* of the numbers is 84,
$6x + x = 84$
$7x = 84$
$x = 12$, the less number,
$6x = 72$, the greater number.

2. A man bought a horse, a carriage, and a harness for \$350. The carriage cost 3 times as much as the harness, and the horse cost twice as much as the carriage. What was the cost of each?

Let x = the number of dollars the harness cost;
then $3x$ = the number of dollars the carriage cost,
and $6x$ = the number of dollars the horse cost.
Since they *all* cost \$350,
$x + 3x + 6x = \$350$
$10x = \$350$
$x = \$35$, the cost of the harness,
$3x = \$105$, the cost of the carriage,
$6x = \$210$, the cost of the horse.

3. The difference between two numbers is 48, and the greater number is five times the less. Find the numbers.

Let x = the less number;
then $5x$ = the greater number.
Since the *difference* between the numbers is 48,
$5x - x = 48$
$4x = 48$
$x = 12$, the less number,
$5x = 60$, the greater number.

4. A father's age exceeds the age of his son by three times the son's age, and the sum of their ages is seventy-five years. What is the age of each?

Let x = the son's age;
then $3x$ = the difference between their ages,
and $4x$ = the father's age.

Since the *sum* of their ages is 75 years,

$$4x + x = 75$$
$$5x = 75$$
$$x = 15, \text{ the son's age,}$$
$$4x = 60, \text{ the father's age.}$$

To the Teacher. — During the study of the following problems, teach very carefully definitions, principles, and rules, 1 . . . . . 55. Be sure that students know the law of the algebraic notation, and the signification of coefficients and exponents.

PROBLEMS.

1. A man paid thirty-five dollars for a coat and shoes. If the coat cost 6 times as much as the shoes, what was the cost of the shoes?

2. The sum of two numbers is 126, and the larger number is 8 times the smaller. Find the larger number.

3. A man sold a horse and carriage for $ 500, receiving three times as much for the horse as for the carriage. How much did he receive for the carriage?

4. A has three times as many sheep as B, and both have one hundred eighty. How many sheep has A?

5. There are 795 pupils in a school, and there are twice as many girls as boys. How many girls are there?

6. An estate of twenty-four thousand dollars was divided between a son and daughter, the son having 3 times as much as the daughter. How much did the son receive?

7. The greater of two numbers is six times the smaller, and their difference is 425. Find the larger number.

8. A, B, and C own 750 acres of land. B owns three times as many acres as A, and C owns one-half as many acres as A and B together. How many acres has C?

9. My house cost 3 times as much as the lot. If the house cost five thousand dollars more than the lot, what did the house cost?

10. A man's salary was doubled each year for three years. If he received five thousand two hundred fifty dollars in the three years, what was his salary the second year?

11. A farmer raised 1512 bushels of grain. He had one-fifth as many bushels of rye as oats, and twice as many bushels of corn as oats and rye together. How many bushels of rye and oats did he have?

12. A and B are 81 miles apart. They travel toward each other until they meet, A traveling twice as fast as B. How far does B travel?

13. A and B own a farm worth $16,800. A has 3 times as much invested as B. How much has A invested?

14. A is six times as old as B, and the difference between their ages is 65 years. Find the age of B.

15. A man paid four times as much for a farm as he paid for a house. If the farm cost $8400 more than the house, how much did he pay for both?

16. In a mixture of 192 bushels of corn and oats, there are twice as many bushels of corn as of oats. How many bushels of corn are there in the mixture?

17. A man earned five times as much as his son. If the son earned six hundred forty-three dollars less than his father, how much did the son earn?

18. A has twice as many sheep as C, and B has four times as many as C. If all have 595, how many has A?

NUMERICAL VALUES.

Find the numerical value of the following expressions when $a=1$, $b=2$, $c=3$, $d=4$, $e=0$, $m=\frac{1}{2}$, and $n=\frac{1}{3}$:

1. $5a+4b+3c+8d$.
2. $8d-7a+8m+6c$.
3. $9c-9n+8d+7c$.
4. $5c+4b-6n+6m$.
5. $7a+5d-3n+6e$.
6. $9d-8e+4m-3a$.
7. $8b+6c-2d+9c^2$.
8. $8m+4e+5d-9a^2$.
9. $7d+6m+8c-9n^2$.
10. $9n+8c-7e+8m$.
11. $3d+4c-2a^2+4m^2$.
12. $4c+6e+8b^2+9a^2$.
13. $2ab+8m-3n^2+2c^2$.
14. $5ad+8a^2-2b-8m^2$.
15. $4cn+2d^2-7e-4m^2$.
16. $6am+9a^2+2bc-3m^2$.
17. $bcd-9n^2-4a^2+6am$.
18. $4ad-6n^2+2dm-2b^2$.
19. $abc+2d^2-6mn+2c^2$.
20. $3cd-3a^2-3mn+4b^2$.
21. $bcd-5e^2-4m^3+3cn$.
22. $5ab+6c^2-4dm+8a^2$.
23. $6b^3-adm+5bc-6n^2$.
24. $7d^2-4cd+5b^3-6am$.
25. $8a^4+6mn-2b^2+bcd$.
26. $8b^2-5am+9bn-2ac$.
27. $5cd-8m^3+9a^3-6cn$.
28. $8ad-7e^2-3bm+2dn$.
29. $4bc+7d^2-9n^3+7ab$.
30. $7a^4+9ad-2bm+5an$.
31. $6bn+5e^3-7cm^2+8ad$
32. $5bd+6a^3c-4bm+4dn$.
33. $6m+abc+b^2d^3-2d^2m^3$.
34. $emn+9d+8e^3n^2-a^4b^3$.
35. $5b^4m^2-9n+bcm+c^4n^2$.
36. $d^3m^2+5c-4e^2n^3+acd$.
37. $cdm+3c^3n^2+a^4b^5-7e$.
38. $8m+5b^3d^2-a^5c^3-men$.
39. $a^3c^3+7m-4d^2m^3-ben$.
40. $bcd+9c-4a^2m^2+b^3c^3$.
41. $9n+cdm+d^4e^4-2a^3b^2$.
42. $c^3d^2-acn+8m+2d^4m^2$.
43. $3c^3d^2+emn+3n+a^4d^3$.
44. $7d^3m^2+3n-4e^2m^3+ac$.

45. $4c^3d^2 - bm^2n \times a^2e^2 - 4a^3bcd + c^3d^2m - 8c^3 + 2m^4$.

46. $a^4bd \times c^2n^2 + 8b^3c^3 - b^2c^3mn - 3d^2 + 4n^2 - 5a^2ecd$.

47. $ab^3d + 2c^3n^2 \times a^2b^3 - 8d^2 + 4b^3 + a^4b^3c^3d + 9ce^2mn^3$.

48. $cd^2m - 8a^3b^3 + d^2m^2 + 9c^3 \times 2b^2 + 6a^3dm^2n - b^3c^3d^2n$.

49. $9b^2n + (a^3d^2m^2 + cm)b^2c - emn - (2b^2d^2 - 7c^4n^3 - 8cm)n$.

50. $m^3 \times d^3 + 7bcm + 9a^3c^3 - a^2c^2e^2 + c^2d^2 + b^4m + a^5 \times d^3$.

51. $7bde + 8bc^3m^2 + 6a^3b^2 \times d^2m + b^3c^3 + m^2 - 5c^3dn^2 + 8c^2m^3$.

52. $5c^3d - 9a^4b^3 + 4d^2m^2 + 9c^2n^2 \times 7d^2m^2 - abc \times dem + 4a^4bc^2$.

53. $6abd - (36mn - 4a^3) + 4b(3c^2n^2 + abd) + (4am + c^2m)ad^2$.

54. $7c^3d^2 - n(4abm + b^3) - c^3n^2 \times 7ad - (250 + d^2 \times 8)n - 4a^2n^2$.

55. $5bcd - (2a^2d^2 - b^3m)n + 9d^2m + 4a^2c^2 + m^2n^2 - 5be^2 \times 7c^3d^3$.

56. $3a^2d^2m^2 + e(4ac + bd) + 5b^3dm^2 - a^4d^3m^2 + (m^2 + n^2)c^3d^3$.

Find the numerical value of the following expressions when $a = \frac{1}{2}$, $b = \frac{1}{3}$, $c = \frac{1}{5}$, $d = 2$, $m = 10$, $n = 5$, and $x = 1$:

1. $\dfrac{a+x}{a-c}$.
2. $\dfrac{b+c}{x-c}$.
3. $\dfrac{n-1}{b+x}$.
4. $\dfrac{1}{a} + \dfrac{1}{b}$.
5. $\dfrac{a}{c} - \dfrac{b}{2}$.
6. $\dfrac{n+x}{a-b}$.
7. $\dfrac{x^2+6}{a^2+b}$.
8. $\dfrac{n^2-9}{a^2-c}$.
9. $\dfrac{n}{a} + \dfrac{1}{b^2}$.
10. $\dfrac{x^2+n^2}{a^2+b^2}$.
11. $\dfrac{m^2+n^2}{d^3+b}$.
12. $\dfrac{d^2+x^2}{a^2+c^2}$.
13. $\dfrac{2a+3b}{d^2+x^2}$.
14. $\dfrac{5m+5c}{a^2+b^2}$.
15. $\dfrac{5n^2}{b^2} - \dfrac{6d^2}{ab^2}$.
16. $\dfrac{8a^2}{c^2} - \dfrac{6n^2}{d^2}$.
17. $\dfrac{c^2m^2}{4b^2} + \dfrac{3x^2}{a^2}$.
18. $\dfrac{10c+10d}{3b^2+5c^2}$.
19. $\dfrac{2m^2}{a^2d^2} - \dfrac{4nx}{3b^3}$.
20. $\dfrac{cdm}{ab} + \dfrac{4mx}{bc}$.
21. $\dfrac{5cn}{b^2c^2} - \dfrac{3dm}{2a^3}$.

Find the numerical value of the following expressions when $a = 10$, $b = 8$, $c = 5$, $d = 4$, $e = 3$, and $x = 1$:

1. $a(b + c)$.
2. $(e - x)\,ac$.
3. $abx(a + c)$.
4. $(a - e)\,d^2$.
5. $dx^2(c + e)$.
6. $(a - c)\,5\,ex$.
7. $(b - c)\,ax^2$.
8. $(e + x)\,cd^2$.
9. $(a + 2\,d)\,cx^2$.
10. $a^2e^2(a - b)$.
11. $(a - x)\,d^2e^2$.
12. $2\,c^2x^2(c + x)$.
13. $(a + c)(b - d)^2$.
14. $e^2x^2(a - b)^2$.
15. $2\,dex(c - d)^2$.
16. $4x(b - c)(e + d)$.
17. $(x^2 + c^2)\,2\,ce^2$.
18. $(a^2 - b^2)3ce^2x^2$.

Find the numerical value of the following expressions when $a = 8$, $b = 6$, $c = 5$, $d = 3$, $e = 0$, and $m = \frac{1}{2}$:

1. $(a + b)\,c - d$.
2. $(a + b)(c - d)$.
3. $(b + c)\,2\,d^2 + 4\,a^2m$.
4. $(c^2 + d^2)(b - d)$.
5. $(a^2 + c^2)(m^2 - de^2)$.
6. $3\,e(a + m)(b - d)$.
7. $(a - b + c)(d - m)$.
8. $(b + m)\,d(b - c)\,4\,a^2$.
9. $(b^2 + c^2 + d^2) - (a - m)$.
10. $(c^2 + d^2)(b^2 - c^2)\,2\,am$.
11. $(ac + b + d) - m^2(c^2 - d^2)$.
12. $3\,bdm(a - c + d) - 5\,a^2e^2$.

Find the numerical value of the following expressions when $a = 8$, $b = 7$, $c = 6$, $d = 5$, $e = 4$, $m = 3$, $n = 2$, and $x = 1$:

1. $(a^2 + n^2)(c - m)^2$.
2. $(b + x)\,m(c^2 - m^2)$.
3. $(em + x^2)\,n(b - d)^2$.
4. $(d^2 - n^2)(b - 5\,x^2)^2$.
5. $(m^2 - n^2)(c - e^2)^2$.
6. $(b^2 - x^2)\,8\,(9\,d - e^2)$.
7. $a^n + b^{m-1} + d^{m-n}e^2$.
8. $d^m - a^{b-d} + c^{n+1}ex^2$.
9. $x^m + d^{n+1} + e^{m+1}x^{2n}$.
10. $a^2 - 2(m - x)\,3(b - d)$.
11. $9\,a - (a - c)\,5(e^2 - m^2)$.
12. $a^{n+1} - m(a - b)\,c(d - 2x)$.

TO THE TEACHER. — Teach 56 . . . . 62.

ADDITION.

1.	2.	3.	4.	5.
$5ax$	$-7xy$	$6ac$	$3ab$	$5bc$
$4ax$	$-2xy$	$-2ac$	$-9ab$	$-bc$

6.	7.	8.	9.	10.
$4ab$	$-7cd$	$-2ax$	$5ac$	$9an$
ab	$-cd$	$-8ax$	ac	an
$3ab$	$-5cd$	$-ax$	$2ac$	$7an$
ab	$-2cd$	$-5ax$	$9ac$	an

11.	12.	13.	14.	15.
$4cx$	$-am$	$6bc$	cd	$5xy$
$3cx$	$4am$	$-2bc$	$-9cd$	$9xy$
cx	$-9am$	$5bc$	$5cd$	$-xy$
$5cx$	$-3am$	bc	$-8cd$	$8xy$

16.	17.	18.	19.	20.
$7ad$	$-9xy$	$8ab$	ac	$9bc$
ad	$4xy$	$-4ab$	$-7ac$	$-4bc$
$4ad$	$-xy$	$-ab$	$-ac$	bc
ad	$2xy$	$7ab$	$2ac$	$-2bc$
$5ad$	$-6xy$	$-4ab$	$3ac$	bc
$2ad$	xy	ab	$-9ac$	$-4bc$
$6ad$	$5xy$	$-2ab$	$4ac$	$-2bc$

21.	22.	23.	24.	25.
$2ab$	$3mn$	abc	$5ax$	abd
$3cd$	$-2ab$	xyz	$-3cy$	$-4xy$

26.	27.	28.	29.	30.
$5ax$	$7ac$	$15x$	acx	$4ab$
$2cd$	$-bd$	$-2ay$	$22y$	$-3cd$

31.	32.	33.	34.	35.
$7\,ax^2$	$5\,mn^2$	$-7\,cd$	$3\,cx$	$-7\,x^2y$
$-9\,ax^2$	$-\,mn^2$	$4\,cd$	$-2\,cx$	$9\,x^2y$
$-6\,ax^2$	$-8\,mn^2$	$-\,cd$	$9\,cx$	$5\,x^2y$
$8\,ax^2$	$4\,mn^2$	$-8\,cd$	$-8\,cx$	$-8\,x^2y$
$-7\,ax^2$	$6\,mn^2$	cd	$6\,cx$	$-4\,x^2y$
ax^2	$3\,mn^2$	$-6\,cd$	$-3\,cx$	$9\,x^2y$
$5\,ax^2$	$-\,mn^2$	$2\,cd$	$-4\,cx$	$-5\,x^2y$

36.	37.	38.	39.	40.
$7\,bc^2$	$8\,ax^2$	$-4\,ax$	xy	$-4\,a^2d$
$-3\,bc^2$	$-3\,ax^2$	$-\,ax$	$-6\,xy$	a^2d
$6\,bc^2$	$5\,ax^2$	$3\,ax$	$5\,xy$	$-2\,a^2d$
bc^2	$-2\,ax^2$	$-7\,ax$	$-\,xy$	$-\,a^2d$
$9\,bc^2$	$7\,ax^2$	$5\,ax$	$4\,xy$	$9\,a^2d$
$-5\,bc^2$	$-\,ax^2$	$-9\,ax$	$-2\,xy$	$-7\,a^2d$
$-\,bc^2$	ax^2	ax	$-9\,xy$	$3\,a^2d$

41.	42.	43.	44.	45.
acx	$-5\,ab$	$3\,mx$	$-2\,ax$	$-6\,ac$
$-2\,bd$	$-3\,cd$	$-\,ny$	$-5\,by$	$-\,by$

46.	47.	48.	49.	50.
$2\,ab$	$-2\,bd$	$-4\,ax$	$7\,ab$	$-\,cd$
$-\,xy$	$3\,ac$	$-\,by$	$-\,24$	$3\,ab$

51.	52.	53.	54.	55.
$5\,ax$	$-3\,cd$	$5\,bx$	$-5\,ab$	$-7\,xy$
$-\,ax$	$-2\,ab$	$-\,xy$	$-2\,cd$	$-2\,bc$
$2\,by$	$7\,cd$	$-4\,bc$	$8\,ab$	$6\,xy$

56.	57.	58.	59.	60.
$5\,ac$	$4\,ab$	$2\,ax$	$4\,cd$	$-\,cy$
$-\,ax$	$-3\,ab$	$-\,ac$	$-8\,cd$	$5\,xy$
$4\,ac$	$-\,12$	$3\,ax$	ab	$-4\,xy$

To the Teacher. — Teach how to add or unite terms that are partly similar. See 63.

Add the following with reference to the similar parts:

1.	2.	3.	4.	5.
ax	ay	ax	mx	mx
bx	cy	x	$-nx$	$-x$

6.	7.	8.	9.	10.
$-y$	$2x$	bx	a	ac
ay	$2y$	$-x$	ax	$-bc$

11.	12.	13.	14.	15.
ax	ac	$4y$	ax	$2ad$
x	$4c$	cy	bx	$-3bd$
ax	$-9c$	$-y$	$-x$	$3ad$

16.	17.	18.	19.	20.
ax^2	$6ay$	$2axy$	$4ay^2$	$-3axy^2$
$-3bx^2$	$3by$	$7xy$	$-2by^2$	$7bxy^2$
$-2ax^2$	$-2ay$	$-axy$	$-ay^2$	$4axy^2$
$5ax^2$	$-9by$	$-8xy$	$-2ay^2$	$-8bxy^2$

21.	22.	23.	24.	25.
$a(c+d)$	$a(x-y)$	$a(x+y)$	$b(x-1)$	$3a(x+y)$
$b(c+d)$	$(x-y)$	$-c(x+y)$	$-(x-1)$	$(x+y)$

26.	27.	28.	29.	30.
$a(b+4)$	$a(c-1)$	$c(x+y)$	$a(b-2)$	$(x-1)$
$3(b+4)$	$-(c-1)$	$-3(x+y)$	$(b-2)$	$-c(x-1)$

31.	32.	33.	34.	35.
$2a(x-1)$	$-(b+c)$	$3a(x-3)$	$-b(c+2)$	$4a(x+y)$
$4(x-1)$	$5a(b+c)$	$a(x-3)$	$3b(c+2)$	$6(x+y)$
$-a(x-1)$	$5(b+c)$	$-2a(x-3)$	$-(c+2)$	$-2a(x+y)$
$-(x-1)$	$-2a(b+c)$	$(x-3)$	$2b(c+2)$	$-(x+y)$

To the Teacher. — Teach addition of polynomials.

Express the following in their simplest form:

1. $2x-4y-6x-5z+3y+3z-2y+5x-z+3y.$
2. $4b+3c-2d-5c-2b+8d+3b+3c-c-6d.$
3. $5x-3y-4z+5y-3z-3u-2x+9z-6y+4x.$
4. $3a-4c+5d+7c-4e-2a-2d-5f-3c+3e.$
5. $5b-3c+6d-4e-9d+5c+3d-2b+3f-6c.$
6. $2x+3y+2y-3z-3x+2z+2x+3u-4y-6z.$
7. $6a+4b-7c-5b+3d-2a+7c-3d+7b+5m.$
8. $7xy-3y+z+y+xy-z-6xy+z-3x-y+7z-y.$
9. $5ac+2d-e-d+ac-e-3ac-2f+e+d+5e-d.$
10. $5bx-3cx-4dx+xy+cx+5dx+2cx-2xy-4bx.$
11. $7a-5b+4c-2c-a-d+5c+5b-2a+6d-3a.$
12. $5ab+4ab^2-5c-9ab^2+3c-4ab+4c+3ab^2-6c.$
13. $4ac-2ab-3ay+4ab-ax-4ay-4ac+7ay-ab.$
14. $3a+5b-4c+6c-a+3d+2a-c-6b-2d-3a.$
15. $7x^2y^2-3x^2y^3-7z^2+5x^2y^3-9x^2y^3-2u^2+2x^2y^3+9z^2.$
16. $7ac-8ax+5ay+3ax-3xy-4ac-4ay+5ax-ac.$
17. $9a^2b^3+4a^3b^2-3b^2-3a^3b^2-2c^3-7a^2b^3+3b^3-2a^3b^3.$
18. $5ab-4ac+7ax-4xy-3ab-4ax+4ac-2ab+xy.$
19. $7ab-5ac+3ad+2ac-2ae-5ab+2ac-4ad-ab.$
20. $5ac-7ax+4ax-3ay-3ac-2ay+7ax+6ay+ac.$
21. $5ax+2ay-7az-3xy+4ay+3az-7ay+5az+ax.$
22. $3ab+5ac-3ac-5ad+4ad+7ab-8ab-3ac-ad.$
23. $4ac-7ae+7ad-3mn-7ac-5ad+9ae+3ac+ad.$
24. $5xy-3xy^2+3z-8xy^2-5z-4xy+8z-6xy^2-7z+3xy.$
25. $4ab-5ab^2-5c-2ab+2c+7ab^2-3c-3ab^2+7c+5ab.$

1. Add $3x^3-5x^2+2x-6$, $x^3+4-6x+2x^2$, $3x^3-3x-x^3+8$, $4x+4x^3-4-2x^2$, and $3x^2+5x-2-7x^3$.

2. Add $4a^3-8a^2+3a-5$, $5a^2+9-7a-a^3$, $-a^2+7a^3-7+4a$, $6a^2-3a-8a^3+6$, and $a-4a^2-3+3a^3$.

3. Add $ab-\frac{1}{2}ac+\frac{1}{4}ad-\frac{1}{8}ae$, $ac+\frac{5}{6}ae-\frac{1}{2}ad+\frac{3}{4}ab$, $ad+2ac+\frac{1}{2}ae+\frac{1}{4}ab$, and $ae+ac+ad+ab$.

4. Add $2(a+b)+7c$, $(a+b)-2c$, $c-3(a+b)$, $(a+b)-5c$, $-c-(a+b)$, and $8c+2(a+b)$.

5. Add $\frac{1}{4}ax-\frac{1}{2}ay+\frac{2}{3}bx-2by$, $ay+by+\frac{1}{2}ax$, $ay-\frac{1}{2}by+ax-bx$, and $\frac{2}{3}by-\frac{3}{4}ay+\frac{3}{4}bx+\frac{1}{3}ax$.

6. Add x^3+3x^2-5x+7, $-x^2-4+2x-3x^3$, $-x+5x^3+3-5x^2$, $4x^2-1-4x-x^3$, and $8x+2-2x^3-x^2$.

7. Add $2(a+c)+3(b-c)-bc$, $4bc-(a+c)-(b-c)$, $7(a+c)-2bc+5(b-c)$, and $8(a+c)-bc-(b-c)$.

8. Add $\frac{1}{3}a^2x-\frac{1}{2}ax^2+\frac{1}{4}ax-\frac{2}{3}ay$, $a^2x+2ay-ax$, $\frac{1}{2}ax-\frac{1}{2}a^2x+ax^2$, and $2a^2x-\frac{1}{3}ax^2-\frac{5}{6}ay$.

9. Add $a(a+b)+a(a-b)-a(a+c)$, $-a(a-b)+(a+b)+a(a-c)+a(a+c)$, and $b(a-b)+2(a+c)$.

10. Add $a(a+b)+3(b+c)+(b-c)$, $(b+c)+3(a+b)+b(b-c)$, and $a(b+c)-(a+b)-5(b-c)$.

11. Add $\frac{7}{8}x^3y^3+x^2y^3-\frac{3}{8}xy+\frac{1}{2}xz$, $xy-x^2y^3-xz$, $\frac{3}{4}xz-\frac{3}{4}xy-\frac{3}{8}x^2y^3$, and $xy+\frac{5}{6}x^2y^3+\frac{3}{4}x^3y^2$.

12. Add $ab+ac+ae-2bc$, $-ac+bc-ab$, $\frac{5}{6}ac-\frac{1}{3}bc+ab-\frac{5}{4}ae$, and $ae-2ac+\frac{1}{2}bc-\frac{3}{8}ab$.

13. Add $(a+c)+2a(b+c)$, $b(b-c)+a(a+c)-(b+c)$, $(a+c)-(b-c)-a(b+c)$, and $4(b-c)+(a+c)$.

EXERCISES IN ALGEBRAIC EXPRESSION.

1. Write six times the product of x and y, increased by three times the square of x.

2. Write three times the sum of a square and b square, diminished by five times the product of a, b, and c.

3. If a represents an odd number, what will represent the next larger odd number?

4. If x represents an even number, what will represent the next smaller even number?

5. If a man can perform a piece of work in x days, what part of it can he do in one day?

6. Write an expression for the sum of three consecutive numbers of which x is the smallest.

7. How many square yards are there in a ceiling which is x feet long and y feet wide?

8. What is the interest on five hundred dollars for a years at x per cent per annum?

9. Write an expression for the sum of four consecutive even numbers of which x is the largest.

10. If a man can perform a piece of work in x days, what part of it can he do in four days?

11. When a represents an integer, does $2a - 1$ represent an even number or an odd number? Show why.

12. At six per cent per annum, what is the interest on three thousand dollars for x years?

13. Write an expression for the sum of five consecutive odd numbers of which x is the middle one.

14. In eight years a man will be x years old. How old was he eight years ago?

15. A man bought x sheep at a dollars a head and had b dollars left. How much money had he at first?

EQUATIONS AND PROBLEMS.

To the Teacher. — Teach how to transpose a term from one member of an equation to the other, also how to solve such equations as the following:

1. $4x - 4 = 2x + 8$.
2. $5x + 3 = 2x + 9$.
3. $9x - 8 = 4x + 7$.
4. $7x + 4 = 3x + 8$.
5. $6x - 3 = 5x + 4$.
6. $8x + 1 = 6x + 9$.
7. $7x - 6 = 4x + 9$.
8. $3 - 3x = 9 - 5x$.
9. $1 + 2x = 9 - 2x$.
10. $5 - 4x = 7 - 6x$.
11. $3x - 7 = 8 - 2x$.
12. $1 - 7x = 9 - 9x$.

1. My house and lot cost \$ 17,500, the house costing 4 times as much as the lot. Find the cost of the house.

2. Twice a number increased by 68 is equal to three times the number diminished by 57. Find the number.

3. The sum of two numbers is 112, and their difference is 36. What is the larger number?

4. A horse and carriage cost \$ 365. If the horse cost \$ 85 more than the carriage, what was the cost of the horse?

5. A, B, C, and D have 280 sheep. B has twenty more than A, C has twenty more than B, and D has twenty more than C. How many have A and B?

6. A man paid $\frac{1}{4}$ of a debt and still owes \$ 90 more than he paid. How much of the debt did he pay?

7. The sum of the ages of father, mother, and son is 105 years. The mother is twice as old as the son, and five years younger than the father. Find the father's age.

8. A has twice as many acres of land as B, and B has three times as many acres as C. If together they have 2600 acres, how many acres have B and C?

9. Separate 148 into two parts such that the greater shall exceed the less by 34.

10. The sum of two numbers is 259, and their difference is five times the smaller. Find the larger number.

11. A man lost $\frac{1}{8}$ of his money, but he still has $5800 more than he lost. How much has he left?

12. Three farmers sold 2080 bushels of potatoes. A sold $\frac{1}{4}$ as many bushels as B, and C sold $\frac{3}{5}$ as many bushels as A and B together. How many bushels did B sell?

13. A father and son earn $107 a month. If the son's wages were doubled, he would receive only $11 less than his father. How much does the son receive?

14. A man built a house costing five times as much as the lot. If the lot cost fourteen thousand dollars less than the house, how much did the lot cost?

15. Three men raised 1925 bushels of corn. A had three times as many bushels as B, and 175 bushels more than C. How many bushels did A and B raise?

16. A has one-sixth as much money as B, and C has $2\frac{1}{7}$ times as much as A and B. If C's money exceeds B's by $4500, how much has B?

17. The sum of the ages of father and son is 72 years, and the difference between their ages is twice the son's age. Find the age of the son.

18. A man has four thousand fifty dollars in four banks. He has twice as much in the first bank as in the second, twice as much in the third as in the first and second, and twice as much in the fourth as in the other three. How much money has he in the third bank?

TO THE TEACHER. — Teach 65 . . . . 71.

SUBTRACTION.

1.	2.	3.	4.	5.	6.
$7a$	$-9b$	$3c$	$-6x$	$9a$	$-7b$
$3a$	$-2b$	$5c$	$-7x$	$-4a$	$8b$
7.	**8.**	**9.**	**10.**	**11.**	**12.**
$4x$	$-3c$	$9a$	$-7b$	$-8c$	$5b$
$5x$	$-8c$	$8a$	$-6b$	$3c$	$-8b$
13.	**14.**	**15.**	**16.**	**17.**	**18.**
$9c$	$-7x$	a	$-5c$	$7b$	$-6x$
c	$-x$	$9a$	$-6c$	$-2b$	$8x$
19.	**20.**	**21.**	**22.**	**23.**	**24.**
a	$-c$	$8x$	$-4a$	$-7d$	$4e$
$2a$	$-7c$	$7x$	$-3a$	$3d$	$-9e$
25.	**26.**	**27.**	**28.**	**29.**	**30.**
$8x$	$-7c$	$4b$	$-4a$	$8x$	$-5d$
$3x$	$-2c$	$8b$	$-5a$	$-4x$	$7d$
31.	**32.**	**33.**	**34.**	**35.**	**36.**
$6c$	$-2a$	$7y$	$-9d$	$-7e$	$3a$
$7c$	$-6a$	$6y$	$-8d$	$5e$	$-8a$
37.	**38.**	**39.**	**40.**	**41.**	**42.**
$7b$	$-2x$	d	$-3c$	$9a$	$-4e$
b	$-x$	$7d$	$-4c$	$-5a$	$9e$
43.	**44.**	**45.**	**46.**	**47.**	**48.**
d	$-a$	$6x$	$-8b$	$-9e$	$5c$
$2d$	$-5a$	$5x$	$-7b$	$7e$	$-9c$

49.	50.	51.	52.	53.	54.
$7ac$	$-7x^2$	$6a$	$-3bc$	$9b^2$	$-4ab^2$
$2ac$	$-3x^2$	$5a$	$-4bc$	$-4b^2$	$7ab^2$

55.	56.	57.	58.	59.	60.
$3bc$	$-3c^2$	$3x$	$-7ab$	$-8x^2$	$4bc^2$
$8bc$	$-8c^2$	$4x$	$-6ab$	$3x^2$	$-7bc^2$

61.	62.	63.	64.	65.	66.
$9ab$	$-6a^2$	$2b$	$-ac$	$7a^2$	$-ac^2$
ab	$-a^2$	b	$-2ac$	$-a^2$	$8ac^2$

67.	68.	69.	70.	71.	72.
ac	$-a$	a	$-2ax^2$	$-7b$	xy^2
$9ac$	$-7a^2$	$2a$	$-ax^2$	b	$-8xy^2$

73.	74.	75.	76.	77.	78.
$8bc^2$	$-9b^2$	$9c$	$-5bc^2$	$8c$	$-7ax^2$
$3bc^2$	$-2b^2$	$2c$	$-6bc$	$-7c^2$	$9ax^2$

79.	80.	81.	82.	83.	84.
$4ab^2$	$-7x$	$7b$	$-8ac^2$	$-8a$	$8ab^2$
$9ab^2$	$-9x^2$	$8b$	$-7ac^2$	$6a$	$-9ab^2$

85.	86.	87.	88.	89.	90.
$6xy^2$	$-8c$	$2x$	$-ax^2$	$6x$	$-xy^2$
xy^2	$-c^2$	x	$-2ax^2$	$-x^2$	$7xy^2$

91.	92.	93.	94.	95.	96.
ax^2	y	a	$-2xy^2$	$-9c$	$7ac^2$
$7ax^2$	$-6y^2$	$4a$	$-xy^2$	c^2	$-8ac^2$

To the Teacher. — Teach 72.

Subtract with reference to the similar parts:

1.	2.	3.	4.	5.	6.
ax	$-ay$	ac	$-ax$	ac	y
bx	$-2y$	c	$-x$	$-bc$	$-cy$

7.	8.	9.	10.	11.	12.
bc	$-ax$	d	bx	$-c$	ay
$3c$	$-bx$	cd	$-x$	$-bc$	$-2y$

13.	14.	15.	16.	17.	18.
ac	$-ax$	by	ac	$-am$	c
bc	$-3x$	y	$-bc$	$-m$	$-ac$

19.	20.	21.	22.	23.	24.
ax	$-ac$	y	cx	$-n$	ad
$4x$	$-bc$	cy	$-x$	$-cn$	$-4d$

25.	26.	27.	28.	29.	30.
am	$-bc$	ad	bx	$-ay$	n
cm	$-5c$	d	$-dx$	$-y$	$-an$

31.	32.	33.	34.	35.	36.
ad	$-cm$	n	ac	$-x$	bc
$5d$	$-am$	mn	$-c$	$-cx$	$-5c$

37.	38.	39.	40.	41.	42.
cy	$-cx$	ay	mx	$-ad$	b
dy	$-4x$	y	$-nx$	$-d$	$-ab$

43.	44.	45.	46.	47.	48.
be	$-dy$	c	am	$-x$	ay
$6e$	$-cy$	ac	$-m$	$-bx$	$-7y$

1. From $a-b+c+4$ subtract $b+c-a$.
2. Subtract $u+y+x-z$ from $x+2y-z$.
3. From $a+c+2x-y$ subtract $2x-5+a$.
4. Subtract $2c+y-2x$ from $3c+d+x+6$.
5. From $5b+2c+d$ subtract $c+8-4d-6b$.
6. Subtract $5z-4+6x$ from $7x-8y+5z-5$.
7. From $7c-8d-5e$ subtract $3d+f-4c-4e$.
8. Subtract $d-4+3ab-6c$ from $8ab-5c+4d$.
9. From $4ax+6ay-z+8$ subtract $5ay-z+9ax$.
10. Subtract $8de+7+5bc$ from $6bc-4b+8de+f$.
11. From $4cx+7by-2xy-9$ subtract $7by+8+3cx$.
12. Subtract $3ab+6-7ac-3ax$ from $4ab-6ac-am$.
13. From $5ax-7ay-3xy-23z$ subtract $4xy-7-7ay$.
14. Subtract $5e-8be-2bc$ from $4bc+2bd-7be-8de$.
15. From $6am-5an+4ar-7rs$ subtract $125+6am-9an$.
16. Subtract $4ax-2xy+7ab$ from $4ax-5ac-3xy+8ab$.
17. From $4ay-7xy-3by+5xz$ subtract $6xy+6xz-3by$.
18. Subtract $3a^2x-8-2a^2y$ from $3a^2x+3ax-2a^2y-4az$.
19. From $8x^2y-8xy^2+3xz$ subtract $7x^2y-2xz+7-9xy^2$.
20. Subtract $8x^2y-6xy-36x^3$ from $25x^3+9x^2y+7xy-8$.
21. From $7ab-5ab^2+24b^3$ subtract $7ab-6ab^2-7-37b^3$.
22. Subtract $7bc^2-abd+9b^2c$ from $4b^2c+8bc^2-abd-9$.
23. From $8a^2x-8a^2y+187$ subtract $z+7a^2y+243+7a^2x$.
24. Subtract $2xyz-93y^3-8+29x^3$ from $48x^3-7y^3+xyz$.
25. From $16a^2c+ac^3+d$ subtract $30ac^3+7-25d+73a^2c$.
26. Subtract $9xy^3+z-434-22x^2y$ from x^2y+8xy^3-256.

In such examples as the following, write all the polynomials, similar terms in a column, writing those to be subtracted with their signs changed, and then add.

1. From the sum of $2a+b-c+d$ and $3d+4a-3b$ subtract $2b+4+4d+7a$.

2. Subtract $3z-2u+2x$ from the sum of $3x-4y+3z-u$ and $8+5y-x-z$.

3. From $b-c+d-4e$ subtract the sum of $2d-7e+7b-3c$ and $3e-6b-d$.

4. Subtract the sum of $2b+3y-7x-5a$ and $5x-b-4y-a$ from $a+b-3x$.

5. From the sum of $4x+y-2z$ and $4u+3y-7x-2z$ subtract $4u+4y-5z+5-3x$.

6. Subtract $3a-2d-b-4c$ from the sum of $4a-2b+3c+d$ and $b-3d+5a-2c$.

7. From $3x-4y+3z-u$ subtract the sum of $z+x-8-5y$ and $3z+2x-2u$.

8. Subtract the sum of $5m-a-9n$ and $5b+5n+a-4m$ from $a+4b+6m-4n$.

9. From the sum of $4c+d-3e$ and $7e-f-9d-5c$ subtract $e-2d-2f-c$.

10. Subtract the sum of $\frac{1}{4}x-\frac{1}{2}y-c$ and $z-\frac{1}{2}c$ from the sum of $\frac{1}{2}a-\frac{1}{8}c-\frac{1}{4}x$ and $\frac{3}{4}y-\frac{1}{2}x+z-\frac{5}{6}a$.

11. From the sum of $m+\frac{1}{4}n-\frac{2}{3}r$ and $r-4s-n-2m$ subtract the sum of $8-3s-\frac{1}{2}n$ and $\frac{3}{4}r-\frac{1}{4}n$.

12. Subtract the sum of $3a+e+c$ and $\frac{5}{6}d+\frac{3}{4}b$ from the sum of $4a-\frac{7}{8}b-c$ and $2c+\frac{2}{3}d+2b+e$.

13. From the sum of $2ab - ac + 2bc$ and $2ac - bc - 3ab$ subtract the sum of $3ac - 4bc - ab$ and $2abc - 2ab - 2ac$.

14. Subtract the sum of $a^2c - ac^2 + a^2b$, $a^2b - ac^2 - a^2c$, and $ab^2 + a^2c + ac^2$ from $a^2b + a^2c - ac^2$.

15. From the sum of $3x^2 - 2x + 5$ and $4x + 2y - 4$ subtract the sum of $3x - 2x^2 + 3$ and $y - 2x - 2$.

16. From the sum of $3xy - 2xz + yz$ and $xz + 5z^2 - xy$ subtract the sum of $4xz - 2yz$ and $4z^2 + xy + yz - 5xz$.

17. Subtract the sum of $xy^2 - xz^2 - x^2z$ and $xz^2 - x^2y - x^2z$ from the sum of $xy^2 - x^2y - x^2z$ and $x^2y - xz^2 - x^2z$.

18. From $4ab - 3ac + 2bc$ subtract the sum of $3bc + bd - ac$, $3ab - 3bd - bc$, and $bd - 2ac - ab$.

19. From the sum of $2x^2y - 3x^2z + xz^2$, $2x^2z - 2xz^2 - x^2y$, and $2xz^2 - xy^2 - x^2z$ subtract $xz^2 - xy - 2x^2z$.

20. Subtract the sum of $2x - x^2 - y$, $x^2 + 5 - 2x^2$, and $x - x^2 - 4 - y$ from $2x^2 - 4x^2 + 3x$.

21. From the sum of $-a^2b + a^2c - ab^2$ and $ac^2 - 2a^2c + 3a^2b$ subtract the sum of $-a^2c - 2ab^2$ and $2a^2b - 2ac^2 - ab^2$.

22. Subtract $x^2z - xy - 2xy^2$ from the sum of $x^2y - 2xy + xy^2$, $x^2z - 3x^2y - 5xy^2$, and $3xy + x^2z + 2x^2y$.

23. Subtract the sum of $2b^2 - d^2 + 2c^2$, $a^2 + 6 - c^2$, and $2d^2 - b^2 - 5$ from $2a^2 + 3b^2 + c^2 - d^2$.

24. Subtract the sum of $a^2b - 4a^2c + 5ab^2$ and $2a^2b - 2ac^2 - 2a^2c$ from the sum of $2ab^2 + 3a^2b - 5a^2c$ and $4ab^2 - 3ac^2 - a^2b$.

25. Subtract $2xz^2 - 2xy^2 - x^2y$ from the sum of $xy^2 - xz^2 + x^2y - x^2z$, $-3x^2y - 2xy^2$, and $4x^2z + 3xz^2 + x^2y$.

To the Teacher. — Teach 73.

Remove the signs of grouping and simplify:

1. $a-b-(b-2a)$.
2. $x-(y-x+2y)$.
3. $a+(b-\overline{2a-b})$.
4. $5x+y-(x+y)$.
5. $c+d-(-c+d)$.
6. $a+(-a-\overline{a+b})$.
7. $x-(-x-\overline{x-y})$.
8. $a-b-(-a+b)$.
9. $3x-y-x-(-y)$.
10. $c+(-c-\overline{4c-d})$.
11. $a+4b-(-a+b)$.
12. $a-3b-(-a+b)$.
13. $7c-(-2c-\overline{c+d})$.
14. $7b-(-c-\overline{4b+c})$.
15. $5x-y-(-2x+y)$.
16. $5x+y-(-4x-y)$.
17. $8a-(-4a+\overline{b+c})$.
18. $9a-(-\overline{b-8c+d})$.
19. $5c-(-b+4c)-2c$.
20. $3m-n-(-n+m)$.
21. $8x-\{y-\overline{3z-(y-5z+x-6y)-4z}\}$.
22. $4a-[b-(3c-\overline{a+2c-4b})+a+3b]$.
23. $(3c-\overline{5d+4-7e})-(2c+\overline{4d+5}+3e)$.
24. $5a-[3b+\overline{2a-(5-5b)}-\overline{4a-4}-9b]$.
25. $5b-\overline{5c-[5+3b-(7-c+7b)-4]-4c}$.
26. $2x-\{y-\overline{3x+(y-6z+x)-z}+3y-\overline{4z}\}$.
27. $5a-\{7b+\overline{4c-(3a+8b+c)+7a}\}-\overline{b-c}$.
28. $7x-\overline{3y+z-[4x+(2z+4y-2)-9x]+y}$.
29. $9c-\{-\overline{4c-(7-2a+e)-3c+(4a-2e)}+6\}$.
30. $x-\{-\overline{2y-(4z+8x)-y-4x+(z-3y)}-3x\}$.
31. $9a-[-\{b-\overline{c-(6b-c+3a-9c)-7b+5a}\}]$.
32. $5x-[7x-\{2x-(3x-\overline{4x-8})\}-(5x-\overline{4x-7})]$.
33. $9c-[-\overline{8d-(5-2e+6d)+4c}]-\overline{7-(3e+5c)}$.
34. $3a-[4a-\{a-(6a-\overline{2a+5})\}-(7a-\overline{3a+13})]$.
35. $4c-[3c-(5c-x)-\{2c-(4c+3x)-(5c-\overline{x+4})\}]$.

EXERCISES IN ALGEBRAIC EXPRESSION.

1. Write seven times the third power of x, diminished by three times the sum of $2a$ and $5b$.

2. If x represents an odd number, what will represent the next smaller odd number?

3. If A can do a piece of work in x days and B can do it in y days, what part of it can both do in one day?

4. Write four times the product of a square and x cube, increased by a times the quantity $2x - 3y$.

5. What is the interest on x dollars for a years at five per cent per annum?

6. In how many days can m men do as much work as n men can do in a days?

7. Write an expression for the sum of four consecutive numbers of which x is the largest.

8. If a man can perform a piece of work in five days, what part of it can he do in x days?

9. How many square yards are there in a floor which is x yards long and y feet wide?

10. If $2x - 1$ represents an odd number, what will represent the next larger odd number?

11. How many days will it take a man to build m yards of wall, if he builds n feet a day?

12. How long will it take a person to walk x miles, if he walks at the rate of 15 miles in a hours?

13. Write an expression for the sum of three consecutive odd numbers of which x is the smallest.

14. How many square feet are there in a piece of paper a yards long and x inches wide?

15. A has y sheep, B has 8 less than A, and C has 15 more than A and B together. How many have all?

PROBLEMS.

1. The greater of two numbers is seven times the less, and their sum is 184. What is the larger number?

2. A is three times as old as B and 8 years older than C. The sum of their ages is 90 years. Find C's age.

3. In a company of 98 persons it was found that there were twice as many women as men, and twice as many children as women. How many children were there?

4. The sum of two numbers is 240. If the smaller number were doubled, it would still be six less than the larger number. Find the larger number.

5. The sum of two numbers is 224, and their difference is six times the smaller number. Find the larger number.

6. After a farmer had sold one-sixth of his wheat, he still had four hundred thirty-two bushels more than he sold. How many bushels had he left?

7. The sum of two numbers is eighty-four, and three times the smaller number exceeds twice the larger number by twelve. Find the larger number.

8. A, B, and C together earn two thousand eight hundred dollars. A's salary is $200 more than B's, and $300 less than C's. Find C's salary.

9. A man rode 118 miles in three days, riding 6 miles more the second day than the first, and 4 miles more the third day than the second. How far did he ride the third day?

10. A boy bought oranges at four cents apiece and had 11 cents left; but if he had bought them at six cents apiece, he would have needed 21 cents more to pay for them. How many oranges did he buy?

11. A and B together earn $175 a month; A and C, $225; B and C, $315. How much do all earn?

12. The sum of three numbers is 142. The second exceeds the first by 9, and the third is 17 less than the second. Find the sum of the second and third.

13. Three men raised 1530 bushels of oats. A had three times as many bushels as C, and 248 bushels more than B. How many bushels did B raise?

14. A has three times as much money as B; but if A gives B six dollars, they will then have equal amounts. How much money have both?

15. Each of two boys has the same number of marbles. If John buys 35 more and Frank loses 15, John will then have twice as many as Frank. How many have both?

16. A boy has two dollars forty cents in dimes and 5-cent pieces. If he has six times as many 5-cent pieces as dimes, how many coins has he?

17. A has one-third as many sheep as B. If A should double his flock and B should sell eighty, they would then have the same number. How many has B?

18. A has one-fourth as much money as B, and C has $2\frac{1}{2}$ times as much as A and B. If C's money exceeds B's by three thousand six hundred dollars, how much have all?

19. Frank has one-fourth as many marbles as John. If John loses 206 and Frank loses 14, they will each have the same number left. How many marbles have both?

20. A man sold forty acres of land in two equal parts for three hundred sixty dollars, receiving twice as much per acre for the second as for the first. How much did he receive per acre for the second piece?

To the Teacher. — Teach 74 . . . . 81.

MULTIPLICATION.

1.	2.	3.	4.	5.	6.	7.
$4a$	$-5x$	$-7c$	$-8b$	$6a$	$-7y$	$-5x$
$3a$	$-4x$	$5c$	$-6b$	$4a$	$-4y$	$4x$

8.	9.	10.	11.	12.	13.	14.
$5c^2$	$-6b$	$-7a^3$	$4x$	$3m^2$	$-4a^4$	$-8y$
$3c$	$9b^2$	$-5a^2$	$-3x$	$2m^2$	$-7a$	$2y^3$

15.	16.	17.	18.	19.	20.	21.
ab	$-ax$	$-ac$	xy	a^2b	$-ab$	$-a^2b$
ab	$-ay$	ac^2	$-xy^3$	ab^2	cd	$-ab^4$

22.	23.	24.	25.	26.	27.	28.
c^3d	$-am^3$	$-y^4$	x^2y	ac^2	$-6b^2$	$-ab$
cd^2	m^2	$-7y^3$	$-x^3y$	ab^2	$-5a^2$	xy

29.	30.	31.	32.	33.	34.	35.
$7a^3$	$-a^2x$	$-bc^3$	xy^3	a^2x	$-4c^3$	$-ab$
$5b^2$	$-a^3x$	ac	$-xz^2$	ax^2	$3d^2$	$-xy$

36.	37.	38.	39.	40.	41.	42.
$4a^m$	$-5c$	$-4x^m$	$6b^m$	ac^3	$-a^mx$	$-b^3c$
$3a$	$2c^m$	$-5x^n$	$-7b^2$	ac^n	$-ax^n$	bc^n

43.	44.	45.	46.	47.	48.	49.
$4c^2$	$-ab^m$	ab^n	$-x^ny$	$9a^m$	$-bc^n$	$-xy$
$6c^m$	$-ab^2$	$-b^3c$	xy^m	$5a^n$	b^mc	$-5x^r$

50.	51.	52.	53.	54.	55.	56.
a^{m+1}	$-c^{m-n}$	$-x^{m+3}$	b^{m+1}	a^{n+1}	$-x^n$	$-b^2$
a^{m-1}	c^n	$-x^{m-2}$	$-b^2$	a^n	$-x^{n+2}$	$-b^{n+1}$

To THE TEACHER. — Teach how to multiply a polynomial by a monomial. See 82.

Multiply:

1. $3x-y$ by $-2y$.
2. $ab+bc$ by ac.
3. $2x+y^2$ by $-4x$.
4. $4a-5x$ by $3a$.
5. $4a-3c$ by $-ac$.
6. $3x+4y^2$ by $5x^2$.
7. $7b+5y$ by $-4b$.
8. $2ac-3c^2$ by a^2c^2.
9. $5x-4y$ by $-5z$.
10. $3xy+2y^2$ by x^2y^2.
11. $3b^2+5x^2$ by $-bx$.
12. $7ac-5bc$ by $3bc^2$.
13. $6cd-c^3d^3$ by $-cd$.
14. $6am+2bn$ by $4mn^2$.
15. $4ab^2+3a^2b$ by $-2ab$.
16. $5c^3d-4cd^2$ by $3c^2d^2$.
17. $3a^2x-5ax^2$ by $-3a^2x$.
18. $4ax^2+5a^2x$ by $6a^2x^2$.
19. $5x^3y^2+2x^2y$ by $-5ax^2$.
20. $7x^2y^3-2x^2y$ by $3x^2y^4$.
21. $7a^3b^2-4c^3d^3$ by $-abcd$.
22. $5a^2x^2+3b^2y^2$ by $abxy$.
23. $4a-c+3x-2y+xy$ by cx.
24. $3x+2y-5b+6c-cx$ by by.
25. $4b+2c-a+3y-xy$ by $3b^2y$.
26. $ax-3a+2b-5x+4y$ by a^2y.
27. $5a-ab+3b-4c+7b^2$ by $2a^2b$.
28. $b^3c^3+2b-2c^3+b^3c^3-4c$ by $3b^3c^3$.
29. $2c+2c^3-a^2c+3a-a^2c^2$ by $4ac^3$.
30. $x^2y-3x+5y^3-x^3y^3+3y$ by $2xy^2$.
31. $a^3b+ab^2-5a+3b-4b^2$ by $3a^2b^2$.
32. $3x-ax^2+4a-a^2x+2x^2$ by $5a^3x^2$.
33. $3a^2-a^3b^3+5b-b^2c^3+7a$ by $a^2b^2c^2$.
34. $x^2y^4+2z^2-4x+x^3z^3-5y$ by $x^4y^2z^3$.
35. $2x^3y-3x^2y^2+4xy^3-3y+4z$ by $4xy^2z$.
36. $3a^4-2a^3b+3a^2b^2-2ab^3+2b^4$ by $4a^2b^3c^2$.

To THE TEACHER. — Teach how to multiply a polynomial by a polynomial. See 83.

Multiply:

1. $2x+y$ by $x+4y$.
2. $a-3x$ by $5a-x$.
3. $3b+2y$ by $b-y$.
4. $4a-c$ by $a+2c$.
5. $4a-3b$ by $3a-b$.
6. $3b+x$ by $2b-5x$.
7. $ac+xy$ by $ac+xy$.
8. $ab+cd$ by $ab-cd$.
9. $ax+cy$ by $ax-cy$.
10. $bx-dy$ by $bx+dy$.
11. $2x-5y$ by $3x+2y$.
12. $7a+5c$ by $3a-4c$.
13. $5a+4c$ by $3a-2c$.
14. $3c-5y$ by $6c+2y$.
15. $7x-5y$ by $2x-4y$.
16. $9a+2b$ by $2a+9b$.
17. $8b+3c$ by $5b-4c$.
18. $8x-3y$ by $4x-7y$.
19. $4x^2-2y^2$ by $5x+3y$.
20. $5a^2-4b^2$ by $3a-5b$.
21. $7x^3+5y^3$ by $2x-4y$.
22. $4c^3-6x^3$ by $5c-7x$.
23. $3a^3+5a-6$ by $4a^2-5$.
24. $5x^4-3x^3-2x^2$ by x^2-x.
25. $4x^3-2x^2+5x$ by $5x-3$.
26. $3c^3-2c+4$ by c^2-c+1.
27. $4a^2+5a-3$ by $3a^2-a+5$.
28. $3x^3+2x^2-4x+1$ by $5x+4$.
29. $x^3y-x^2y^2-3xy^3$ by $3x^2y-xy^2$.
30. $3ab+2b^2+a^2$ by $2a^2-3ab+b^2$.
31. $5ac-2a^2+3c^2$ by $3a^2+c^2-2ac$.
32. $3x^3-2x+3x^2-5$ by $3+2x^3-4x$.
33. $3xy^2+x^3-3x^2y-y^3$ by $y^2+2xy+x^2$.
34. $3m^2-2m+4-2m^3$ by $2m^2-4-3m$.
35. $2x^4+3x-2x^3-x^2-3$ by $3x+x^2+3$.
36. $3a^3+2a-4a^2+1$ by $2a^3-a-3a^2-4$.
37. $9x^3+2xy^2+3x^2y+4y^3$ by $4x^2+y^2-3xy$.

38. $4x^2-3x-2$ by x^2+x-3.

39. $3a^2-5a+3$ by a^2-4a+2.

40. $a^2+ab+2b^2$ by a^2-ab-b^2.

41. $x^2-3xy+5$ by $x^2+4xy-3$.

42. $b^2-4bc+c^2$ by $b^2-2bc-c^2$.

43. $a^4+b^4-a^2b^2$ by $a^4+b^4+a^2b^2$.

44. $3x^5+2x^3-4x^3-6$ by x^3-5.

45. $12x^3+4-3x$ by $6+5x^3-4x$.

46. $5x^2-12xy+22x^2y^2$ by $4y+3x$.

47. $2a+3+4a^2$ by $3a+4a^3-2a^2$.

48. $3x+1+x^3$ by $4x^3-2x+1-x^2$.

49. $2a^2-3b^2+4c^2$ by $6a^2+2b^2-4c^2$.

50. $a^3-3a^2c+5ac^2$ by $3a^2c-ac^2-6a^3$.

51. $a^4-a^2+3a-2a^3+1$ by a^2+1-a.

52. $3x^2-2xy+5y^2$ by $4x^2+2xy-3y^2$.

53. $3x^2+6y^2-4xy$ by $3x^2+6y^2+4xy$.

54. $2x^2-4x+2x^2+8$ by $3x^2-6-2x$.

55. $x^2+y^2+1-xy-x-y$ by $x+1+y$.

56. $5x^2+4-6x$ by $2x^2-4x+3-12x^2$.

57. $a^2+b^2+a-ab+b+1$ by $a-1+b$.

58. $a^2+b^2+c^2-ab-ca-cb$ by $a+b+c$.

59. $x^2+2xy+y^2+z^2$ by $x^2-2xy+y^2-z^2$.

60. $3x^2-x^4+4-x^3+2x$ by $3x^2+5-4x$.

61. $a^2-ac+c^2-bc+b^2+2ab$ by $a+b+c$.

62. $2m^3+3mn^2-4m^2n-n^3$ by m^2+n^2-mn.

Simplify the following:

1. $(a+x)(a-x)(a-x)(a+x)$.
2. $(1-x)(1+x)(1+x)(1+x)$.
3. $(1+a^4)(1+a^2)(1+a)(1-a)$.
4. $(x^2-2xy+y^2)(x^2+2xy+y^2)$.
5. $(a^4+x^4)(a^2+x^2)(a+x)(a-x)$.
6. $(a^2-c^2)(a^2-c^2)(a^2-c^2)(a^2-c^2)$.
7. $(a+c+x)(a+c-x)-(2ac-x^2)$.
8. $8(a-2c)(a+2c)-3(a-5c)^2+6a^2$.
9. $5(2a-x)^2-2(2a+2x)(2a-2x)-5x^2$.
10. $(a-3c)(c-2a)-(a-4c)(2c-a)+8ac$.
11. $(3x-2y)3(3x+2y)-3(4x+3y)^2-7xy$.
12. $6(x+2y)(x-2y)-(x-4y)^2+8(x^2+5y^2)$.
13. $(x-4)(x-2)-2x(x+4)+5(x+3)(x+4)$.
14. $(x+y)(y-z)-(z-u)(u+x)-(x+z)(y-u)$.
15. $(a-x)(x-y)+(x-y)(y-a)+(y-a)(a-x)$.
16. $(x^m+y^n)(x+y)$.
17. $(a-b)(a^m-b^m)$.
18. $(a^m-c^n)(a^m+c^n)$.
19. $(x^m+y^n)(x^n+y^m)$.
20. $(a^m+x^m)(a^n+x^n)$.
21. $(a^m-c^m)(a^m-c^m)$.
22. $(x^m-y^m)(x^{-1}+y^{-1})$.
23. $(m^{p-1}-n^{p-1})(m^2-n^2)$.
24. $(a^{2n}+c^{2n})(a^{-n}+c^{-n})$.
25. $(x^{m+1}-y^{m+1})(x^{-m}+y^{-m})$.
26. $(a^m-b^n)(a^{-2m}-b^{-2n})$.
27. $(x^{m+2}+y^{m+3})(x^{-2}-y^{-3})$.
28. $(4a^{2n}-2a^{2n}+a^n-1)(3a^n+1)$.
29. $(2a^{2m}-3a^mc^m+c^{2m})(2a^m+3c^m)$.
30. $(3x^m+2x^{m-1}-4x^{m-2}-1)(3x+1)$.
31. $(4a^n-3a^{n-1}+2a^{n-2}+3a^{n-3})(2a-1)$.

To the Teacher. — Teach 84 . . . . 90.

FORMULAS.

Write the following products by inspection:

1. $(a+c)(a+c)$.
2. $(x-y)(x-y)$.
3. $(b+c)(b-c)$.
4. $(a+b)(a+b)$.
5. $(a-x)(a-x)$.
6. $(a-x)(a+x)$.
7. $(e+c)(e+c)$.
8. $(x+y)(x-y)$.
9. $(c+d)(c+d)$.
10. $(a-d)(a-d)$.
11. $(b-x)(b+x)$.
12. $(x+y)(x+y)$.
13. $(a-b)(a-b)$.
14. $(c+d)(c-d)$.
15. $(r+s)(r+s)$.
16. $(b-c)(b-c)$.
17. $(a-b)(a+b)$.
18. $(a+x)(a+x)$.
19. $(b-x)(b-x)$.
20. $(e+c)(e-c)$.
21. $(b+c)(b+c)$.
22. $(a-c)(a-c)$.
23. $(a+1)(a-1)$.
24. $(b+8)(b+8)$.
25. $(b-4)(b-4)$.
26. $(x-2)(x+2)$.
27. $(a+7)(a+7)$.
28. $(3-x)(3-x)$.
29. $(c+3)(c-3)$.
30. $(x+6)(x+6)$.
31. $(2-a)(2-a)$.
32. $(b-4)(b+4)$.
33. $(c+5)(c+5)$.
34. $(1-c)(1-c)$.
35. $(a+5)(a-5)$.
36. $(x+4)(x+4)$.
37. $(a-7)(a-7)$.
38. $(x-6)(x+6)$.
39. $(a+3)(a+3)$.
40. $(c-6)(c-6)$.
41. $(c+7)(c-7)$.
42. $(c+1)(c+1)$.
43. $(5-a)(5-a)$.
44. $(b-8)(b+8)$.
45. $(2+b)(2+b)$.
46. $(1-x)(1+x)$.
47. $(x-9)(x-9)$.
48. $(x+7)(x-7)$.

49. $(2a-b)(2a-b)$.
50. $(a+3b)(a-3b)$.
51. $(4x+y)(4x+y)$.
52. $(a-4x)(a+4x)$.
53. $(a+2x)(a-2x)$.
54. $(3b+c)(3b+c)$.
55. $(x-4y)(x-4y)$.
56. $(5a-c)(5a+c)$.
57. $(a+4b)(a+4b)$.
58. $(3c-d)(3c-d)$.
59. $(2x+3)(2x-3)$.
60. $(1+3c)(1+3c)$.
61. $(4a-6)(4a-6)$.
62. $(3b-1)(3b+1)$.
63. $(2x+7)(2x+7)$.
64. $(1-9a)(1-9a)$.
65. $(5c+3)(5c-3)$.
66. $(4x+4)(4x+4)$.
67. $(2a-bc)(2a-bc)$.
68. $(ax-3y)(ax+3y)$.
69. $(ac+4c)(ac+4c)$.
70. $(3x-xy)(3x-xy)$.
71. $(8x+xy)(8x-xy)$.
72. $(bc+5d)(bc+5d)$.
73. $(3a-ab)(3a-ab)$.
74. $(ab-7c)(ab+7c)$.
75. $(6a+ac)(6a+ac)$.
76. $(xy-4x)(xy-4x)$.
77. $(8a-9b)(8a+9b)$.
78. $(2x+3y)(2x+3y)$.
79. $(4a-2c)(4a-2c)$.
80. $(7b+6c)(7b-6c)$.
81. $(5a+3x)(5a+3x)$.
82. $(4x-5y)(4x-5y)$.
83. $(9b-7c)(9b+7c)$.
84. $(2c+5d)(2c+5d)$.
85. $(2a-ac^3)(2a-ac^3)$.
86. $(x+7xy^2)(x-7xy^2)$.
87. $(5c+ac^2)(5c+ac^2)$.
88. $(b-3ab^3)(b-3ab^3)$.
89. $(9a-bc^2)(9a+bc^2)$.
90. $(x+6yz^2)(x+6yz^2)$.
91. $(7c-bc^3)(7c-bc^3)$.
92. $(8a+ax^2)(8a-ax^2)$.
93. $(a-4ab^2)(a-4ab^2)$.
94. $(8x-xy^2)(8x+xy^2)$.
95. $(3a^n+2b^n)(3a^n+2b^n)$.
96. $(5x^m-2y^n)(5x^m-2y^n)$.
97. $(6a^m-7c^n)(6a^m+7c^n)$.
98. $(4a^m+3a^n)(4a^m+3a^n)$.
99. $(7x^m-2x^n)(7x^m-2x^n)$.
100. $(2x^m+7y^n)(2x^m-7y^n)$.

To the Teacher. — Teach 91 and 92.

Write the following products by inspection:

1. $(x+3)(x+2)$.
2. $(x-5)(x-2)$.
3. $(x+4)(x+1)$.
4. $(x-6)(x-3)$.
5. $(x+5)(x+4)$.
6. $(x-7)(x-2)$.
7. $(x+6)(x+1)$.
8. $(x-6)(x-4)$.
9. $(x+4)(x+2)$.
10. $(a-7)(a-8)$.
11. $(x+7)(x+3)$.
12. $(a-8)(a-1)$.
13. $(x+6)(x+5)$.
14. $(a-7)(a-5)$.
15. $(x+8)(x+4)$.
16. $(a-9)(a-1)$.
17. $(x+9)(x+6)$.
18. $(a-8)(a-3)$.
19. $(x+7)(x-4)$.
20. $(a+2)(a-6)$.
21. $(x-1)(x+6)$.
22. $(a-5)(a+4)$.
23. $(x+6)(x-5)$.
24. $(x+2)(x-3)$.
25. $(a-4)(a+5)$.
26. $(x-7)(x+3)$.
27. $(x+5)(x-1)$.
28. $(a+7)(a-8)$.
29. $(a-6)(a+7)$.
30. $(x-8)(x+5)$.
31. $(x+4)(x-2)$.
32. $(a+5)(a-9)$.
33. $(a-3)(a+4)$.
34. $(x-8)(x+6)$.
35. $(x+6)(x-2)$.
36. $(a+4)(a-9)$.
37. $(x+9)(x+2)$.
38. $(x-9)(x+2)$.
39. $(x-7)(x-4)$.
40. $(a+7)(a-2)$.
41. $(x+9)(x+5)$.
42. $(x+6)(x-9)$.
43. $(x-8)(x-3)$.
44. $(a-1)(a+7)$.
45. $(x+7)(x+9)$.
46. $(a-9)(a+7)$.
47. $(x-9)(x-4)$.
48. $(a+6)(a-4)$.
49. $(x+8)(x+2)$.
50. $(a+3)(a-9)$.

51. $(x+5)(x+1)$.
52. $(a-1)(a-7)$.
53. $(a-5)(a+2)$.
54. $(x-4)(x+9)$.
55. $(x+4)(x+3)$.
56. $(a+2)(a-6)$.
57. $(c-5)(c+7)$.
58. $(b-4)(b-5)$.
59. $(y+4)(y+8)$.
60. $(a+8)(a-7)$.
61. $(x-6)(x+5)$.
62. $(c-3)(c-1)$.
63. $(b+6)(b+2)$.
64. $(y+3)(y-4)$.
65. $(x-8)(x+9)$.
66. $(a-5)(a-3)$.
67. $(c+6)(c+7)$.
68. $(x-7)(x+6)$.
69. $(a+7)(a-3)$.
70. $(y-4)(y-3)$.
71. $(x^2+5)(x^2+3)$.
72. $(a^2+4)(a^2-9)$.
73. $(c^3+9)(c^3-2)$.
74. $(b^2-7)(b^2-1)$.
75. $(a^2+6)(a^2+1)$.
76. $(x^3-5)(x^3+4)$.
77. $(y^3-3)(y^3+8)$.
78. $(c^2-5)(c^2-3)$.
79. $(4x+2)(4x+3)$.
80. $(3a+5)(3a-6)$.
81. $(1+4x)(1+3x)$.
82. $(1-5a)(1+4a)$.
83. $(5c+6)(5c-2)$.
84. $(8y-3)(8y-2)$.
85. $(1-6c)(1+8c)$.
86. $(1-4b)(1-6b)$.
87. $(7b+4)(7b+1)$.
88. $(6x-9)(6x+3)$.
89. $(1+5y)(1+4y)$.
90. $(1+7x)(1-8x)$.
91. $(3a-3)(3a+8)$.
92. $(9c-4)(9c-1)$.
93. $(1+9a)(1-8a)$.
94. $(1-2c)(1-7c)$.
95. $(4x+6)(4x+4)$.
96. $(5y+6)(5y-7)$.
97. $(1+7x)(1+3x)$.
98. $(1-4a)(1+3a)$.
99. $(6b+7)(6b-5)$.
100. $(3x-3)(3x-4)$.
101. $(1-7y)(1+8y)$.
102. $(1-3x)(1-6x)$.

EXERCISES IN ALGEBRAIC EXPRESSION.

1. Write the third power of a, plus three times the product of b square multiplied by x, diminished by m times the square of the binomial $a - x$.

2. If $2x + 1$ represents an odd number, what will represent the next smaller odd number?

3. At n cents a square foot, how much will it cost to plaster the ceiling of a room a yards long and b feet wide?

4. What is the interest on a dollars for eight years at x per cent per annum?

5. If A can do a piece of work in x days and B can do it in y days, what part of it can both do in 3 days?

6. If x represents an integer, does $2x + 2$ represent an even or an odd number? Show why.

7. Write an expression for the sum of five consecutive numbers of which x is the middle one.

8. If it takes a men x days to do a piece of work, how many days will it take one man to do it?

9. If x represents the number of tens in a number and y the number of units, what will represent the number?

10. At n cents a square yard, how much will it cost to plaster the four walls of a room a feet long, b feet wide, and c feet high?

11. Write an expression for the sum of three consecutive even numbers of which x is the smallest.

12. While the minute-hand of a clock is passing over x spaces, how many spaces does the hour-hand pass over?

13. A merchant bought a pieces of silk at n cents a yard, and b pieces of another kind at m cents a yard. Express the cost in dollars, if each piece contained x yards.

14. At c dimes a square yard, how much will it cost in dollars to carpet a room a yards long and b feet wide?

PROBLEMS.

1. A has twice as much money as B, and B has twice as much as C. If they all have $455, how much has C?

2. A has twice as many sheep as B and 35 less than C. If all have 635, how many has A?

3. The sum of the ages of A, B, and C is one hundred forty-eight years. A is three times as old as C, and eight years younger than B. How old is B?

4. A horse, carriage, and harness cost $292. The horse cost ninety dollars more than the harness, and the carriage cost twenty-three dollars less than the horse. Find the cost of the horse and carriage.

5. A is three times as old as B, but ten years ago A was 5 times as old as B. Find A's age.

6. A lady spent $\frac{1}{3}$ of her money for a dress, when she found that she had left $144 more than she had spent. How much had she left?

7. Three men engage in business with a capital of $14,000. B invests one-half as much as A, and four hundred dollars more than C. How much has A invested?

8. Six boys and fifteen men earn two hundred sixty-four dollars a week. If each man earns four times as much as each boy, how much do the six boys earn per week?

9. A farmer sold his corn and wheat for $600; his corn and oats for $400; his wheat and oats for $520. How much did he receive for all his grain?

10. I bought 2 carriages for $305. At the same prices, 3 of the poorer ones would cost $15 more than 2 of the better ones. Find the cost of the better carriage.

11. There are three times as many pupils in one school as in another. If eighty pupils be transferred from the larger school to the smaller, the larger will still have twice as many as the other. How many pupils are there in both schools?

12. A man has three times as much money in the bank as he has in his safe. If he draws $175 from the bank and puts it into his safe, he will have the same amount in each place. How much has he in the bank?

13. The sum of the ages of mother and daughter is 60 years, and the difference between their ages is three times the daughter's age. Find the mother's age.

14. A harness cost one-third as much as a carriage, and a horse cost one and one-fourth times as much as harness and carriage. The horse cost forty-eight dollars more than the carriage. Find the cost of all.

15. A father and two sons earn $111 a month, the two sons receiving the same wages. If the sons' wages were doubled, they would receive only $3 less than their father. How much does the father earn per month?

16. A man bought two horses at the same price. He sold one at a profit of $45 and the other at a loss of $115, receiving twice as much for one as for the other. How much did both horses cost?

17. A farmer bought 45 sheep and had $30 left. If he had bought 60 sheep at the same price, he would have needed fifteen dollars more to pay for them. How much money had he?

18. A man paid a bill of $11.10 in quarters, dimes, and 5-cent pieces, giving three times as many dimes as 5-cent pieces, and twice as many quarters as dimes. How many coins did he give in payment?

To the Teacher. — Teach 93 . . . . 99.

DIVISION.

1. $4\,ab^2) 12\,a^3b^3$.
2. $3\,a^2b) -15\,a^4b^3$.
3. $-abc) -6\,ab^3c^3$.
4. $2\,bc^3) 16\,b^2c^4$.
5. $5\,x^3y) -30\,x^5y^4$.
6. $-xyz) -5\,x^2yz^4$.
7. $3\,xy^2) 18\,x^4y^4$.
8. $4\,a^2x) -28\,a^2x^2$.
9. $-acx) -7\,ac^3x^4$.
10. $5\,cx^3) 15\,c^3x^4$.
11. $2\,b^3c) -18\,b^3c^3$.
12. $-bxy) -4\,b^2xy^4$.
13. $6\,b^2) 18\,ab^3c^3$.
14. $7\,x) -21\,a^2x^2y^3$.
15. $-axy) -3\,ax^2y^3$.
16. $7\,ac^3) 14\,a^3c^3$.
17. $6\,x^2y) -24\,x^3y^2$.
18. $-bcx) -2\,b^3cx^3$.
19. $4\,xy^4) 16\,x^4y^4$.
20. $3\,a) -18\,a^3b^3c^3$.
21. $-abx) -6\,ab^3x^4$.
22. $3\,y^3) 15\,xy^3z^3$.
23. $5\,a^2x) -25\,a^3x^4$.
24. $-xyz) -8\,x^3yz^2$.
25. $2\,ac^3) 18\,a^2c^4$.
26. $4\,b) -20\,a^2b^3c^3$.
27. $-aby) -7\,ab^2y^2$.
28. $5\,x^3) 10\,ax^2c^2$.
29. $2\,c^3d) -14\,c^4d^2$.
30. $-abx) -4\,a^4bx^3$.
31. $6\,xy^3) 12\,x^3y^3$.
32. $7\,a) -28\,a^3b^2c^3$.
33. $-bxy) -3\,bx^3y^4$.
34. $7\,b^3) 21\,ab^3c^3$.
35. $6\,x^3y) -24\,x^4y^2$.
36. $-acx) -2\,a^2cx^3$.
37. $4\,bc^3) 20\,b^2c^3$.
38. $3\,d) -27\,a^3c^3d^2$.
39. $-xyz) -6\,xy^2z^2$.
40. $3\,d^3) 27\,bc^3d^4$.
41. $5\,a^3x) -20\,a^3x^2$.
42. $-abx) -8\,a^2bx^2$.
43. $2\,ax^3) 18\,a^3x^4$.
44. $4\,y) -16\,a^2x^2y^3$.
45. $-axy) -7\,ax^2y^2$.
46. $5\,c^3) 25\,a^3bc^3$.
47. $2\,x^2y) -18\,x^3y^3$.
48. $-bcy) -4\,b^3cy^3$.
49. $6\,bx^3) 36\,b^3x^3$.
50. $7\,z) -21\,x^3y^3z^2$.
51. $-xyz) -3\,xy^4z^4$.
52. $7\,y^3) 42\,xy^3z^3$.
53. $6\,a^2y) -24\,a^2y^3$.
54. $-bxy) -2\,b^3xy^3$.
55. $4\,ax^3) 20\,a^3x^4$.
56. $3\,b) -27\,a^3b^3c^3$.
57. $-acx) -6\,ac^4x^2$.
58. $3\,ab^3) 15\,a^nb^n$.
59. $5\,b^3c) -35\,b^nc^n$.
60. $-bcx) -8\,b^ncx^n$.
61. $2\,x^3y) 14\,x^ny^n$.
62. $4\,a^nx) -32\,a^2x^n$.
63. $-abc) -7\,ab^nc^n$.
64. $5\,b^nc) 15\,b^3c^n$.
65. $2\,x^3y) -24\,x^ny^n$.
66. $-ad) -4\,a^nbcd^n$.
67. $6\,a^nx) 36\,a^3x^n$.
68. $7\,b^nc) -35\,b^3c^n$.
69. $-by) -3\,ab^nxy^n$.
70. $7\,c^nx) 35\,c^nx^3$.
71. $6\,a^3x) -42\,a^nx^n$.
72. $-xy) -2\,ax^ncy^n$.

To the Teacher. — Teach how to divide a polynomial by a monomial.

Divide:

1. $6ac^3 - 8a^2c + 10a^3c^3 - 4ac^2$ by $2ac$.
2. $8x^4y + 4x^3y^2 - 16x^3y + 4x^2y$ by $4x^2y$.
3. $6b^3c^3 + 10b^2c^4 - 8b^3c^2 - 2b^3c^3$ by $2bc^2$.
4. $9a^2b^3 - 3a^3b^4 + 15ab^3 - 6ab^2$ by $3ab^2$.
5. $8x^4y^3 + 4x^3y^3 - 16x^3y^3 + 8x^4y$ by $4x^3y$.
6. $5a^3c^4 + 10a^4c^2 - 5a^3c^3 + 5a^3c^2$ by $5a^2c^2$.
7. $4b^4c^3 + 8b^3c^4d - 16b^3c^3 + 12b^3c^2$ by $4b^2c^2$.
8. $12a^3x^4 + 3a^4x^4 - 6a^2x^3y - 11a^2x^3$ by a^2x^3.
9. $10x^4z^4 - 5x^3yz^5 + 15x^4z^5 - 20x^3z^4$ by $5x^3z^4$.
10. $18a^3c^3 + 12a^3bc^2 - 24a^4c^3 + 6a^3c^2$ by $6a^3c^2$.
11. $32ab^3c^3 - 16b^4c^3d + 16b^3c^4 - 8b^4c^3$ by $8b^3c^2$.
12. $14ab^4c^3 + 28b^4c^3d - 21b^3c^4 + 7b^3c^3$ by $7b^3c^3$.
13. $12b^4x^3y - 18b^3cx^5 + 15b^4x^4 - 6b^3x^3$ by $3b^3x^3$.
14. $15a^4c^3 - 10a^3bc^3d + 25a^3c^3 - 5a^4c^2$ by $5a^3c^3$.
15. $16x^3y^4z + 24ax^3y^3 - 20x^4y^3 + 4x^3y^3$ by $4x^3y^3$.
16. $36x^4y^3 + 18x^3y^4 - 24bx^4y^3z + 6x^4y^3$ by $6x^4y^3$.
17. $18b^nc^3d + 21ab^3c^n - 15b^{2n}c + 6b^nc^3$ by $3b^3c^n$.
18. $18a^4x^3 - 24a^3bcx^4 + 15a^3x^3 - 9a^2x^4$ by $3a^2x^3$.
19. $25a^3b^3c + 20a^4b^4d - 15a^3b^5 - 5a^3b^3$ by $5a^3b^3$.
20. $16a^nb^nc^3 - 24a^3b^nd + 20a^4b^2 - 8a^nb^2$ by $4a^3b$.
21. $14x^3y^n - 28ax^ny^nz + 21xy^{3n} - 7x^3y^n$ by $7x^ny^3$.
22. $8ab^nc^n + 6a^2b^nc^3 - 4a^{2n}b^nc^3 + 2a^nb^3c^n$ by $2a^nb^3c$.
23. $9x^3y^3z^n - 6x^3y^nz^n + 3x^ny^3z^n - 3x^ny^nz^3$ by $3x^ny^nz$.
24. $6bc^nx^m + 8b^3c^mx^3 - 4b^mc^{3n}x^m - 2b^{3n}c^mx^n$ by $2b^mc^3x^n$.
25. $4a^2c^mx^n - 8a^{3n}c^{2n}x^m + 4a^nc^mx^{n-1} - 4acx^{2n-2}$ by $4a^mc^nx$.

To THE TEACHER. — Teach how to divide a polynomial by a polynomial.

Divide:

1. x^2+x-30 by $x+6$.
2. x^2-x-56 by $x-8$.
3. a^2-a-20 by $a-5$.
4. b^2+b-42 by $7+b$.
5. c^2-c-72 by $c-9$.
6. x^2+x-56 by $x+8$.
7. $20+x^2-9x$ by $x-4$.
8. $a^2+7a+10$ by $2+a$.
9. $b^2-4b-21$ by $3+b$.
10. $5y+y^2-84$ by $y-7$.
11. $x^2+6x-16$ by $x-2$.
12. $a^2-7a-78$ by $6+a$.
13. $x^2+13x+36$ by $x+4$.
14. $c^2+11c-12$ by $c-1$.
15. $14a+a^2-72$ by $a-4$.
16. $x^2-13x+12$ by $x-1$.
17. $b^2-17b-18$ by $b+1$.
18. $c^2-32c+60$ by $x-2$.
19. $15x^2+x^4+56$ by $7+x^2$.
20. a^4+15a^2+36 by a^2+3.
21. y^4-19y^2+48 by y^2-3.
22. b^6-15b^3+56 by b^3-7.
23. a^6+50a^3+96 by a^3+2.
24. y^4+47y^2-48 by y^2-1.
25. $a^4+ac^3+a^3c+c^4$ by $a+c$.
26. $b^3+b^2x+bx^2+x^3$ by $b+x$.
27. $a^3-ax^2-a^2x+x^3$ by $a-x$.
28. $a^4+c^4+a^2c^2$ by a^2+c^2+ac.
29. $ac^3-abc-bc^2+b^2$ by $ac-b$.
30. $x^3+3xy^2-3x^2y-y^3$ by $x-y$.
31. $x^2-y^2+z^2-2xz$ by $x+y-z$.
32. $a^2+2xy-y^2-x^2$ by $a+x-y$.
33. $32x^3-6x+1$ by $8x^2+2x-1$.
34. $4a^4-6a^2-8a^3+24$ by $2a-4$.
35. $4b^3+4ab^2-3a^2b-3a^3$ by $b+a$.
36. $16a^4+81x^4-72a^2x^2$ by $2a-3x$.
37. $25x^5-8x-2x^3-x^3$ by $5x^2-4x$.

38. a^3+1 by $a+1$.
39. x^5+1 by $x+1$.
40. x^3+8 by $x+2$.
41. $8-x^3$ by $2-x$.
42. a^4-b^4 by $a-b$.
43. x^5+y^5 by $x+y$.
44. x^3+y^6 by x^2+y^2.
45. $27-c^6$ by $3-c^3$.
46. x^3+8y^3 by $x+2y$.
47. a^3-y^3 by a^2-y^2.
48. $a^{10}+x^{10}$ by a^2+x^2.
49. $x^{12}-y^{12}$ by x^2-y^2.
50. $64x^3-125y^3$ by $4x-5y$.
51. $8a^{12}+729$ by $2a^4+9$.
52. $27x^3+343y^3$ by $3x+7y$.
53. $512a^6-216$ by $8a^2-6$.
54. $a^4-6ac-9a^2-c^2$ by a^2+c+3a.
55. $x^4+9y^4+2x^2y^2$ by x^2+3y^2-2xy.
56. $b^4+16c^4+4b^2c^2$ by b^2+4c^2+2bc.
57. $3cx+2ax+10ac+15c^2$ by $x+5c$.
58. $20ab-12ac-5b^2+3bc$ by $4a-b$.
59. $6x-9x^3-1+4x^4$ by $3x+2x^2-1$.
60. $9y^4+24+50y-67y^2$ by y^2-6+y.
61. $c^4-a^2c^2+2a^3c-a^4$ by c^2-ac+a^2.
62. $9-19a^2+2a^4$ by $2a^3-a+6a^2-3$.
63. $x^4+y^4+4xy^3+4x^2y+6x^2y^2$ by $x+y$.
64. $x^3+y^3+5xy^2+5x^2y$ by x^2+y^2+4xy.
65. $4a+6a^5+3a^3-4-11a^3$ by $3a^2-4$.
66. $y-2x-6y^3+54x^3-3x^2y$ by $2x-y$.
67. $a^4+6a^2x^2+x^4-4ax^3-4a^3x$ by $a-x$.
68. $a^3-a^2+a^6-a^4+2a-1$ by $a+a^2-1$.
69. $27ax^2-25a^3+20a^2x-18x^3$ by $6x-5a$.
70. $a^6-3a^2-1-6a^4$ by $-a-2a^2+a^3-1$.
71. $3a^4+5b^4+3a^2c^2-8a^2b^2-3b^2c^2$ by a^2-b^2.

72. $x^5 + 32$ by $x + 2$.
73. $a^3 + 64$ by $a + 4$.
74. $a^6 - 64$ by $a^2 - 4$.
75. $x^5 - 32$ by $x - 2$.
76. $x^7 - 128$ by $x - 2$.
77. $x^5 + 243$ by $x + 3$.
78. $8x^3 + 1$ by $2x + 1$.
79. $81a^4 - 1$ by $3a + 1$.
80. $27x^3 - 1$ by $3x - 1$.
81. $81x^4 - 1$ by $3x - 1$.
82. $a^4 - 81x^4$ by $a - 3x$.
83. $81x^4 - y^4$ by $3x + y$.
84. $25x^2 - 16y^2$ by $5x + 4y$.
85. $64a^3 + 8b^3$ by $4a + 2b$.
86. $36b^2 - 81c^2$ by $6b - 9c$.
87. $81x^4 - 16y^4$ by $3x + 2y$.
88. $27x^3 + 8y^3$ by $3x + 2y$.
89. $16a^2 - 64b^2$ by $4a - 8b$.
90. $16a^4 - 81b^4$ by $2a - 3b$.
91. $49x^2 - 25y^2$ by $7x + 5y$.
92. $4x^5 - x^3 + 27$ by $4x^2 - 3x^3 + 9 - 6x + 2x^4$.
93. $a^4 - 71a - 36a^2 - 3a^3 - 21$ by $a^2 - 3 - 8a$.
94. $6a^2c^2 - 4ac^3 - 4a^3c + a^4 + c^4$ by $c^2 - 2ac + a^2$.
95. $x^4 + 6x^2y^2 + 4xy^3 + 4x^3y + y^4$ by $y^2 + x^2 + 2xy$.
96. $x^4 - 4xy^3 + y^4 - 4x^3y + 6x^2y^2$ by $x^2 - 2xy + y^2$.
97. $x^2 - 3z^2 + xy - 2y^2 + 2xz + 7yz$ by $x - y + 3z$.
98. $51a + 15a^3 - 18 + 6a^5$ by $7a - 4a^2 + 2a^3 - 2$.
99. $6a^4 - 2a + 23a^2 - 31a^3 - 48$ by $3a^2 + 6 - 5a$.
100. $ax - 3x^2 - ay + 2a^2 - y^2 - 4xy$ by $2a + y + 3x$.
101. $a^5 + 6a^2 - 46a + 7a^3 + 2a^4 - 120$ by $4a + a^2 + 5$.
102. $50m^2 - 32m + 15 + 15m^4 - 32m^3$ by $5 + 3m^2 - 4m$.
103. $-ab - 12c^2 - 6b^2 + 2ac + 2a^2 + 17bc$ by $2a - 4c + 3b$.
104. $4x^2y^2 + 6xy^3 + 10x^3y + 15x^4 - 3y^4$ by $3x^2 - y^2 + 2xy$.
105. $13x + 4x^2 + 6 + x^5 - 6x^3 - 2x^4$ by $x^3 + 3x + 3x^2 + 1$.
106. $a^3 + y^3 + 8x^3 - 6axy$ by $a^2 + y^2 + 4x^2 - 2xy - 2ax - ay$.
107. $53x^2y^3 - 49y^5 + 2x^5 - 9x^4y - 7x^2y^3$ by $2x^2 - 7y^2 - 5xy$.

To the Teacher. — Teach 102 . . . . 109.

Determine what binomial or binomials, if any, will divide each of the following expressions, and give the quotient or quotients in each case:

1. $x^3 - y^3$.	2. $a^3 + 1$.	3. $1 + x^4$.
4. $x^5 + y^5$.	5. $x^7 - 1$.	6. $1 + x^5$.
7. $x^3 - y^6$.	8. $a^4 + 1$.	9. $x^5 - 1$.
10. $x^6 + y^6$.	11. $x^3 - 8$.	12. $8 + x^6$.
13. $x^3 - y^6$.	14. $8 + x^3$.	15. $x^9 - 8$.
16. $a^3 - 27$.	17. $x^3 + y^3$.	18. $a^4 + x^4$.
19. $x^3 + 64$.	20. $a^9 - b^6$.	21. $a^3 + x^6$.
22. $27 - x^6$.	23. $x^3 + 64$.	24. $b^6 - c^3$.
25. $a^5 + 32$.	26. $a^4 + 81$.	27. $x^9 + 64$.
28. $x^9 - 64$.	29. $27x^3 + 1$.	30. $x^5 - 32$.
31. $x^6 - 125$.	32. $216 - a^6$.	33. $1 - 64x^3$.
34. $x^6 + 216$.	35. $8a^9 - 27$.	36. $x^6 + 512$.
37. $8x^3 - 27$.	38. $x^9 + 343$.	39. $a^4 + 625$.
40. $a^3 - 216$.	41. $216 - x^3$.	42. $x^6 - 512$.
43. $343 - x^6$.	44. $a^3 + 343$.	45. $b^{12} - 125$.
46. $x^3 - 512$.	47. $512 + x^9$.	48. $216 - x^{15}$.
49. $8x^3 + 729$.	50. $x^5 - 243$.	51. $729 - 64x^3$.
52. $729 - 8a^6$.	53. $125 + x^9$.	54. $27a^3 - 343$.
55. $x^3 + 1000$.	56. $a^{10} - 243$.	57. $729x^6 + 64$.
58. $1000 - x^9$.	59. $125 - 8x^9$.	60. $343 - 27a^6$.
61. $8x^3 - 27y^3$.	62. $8x^6 + 125$.	63. $81x^4 - 256$.
64. $125x^3 + 8y^6$.	65. $64a^3 - 27$.	66. $16a^4 + 64x^4$.
67. $27a^3 - 512x^6$.	68. $729 + 27x^6$.	69. $729 - 512x^3$.
70. $512x^6 + 27y^3$.	71. $16x^4 - 81y^4$.	72. $216x^4 - 625$.
73. $64a^9 - 343x^6$.	74. $216x^3 + 512$.	75. $81a^4 - 10000$.

EXERCISES IN ALGEBRAIC EXPRESSION.

1. Write five times the square of $a - x$, diminished by the product of the binomials $x - 7$ and $x - 9$.

2. If a represents an integer, when does $a + 1$ represent an even number? When does it represent an odd number?

3. Write an expression for the sum of five consecutive even numbers of which x is the middle one.

4. If A can do a piece of work in 3 days and B can do it in 4 days, what part of it can both do in x days?

5. If x represents the number of hundreds in a number, y the number of tens, and z the number of units, what will represent the number?

6. Write an expression for the sum of four consecutive odd numbers of which x is the largest.

7. What is the interest on eight hundred forty dollars for a months at x per cent per annum?

8. At n cents a square yard, how much will it cost to plaster the walls and ceiling of a room a feet long, b feet wide, and c feet high?

9. If $2x - 5$ represents an odd number, what will represent the next smaller odd number?

10. What algebraic expression will represent the quotient of a number of three figures divided by three times the sum of its digits?

11. A room is x yards long, y feet wide, and a feet high. How many square yards of paper will cover the walls, allowing for 3 windows and 2 doors, each m feet long and n feet wide?

12. A man worked a weeks at b dollars a week, and his son worked c weeks at d dollars a week. They bought m tons of coal at n dollars a ton, and had y dollars left. Write the equation expressing these conditions.

PROBLEMS.

1. A man gave $110 to five boys, giving to each one six dollars more than to the next younger. How much did the oldest boy receive?

2. Three boys sold three hundred fifty-two papers. Frank sold twice as many as John and twenty-eight more than Harry. How many did Harry sell?

3. A man is 32 years older than his son. 12 years ago he was 5 times as old as his son. Find the father's age.

4. Three men invested $9600 in business. A put in $800 more than B, and C invested $400 less than A. How much did A and C together invest?

5. A farmer sold $\frac{5}{8}$ of his potatoes, when he found that he had left 868 bushels less than he sold. Find the value of his whole crop at 65 cents a bushel.

6. A, B, and C together earn $3450. A's salary is one-half of B's, and $450 less than C's. Find C's salary.

7. A lady paid fifty-six dollars for a hat and a dress. The difference in the cost was five times the cost of the hat. Find the cost of the dress.

8. A man paid $195 for his carriage and harness; $315 for his horse and carriage; and $230 for his harness and horse. How much did he pay for all?

9. A, B, and C have seven thousand two hundred fifty dollars. C has one-third as much as B, and B has $600 less than A. How much have A and B?

10. A merchant bought two pieces of silk for $70. At the same prices, four pieces of the cheaper kind would cost $16 more than two pieces of the better kind Find the cost of the better piece.

11. Brown has one-fifth as many acres of land as Jones. If Jones sells Brown 140 acres, Jones will then have twice as many acres as Brown. How many acres have both?

12. A man paid $18,100 for two houses and a farm, paying the same sum for each house. If he had paid twice as much for each house, the two houses would have cost $4700 more than the farm. Find the cost of the farm.

13. James has one-third as many marbles as Frank. If James wins 125 and Frank loses 50, James will then have seven more than Frank. How many have both?

14. I bought two pieces of land at the same price. I sold one piece at a profit of $1600, and the other at a loss of $600, receiving twice as much for one piece as for the other. How much did each piece cost?

15. Three men form a partnership to engage in business. A invests one-half as much as C, and B invests $2\frac{1}{2}$ times as much as A and C. If B's investment exceeds C's by $15,000, what is the whole capital invested?

16. A collection of $14.88 consisted of a certain number of quarters, twice as many dimes as quarters, 3 times as many 5-cent pieces as dimes, half as many 2-cent pieces as 5-cent pieces, and as many pennies as all other coins. How many coins were there in the collection?

17. A lady bought silk at $3 a yard, and had $12 left. If she had bought twice as many yards at two dollars a yard, she would have needed four dollars more to pay for it. How much did she pay for the silk?

18. A farmer sold 20 lambs and 50 sheep for $240. He received twice as much per head for the sheep as for the lambs. How much did he receive for the 50 sheep?

TO THE TEACHER. — Teach 110 . . . . 114.

19. $4a^3x^3 + a^4x^2 + 4a^2x^4$.
20. $81x^6 + 16 - 72x^3$.
21. $64x^2 - 32x + 4$.
22. $\frac{1}{16}a^4 + 16x^2 + 2a^2x$.
23. $4x^4 - 4x^2y + y^2$.
24. $9a^2 + 6a + 1$.
25. $16x^4 + x^6 - 8x^5$.
26. $100 - 40a + 4a^2$.
27. $1 + 64m^4 + 16m^2$.
28. $9a^4b^2 - 30a^2b + 25$.
29. $\frac{4}{9}x^2 + xy + \frac{9}{16}y^2$.
30. $25a^6c^4 - 10a^3c^3 + 1$.
31. $81c^6 + c^4 + 18c^5$.
32. $49x^4 - 98x^2 + 49$.
33. $m^4 - 12m^3 + 36m^2$.
34. $121 + 4a^2c^2 + 44ac$.
35. $\frac{1}{4}x^6 + \frac{4}{9}x^4 - \frac{2}{3}x^5$.
36. $49a^2 - 14ac + c^2$.
37. $16y^2 + 16y^4 + 32y^3$.
38. $9m^2 + 4n^2 - 12mn$.
39. $1 + 18a^2x + 81a^4x^2$.
40. $64c^3 - 16c^7 + c^6$.
41. $\frac{9}{25}a^4 + \frac{1}{4}b^6 + \frac{3}{5}a^2b^3$.
42. $121b^2 - 88b + 16$.
43. $(a + x)^2 + 2(a + x) + 1$.
44. $8(x + y) + (x + y)^2 + 16$.
45. $(a + c)^2 - 16(a + c) + 64$.
46. $9(a - b)^2 - 12c(a - b) + 4c^2$.
47. $100 + 20ab + a^2b^2$.
48. $25x^4 - 50x^3 + 25x^2$.
49. $81a^4 + 4x^2 + 36a^2x$.
50. $\frac{1}{4}m^4 - \frac{1}{2}m^2n^2 + \frac{1}{4}n^4$.
51. $1 + 24ax + 144a^2x^2$.
52. $100x^6 + 16x^4 - 80x^5$.
53. $64 + 9a^4c^2 + 48a^2c$.
54. $49x^2 - 70xy + 25y^2$.
55. $\frac{1}{9}a^6c^4 - \frac{2}{3}a^5c^2 + \frac{1}{9}a^4$.
56. $121m^4 + 22m^2 + 1$.
57. $1 - 40x^4 + 400x^8$.
58. $36a^2 + 25b^2 + 60ab$.
59. $4x^2 - 20xy^2 + 25y^4$.
60. $25a^2 + 16c^2 + 40ac$.
61. $121 - 66m + 9m^2$.
62. $36a^8 + 36a^2 + 72a^5$.
63. $(x - y)^2 + 2ac(x - y) + a^2c^2$.
64. $25(a + x)^2 - 10c(a + x) + c^2$.
65. $9a^4(x - y)^2 + 48a^2(x - y) + 64$.

66. $(a+b)^2-4(a+b)+4$.

67. $6(x-y)+(x-y)^2+9$.

68. $(x+y)^2-8(x+y)+16$.

69. $(a+x)^2+25+10(a+x)$.

70. $12(x+y)+4(x+y)^2+9$.

71. $a^2(b-c)^2-8a(b-c)+16$.

72. $9(x-y)^2-12z(x-y)+4z^2$.

73. $4a^4(a+b)^2+16+16a^2(a+b)$.

74. $9a^2(a-c)^2-12ab(a-c)+4b^2$.

75. $16a^4(x+y)^2+9a^2+24a^3(x+y)$.

76. $9(2b-3x)^2-12a(2b-3x)+4a^2$.

77. $(a+b)^2+(c-d)^2-2(a+b)(c-d)$.

To the Teacher. — Teach 121 . . . . 123.

Case III.

1. a^2-c^2.	2. a^2-1.	3. $1-c^2$.
4. a^2-b^2.	5. b^2-1.	6. $1-b^2$.
7. x^4-y^4.	8. $1-x^4$.	9. y^4-1.
10. a^6-b^6.	11. x^6-1.	12. $1-y^6$.
13. x^3-y^3.	14. b^3-1.	15. $1-x^3$.
16. m^4-n^2.	17. x^6-y^4.	18. a^2-b^8.
19. $4x^2-4$.	20. $9-4x^2$.	21. $9a^4-4$.
22. $9x^6-y^2$.	23. $4x^8-y^2$.	24. a^6-9b^8.
25. $9y^4-16$.	26. $25-9x^6$.	27. $4x^8-49$.
28. $4x^8-25$.	29. $64-9x^2$.	30. $9b^6-81$.
31. $9x^4-4y^2$.	32. $4a^6-b^4c^2$.	33. $x^2y^2-9z^6$.

34. $9x^6 - y^8$.
35. $4x^8 - 1$.
36. $1 - 9x^4$.
37. $4x^{2m} - y^{2n}$.
38. $9a^{2n} - 1$.
39. $1 - 4x^{2n}$.
40. $4a^4x^4 - y^8$.
41. $16 - a^4b^2$.
42. $x^{4n} - 25$.
43. $a^6b^4 - c^2d^6$.
44. $x^2y^6 - 49$.
45. $36 - x^{2n}$.
46. $a^6b^2 - x^4y^8$.
47. $9x^4 - y^6z^2$.
48. $x^{4n} - y^{2n}$.
49. $100x^4 - y^2$.
50. $4a^8 - b^2c^6$.
51. $x^{6n} - y^{4n}$.
52. $a^4 - 144b^2$.
53. $x^6y^2 - 9z^6$.
54. $x^{2m} - y^{2n}$.
55. $81x^8 - 9y^4$.
56. $36x^4 - 81$.
57. $x^{4m} - y^{4n}$.
58. $49x^4 - 4y^6$.
59. $16x^4 - 16$.
60. $x^{2m} - y^{2n}$.
61. $16x^4 - 16y^4$.
62. $a^4 - 625b^4$.
63. $9x^6 - y^{4n}$.
64. $256x^8 - y^8z^8$.
65. $81x^4 - 16y^4$.
66. $x^{4n}y^{4n} - 16$.

To the Teacher. — Teach 124.

1. $(x+y)^2 - 4$.
2. $(a+b)^2 - (c+d)^2$.
3. $9 - (a+c)^2$.
4. $(a+b)^2 - (c-d)^2$.
5. $(x+y)^2 - 1$.
6. $(a-x)^2 - (y+4)^2$.
7. $1 - (b-c)^2$.
8. $(x-y)^2 - (z-3)^2$.
9. $(b+c)^2 - d^2$.
10. $(a+c)^2 - (x+1)^2$.
11. $a^2 - (b+c)^2$.
12. $(a+b)^2 - (2c-d)^2$.
13. $(a-x)^2 - c^2$.
14. $(x-2y)^2 - (z-5)^2$.
15. $b^2 - (a+c)^2$.
16. $(a-b)^2 - (3c+4)^2$.
17. $(a-1)^2 - d^2$.
18. $(2a+x)^2 - (y-z)^2$.
19. $(a+x^2)^2 - 1$.
20. $(3a+c)^2 - (2x+y)^2$.
21. $1 - (x^2+y)^2$.
22. $(a-4b)^2 - (c-7d)^2$.
23. $(x^2-3)^2 - z^2$.
24. $(2x+3y)^2 - (4z+5)^2$.
25. $b^2 - (a-c^2)^2$.
26. $(2-3a)^2 - (4b+6c)^2$.

27. $a^2 + 2a + 1 - b^2$.
28. $x^2 - y^2 + 2y - 1$.
29. $x^2 - 2x + 1 - y^2$.
30. $b^2 - 2c - c^2 - 1$.
31. $x^2 - y^2 + 4x + 4$.
32. $a^2 + 6x - x^2 - 9$.
33. $x^2 + y^2 - 2xy - 4$.
34. $9 - x^2 - y^2 - 2xy$.
35. $b^2 + c^2 - d^2 + 2bc$.
36. $a^2 - 2bc - b^2 - c^2$.
37. $(a^2 - b^2 - c^2)^2 - 4b^2c^2$.
38. $b^2 - a^2 - c^2 + 2ac$.
39. $4x^2y^2 - (x^2 + y^2 - z^2)^2$.
40. $a^2 + x^2 - c^2 - 2ax$.
41. $x^2 - y^2 - 2x + 1 + 2yz - z^2$.
42. $a^2 - c^2 + b^2 + 2ab - d^2 - 2cd$.
43. $a^2 - 2xy - 2ac - x^2 - y^2 + c^2$.
44. $a^2 + b^2 - x^2 - y^2 + 2ab - 2xy$.
45. $a^2 - c^2 - n^2 + m^2 - 2am + 2cn$.
46. $(a - b)(b^2 - c^2) - (b - c)(a^2 - b^2)$.
47. $4(2a - 3b)^4 - 9c^4$.
48. $x^2 + 1 - y^2 + 2x$.
49. $25a^4 - 9(3b - 2c)^4$.
50. $a^2 - 2x - x^2 - 1$.
51. $c^2 - d^2 - 2c + 1$.
52. $b^2 - d^2 + 2d - 1$.
53. $x^2 - z^2 - 6x + 9$.
54. $x^2 - 2y - y^2 - 1$.
55. $x^2 - 4 + y^2 + 2xy$.
56. $a^2 + 2ab + b^2 - 1$.
57. $a^2 + x^4 - 1 + 2ax^2$.
58. $1 - x^2 - y^2 - 2xy$.
59. $m^2 + n^2 - 1 + 2mn$.
60. $a^2 - b^2 - c^2 + 2bc$.
61. $x^2 + y^4 - z^4 - 2z^2 + 2xy^2 - 1$.
62. $a^4 - b^2 - 4c^2 + 9 - 6a^2 + 4bc$.
63. $a^2 - d^2 - y^2 - 2dy + c^2 + 2ac$.
64. $a^4 + c^2 - x^2 - y^4 - 2a^2c + 2xy^2$.
65. $4a^2 + 9b^2 - 9x^2 + 12ab - 4y^2 - 12xy$.
66. $(2a + 3b)^2 - 4c^2 - 3c(2a + 3b - 2c)$.

67. $a^2 - 2b - 1 - b^2$.
68. $x^2 - y^2 + 2x + 1$.
69. $1 - b^2 - c^2 + 2bc$.
70. $x^2 + y^2 - 1 - 2xy$.
71. $1 - a^2 - b^2 - 2ab$.
72. $2ab - a^2 - b^2 + 1$.
73. $a^2 - 4b^2 - 4a + 4$.
74. $9 - 9x^2 - 6xy - y^2$.
75. $4ab - a^2 + 1 - 4b^2$.
76. $a^4 - c^4 + b^4 - 2a^2b^2$.
77. $a^2 + 16c^2 - 16 + 8ac$.
78. $8ac - 4a^2 - 4c^2 + 4$.
79. $a^2 - b^2 + 2b - 1 - 2ac + c^2$.
80. $b^4 - c^2 + 2b^2 - 2cd^2 + 1 - d^4$.
81. $a^2 + 4c^2 - 9x^2 - 6x - 1 + 4ac$.
82. $4a^2 - 4b^2 - c^2 + 4bc - 4a + 1$.
83. $c^4 - b^2 - 4x^2 - 6ac^2 + 4bx + 9a^2$.

To the Teacher. — Teach 126 . . . . 127.

Case IV.

1. $x^3 - y^3$.
2. $a^3 - 1$.
3. $1 - b^3$.
4. $x^5 - y^5$.
5. $a^5 - 1$.
6. $1 - b^5$.
7. $x^7 - y^7$.
8. $a^7 - 1$.
9. $1 - b^7$.
10. $x^8 - y^8$.
11. $a^3 - 8$.
12. $8 - b^3$.
13. $x^8 - y^9$.
14. $a^6 - 8$.
15. $8 - b^9$.
16. $x^3 - 27$.
17. $a^9 - b^3$.
18. $b^6 - c^3$.
19. $27 - x^3$.
20. $a^9 - b^6$.
21. $b^3 - c^{12}$.
22. $x^9 - 64$.
23. $a^3 - 64$.
24. $a^3b^6 - 8$.
25. $x^9 - 27$.
26. $a^6 - 27$.
27. $125 - b^3$.
28. $x^6 - 125$.
29. $27 - a^3$.
30. $b^3 - 216$.
31. $8x^3 - y^{12}$.
32. $8a^9 - 27$.
33. $b^{12} - 125$.

34. $a^5 - x^5$. 35. $x^5 - 1$. 36. $1 - y^5$.

37. $x^9 - y^9$. 38. $8 - x^3$. 39. $y^3 - 8$.

40. $x^9 - y^3$. 41. $x^6 - y^3$. 42. $27 - x^9$.

43. $8x^3 - 27$. 44. $27a^3 - 1$. 45. $x^5 - 32$.

46. $x^3 - 216$. 47. $216 - a^6$. 48. $1 - 64x^3$.

49. $343 - x^6$. 50. $a^9 - 343$. 51. $x^9 - 216$.

52. $x^3 - 512$. 53. $512 - x^9$. 54. $343 - x^3$.

55. $8x^3 - 729$. 56. $x^5 - 243$. 57. $a^6 - 512$.

58. $x^6 - 1000$. 59. $125 - 8x^9$. 60. $729 - 64x^3$.

61. $8x^3 - 125y^3$. 62. $64x^3 - 729$. 63. $27a^3 - 343$.

64. $512a^3 - 27b^6$. 65. $216x^6 - 125y^3$. 66. $729a^3 - 512$.

To the Teacher. — Teach 128 . . . . 130.

Case V.

1. $x^3 + y^3$. 2. $a^3 + 1$. 3. $1 + c^3$.

4. $x^5 + y^5$. 5. $1 + a^5$. 6. $b^5 + 1$.

7. $x^6 + y^6$. 8. $a^6 + 1$. 9. $1 + b^6$.

10. $x^7 + y^7$. 11. $1 + a^7$. 12. $b^7 + 1$.

13. $x^9 + y^9$. 14. $a^3 + 8$. 15. $8 + b^6$.

16. $x^6 + 64$. 17. $x^3 + y^3$. 18. $a^3 + x^6$.

19. $x^5 + 32$. 20. $27 + a^6$. 21. $b^9 + 64$.

22. $x^6 + 216$. 23. $343 + a^5$. 24. $x^3 + 512$.

25. $8x^6 + 729$. 26. $a^{10} + 243$. 27. $x^{12} + 216$.

28. $x^3 + 1000$. 29. $8x^6 + 125$. 30. $64x^2 + 729$.

31. $125x^3 + 8y^6$. 32. $27x^6 + 729$. 33. $343 + 27a^3$.

34. $512x^9 + 27y^6$. 35. $216a^3 + 125b^6$. 36. $729x^3 + 512$.

To the Teacher. — Teach 13 . . . . 131.

Case VI.

1. $x^2+7x+12$.
2. x^2-6x+5.
3. $x^2+7x-18$.
4. a^2-2a-8.
5. $x^2+8x+15$.
6. a^2+8a-9.
7. $x^2-9x+20$.
8. x^2+x-30.
9. $x^2+x-132$.
10. x^2-x-56.
11. $x^2+13x+36$.
12. a^2+a-42.
13. $a^2+17a+30$.
14. x^2-7x+6.
15. $a^2-14a+48$.
16. $1+5x+6x^2$.
17. $x^2+11x-12$.
18. $1-8x-9x^2$.
19. $a^2+11a+30$.
20. x^4-9x^2+8.
21. $x^2-13x+12$.
22. a^4-a^2-20.
23. $a^2-17a-18$.
24. x^4-6x^2-72.
25. $y^2-13y+36$.
26. x^4-4x^2-21.
27. $x^2-4x-320$.
28. y^6+5y^3-84.
29. $x^2-32x+60$.
30. $a^2+9ac+8c^2$.
31. a^4+16a^2+48.
32. $1+6a-72a^2$.
33. x^8+14x^4-72.
34. $1-2a-48a^2$.
35. x^6-5x^3-300.
36. $1-8a+12a^2$.
37. a^4+15a^2+36.
38. $1+7x-18x^2$.
39. a^4-19a^2+48.
40. $x^2+15x+56$.
41. x^4-16x^2+48.
42. $a^2+15a-16$.
43. a^4+11a^2-42.
44. $x^2-16x-17$.
45. x^4-14x^2-32.
46. $a^2+13a+12$.
47. a^6-15a^3+56.
48. $x^2-21x+80$.
49. $x^2+23x+102$.
50. $x^2+16x-80$.

51. $x^2 + 5x - 6$.
52. $x^2 + x - 12$.
53. $x^2 - x - 132$.
54. $a^2 + a - 90$.
55. $a^2 + 2a - 15$.
56. $x^2 - x - 56$.
57. $y^2 - y - 110$.
58. $b^2 - b - 30$.
59. $x^2 + 8x - 48$.
60. $x^6 + x^3 - 72$.
61. $a^2 + 4a - 96$.
62. $1 - 9x + 8x^2$.
63. $x^2 + 17x + 16$.
64. $a^2 - ay - 2y^2$.
65. $b^2 + 15b + 14$.
66. $y^6 - 10y^3 + 9$.
67. $x^2 - 28x - 29$.
68. $1 + 7a + 12a^2$.
69. $a^2 + 12a - 28$.
70. $14x^2 + 9x + 1$.
71. $x^2 - 11x + 24$.
72. $1 + 8x + 12x^2$.
73. $y^2 + 47y - 48$.
74. $x^2 + 25x + 24$.
75. $x^2 - 19x + 84$.
76. $c^6 - 10c^3 - 11$.
77. $b^2 - 17b + 16$.
78. $y^4 - 14y^2 + 40$.
79. $x^2 + 13x + 12$.
80. $x^6 + 27x^3 + 72$.
81. $a^2 + 19a - 20$.
82. $y^4 - 15y^2 + 44$.
83. $x^2 + 23x - 24$.
84. $a^4 + 22a^2 + 96$.
85. $b^2 - 45b - 46$.
86. $1 + 21x - 72x^2$.
87. $a^4 + 16a^2 + 28$.
88. $x^2 - 21x - 100$.
89. $x^4 + 50x^2 + 96$.
90. $y^2 - 26y + 120$.
91. $1 + 3x - 180x^2$.
92. $1 + 11x + 18x^2$.
93. $1 + 12a - 64a^2$.
94. $x^2 + 6ax - 16a^2$.
95. $x^4 - 36x^2 + 320$.
96. $1 - 19c^2 - 20c^4$.
97. $y^4 - 14y^2 - 147$.
98. $a^6 - 32a^3 + 192$.
99. $1 - 8a^2 - 105a^4$.
100. $x^4 - 72x^2 + 512$.
101. $x^2 + 37xy + 36y^2$.
102. $1 - 35a + 300a^2$.

To the Teacher. — Teach 134.

1. $8x^2+x-9$.
2. $7x^2-x-6$.
3. $3x^2-x-2$.
4. $6x^2-x-7$.
5. $6x^2-x-2$.
6. $4x^2+x-5$.
7. $6x^2-x-5$.
8. $7x^2-x-8$.
9. $8a^2+6a-9$.
10. $5x^2-8x+3$.
11. $6x^2+2x-4$.
12. $6a^2-2a-8$.
13. $6x^2+7x+2$.
14. $2x^2+5x+3$.
15. $3x^2+6x+3$.
16. $3x^2+7x-6$.
17. $4x^2+3x-10$.
18. $8x^2-6x-35$.
19. $6x^2+17x+12$.
20. $8a^2+ax-9x^2$.
21. $8x^2+45x-18$.
22. $15b^2-19b-8$.
23. $12x^2+19x+7$.
24. $48a^2-46a-9$.
25. $20b^2-4b-16$.
26. $10x^2-23x-5$.
27. $9a^2-32a-16$.
28. $6x^2+59x-10$.
29. $4x^2+8xy+3y^2$.
30. $10b^2-89b-9$.
31. $12c^2-17c-14$.
32. $18a^2+ax-4x^2$.
33. $12y^2+31y-15$.
34. $12x^2+13x-14$.
35. $14x^2+45x-14$.
36. $12a^2-11a-36$.
37. $9a^2+18ab+8b^2$.
38. $10x^2+19x-15$.
39. $9a^2+8ax-20x^2$.
40. $8a^2-34ac-9c^2$.
41. $9x^2-31xy+12y^2$.
42. $15b^2-4bc-4c^2$.
43. $5a^2+53ac-22c^2$.
44. $8a^2+53ax-21x^2$.
45. $8a^2-97ax+12x^2$.
46. $50x^2+35xy-4y^2$.
47. $15x^2-22xy-9y^2$.
48. $8b^2-37bc-15c^2$.
49. $8b^2-49bx-49x^2$.
50. $12a^2+23ax-9x^2$.
51. $10x^2-29xy+10y^2$.
52. $6x^2-23xy+21y^2$.

To the Teacher. — Teach 137 . . . . 138.

CASE VII.

1. $9a^4 - 15a^2 + 1$.
2. $4a^4 - 13a^2 + 1$.
3. $x^4 + 2x^2y^2 + 9y^4$.
4. $4a^4 - 5a^2b^2 + b^4$.
5. $4a^4 - 53a^2x^2 + x^4$.
6. $9b^4 + 3b^2c^2 + 4c^4$.
7. $9x^4 + 8x^2y^2 + 16y^4$.
8. $a^4 - 26a^2c^2 + 25c^4$.
9. $49x^8 - 50x^4y^4 + y^8$.
10. $36x^4 + 11x^2y^2 + y^4$.
11. $25a^4 - 34a^2x^2 + 9x^4$.
12. $81a^4 - 45a^2c^2 + 4c^4$.
13. $49x^8 - 58x^4y^4 + 9y^8$.
14. $9x^4 - 52x^2y^2 + 64y^4$.
15. $16a^4 - 41a^2c^2 + 25c^4$.
16. $36a^4 + 35a^2x^2 + 25x^4$.
17. $121x^4 + 7x^2y^2 + 16y^4$.
18. $25b^4 - 89b^2x^2 + 64x^4$.
19. $16x^4 - 65x^2y^2 + 49y^4$.
20. $81x^8 + 41x^4y^4 + 25y^8$.
21. $25x^4 - 61x^2y^2 + 36y^4$.
22. $36x^8 - 85x^4y^4 + 49y^8$.
23. $49m^4 + 31m^2n^2 + 64n^4$.
24. $64x^8 + 87x^4y^4 + 49y^8$.

REVIEW OF FACTORING.

1. $a^2 + ac + ay + cy$.
2. $x^4 - x$.
3. $a^2 - m^4 + c^2 + 2ac$.
4. $20x^2 - 8x - 9$.
5. $3ax - 3ay - 2cx + 2cy$.
6. $5ac^4 + 5ac$.
7. $x^7y - 25x^4y^4 + 24xy^7$.
8. $x^4 - z^4 - 2x^2y + y^2$.
9. $(a-c)^2 - 10(a-c) + 25$.
10. $x^4 - 81$.
11. $25x^4 + 65x^2y^2 + 81y^4$.
12. $c + y + cx + xy$.
13. $y^8 - 19y^4z + 84z^2$.
14. $x^{12} + y^{12}$.
15. $(a+b)^2 + 2c(a+b) + c^2$.
16. $x^2 + y^2 - z^2 - 2xy$.
17. $a^4 - a^2c^2 - a^2x^2 + c^2x^2$.
18. $6a^2 + 11a - 72$.
19. $36a^4 + 116a^2x^2 + 121x^4$.
20. $x^6y + xy$.
21. $xy + nx + y + n$.
22. $8c^2 + 31c - 4$.
23. $x^2 - y^2 - n^2 - 2ny + m^2 + 2mx$.

24. $49x^4 + 59x^2y^2 + 100y^4$.
25. $a^8 - 16$.
26. $x^2y^2 - x^2 - 4y^2 + 4$.
27. $x^6 + 4x^3 - 32$.
28. $x^4 - 33x^2y + 272y^2$.
29. $a^2 - c^2 + 4 + 4a$.
30. $64x^4 + 44x^2y^2 + 25y^4$.
31. $c^2 - b^2 - 2ab - a^2$.
32. $(a-b)^2 + (a-b) - 2$.
33. $x^2 - ax - cx + ac$.
34. $ab^2 - 4a + 2b^2 - 8$.
35. $x - 8x^4$.
36. $9x^2 + 80xy - 9y^2$.
37. $x^{10} + y^{10}$.
38. $x^4 - 10x^2y^4 + 24y^8$.
39. $27x^3 - 1$.
40. $(a^2 + 5a)^2 + 12(a^2 + 5a) + 36$.
41. $x^2 - z^2 - 2x - y^2 + 1 + 2yz$.
42. $c^2 - y^2 - d^2 + 2dy - 2cx + x^2$.
43. $a^2 - n^2 - m^2 + 2ab + b^2 - 2mn$.
44. $ax - bx + ay - by + x^2 + y^2 + 2xy$.
45. $81x^4 - 184x^2y^2 + 100y^4$.
46. $7a^5c^2 + 7a^2c^5$.
47. $b^2 - d^2 - 10b + 25$.
48. $12x^2 - 37x - 10$.
49. $x^3 - xy - x + y$.
50. $5a^3 - 5$.
51. $x^3 - x^2y - xy^2 + y^3$.
52. $2c - c^3 + b^2 - 1$.
53. $m + 5 - m^3 - 5m^2$.
54. $x^4 + 12x^2y^2 - 64y^4$.
55. $ac + cx - ax - x^2$.
56. $2y^{13} + 16y$.
57. $121x^4 + 112x^2y^2 + 64y^4$.
58. $20x^2 + 21x - 54$.
59. $x^2z - x^2u - y^2z + uy^2$.
60. $1 + 243a^{10}$.
61. $(x^2 - 3x)^2 - 2(x^2 - 3x) - 8$.
62. $(a^2 - 2x)^2 + 6(a^2 - 2x) + 9$.
63. $2xy + c^2 + d^2 - x^2 - 2cd - y^2$.
64. $c^2 - n^2 + x^2 - 2bn - b^2 + 2cx$.
65. $2ax + 3bx + cx + 2ay + 3by + cy$.

66. $x^5y + 23x^3y^4 + 120xy^7$.
67. $2a^5 - 2a$.
68. $x^4 - 17x^2y^2 + 16y^4$.
69. $5x^3 - 40x^6$.
70. $a^2 + 5a - ac - 5c$.
71. $x + x^{13}$.
72. $2a^2b + ab^2 - b^3$.
73. $12a^2 - a - 11$.
74. $3a^3 - 2a^2c - 3ac^2 + 2c^3$.
75. $12a^2 - 27b^2$.
76. $(x^2 - 2x)^2 - 2(x^2 - 2x) - 3$.
77. $1 - y^2 - 2x + x^2$.
78. $12a^2 - 25ax - 22x^2$.
79. $18x^2 - 32y^2$.
80. $2m + mn - 2 - n$.
81. $40ax^4 + 5ax$.
82. $4x^4 - 45x^2y^4 + 81y^8$.
83. $1 - 13y^2 + 36y^4$.
84. $a^2 + b - 2ab - a + b^2$.
85. $x^2 + 2xy + y^2 + 4x + 4y$.
86. $8(a^2 - 2x) + (a^2 - 2x)^2 + 15$.
87. $(x^2 - 4x)^2 - 10(x^2 - 4x) + 25$.
88. $m^2 - 1 - n^2 + 2n$.
89. $a^6 - 64$.
90. $25x^4 - 26x^2y^2 + y^4$.
91. $m + 32m^6$.
92. $abx^2 + (a^2 + b^2)xy + aby^2$.
93. $9y^2 + 70y - 16$.
94. $(x + y)^2 - (x + y) - 12$.
95. $c^3 + 8c^5 + 15c^7$.
96. $121x^4 + 98x^2y^2 + 81y^4$.
97. $x^{11}y + x^2y^{13}$.
98. $a^2 - m^2 + an - mn$.
99. $2b - b^2 - 1 + a^2$.
100. $a^2x - (a^2 - b^2)y - b^2x$.
101. $64x^3 - 125$.
102. $abx^2 - xy(a^2 + b^2) + aby^2$.
103. $a^5 - 5a^3 - 36a$.
104. $abc^2 + abd^2 + (a^2 + b^2)cd$.
105. $a^4 - a^3c + ac^3 - c^4$.
106. $acx^2 - abx - bcx + b^2$.
107. $x^6 - 35x^3 - 36$.
108. $a^2 + 4c - 1 + b^2 - 2ab - 4c^2$.
109. $m^2 - b^4 - 4cm - 9n^2 + 6b^2n + 4c^2$.
110. $2ax - 3bx - 4cx - 2ay + 3by + 4cy$.

111. $16x^4 - 25x^2y^2 + 9y^4$.

112. $a^{15} + a$.

113. $9x^2 - 52xy - 12y^2$.

114. $14c^2 - 47c - 7$.

115. $(a-b)^2 - 12(a-b) + 36$.

116. $m^{16} - 625$.

117. $49a^4 - 53a^2x^2 + 4x^4$.

118. $a^2 - c^2 - a - c$.

119. $x^6 + 49x^3y^3 + 48y^4$.

120. $27x^{12}y + xy^4$.

121. $ax - ay + nx - ny$.

122. $15a^2 + 2a - 45$.

123. $a^4 - 23a^2c^2 + 112c^4$.

124. $27x^3 - 1331$.

125. $81x^4 - 214x^2y^2 + 121y^4$.

126. $3x^2 + 11x - 20$.

127. $3a + 3b + a^2 + 2ab + b^2$.

128. $ax + cy - x - ay - cx + y$.

129. $(x^2 - 3xy)^2 - 8y(x^2 - 3xy) - 48y^2$.

130. $4a^4 - 9d^2 + 12bd - 12a^2 - 4b^2 + 9$.

131. $(x^2 + 4xy)^2 - 24y(x^2 + 4xy) + 144y^2$.

132. $a^2 - b^2 - 2a + 1$.

133. $x^2y + xy^2$.

134. $a + m + a^2 - m^2$.

135. $a^{16} - c^4$.

136. $a^6c^2 + 15a^4c^4 - 16a^2c^6$.

137. $64a^{11}b + 2ab^6$.

138. $6a^3 - 4a^2 - 9a + 6$.

139. $c - 25c^5 + 24c^{11}$.

140. $(a-c)^2 + (a-c) - 6$.

141. $405x^2y^5 - 5xy$.

142. $2cx - x^2 - c^2 + 1$.

143. $8a^2 - 95a - 12$.

144. $100a^4 - 205a^2c^2 + 81c^4$.

145. $125a^{16}b + ab^4$.

146. $x + 62x^4 - 128x^7$.

147. $m^2 - 64n^6$.

148. $a^2 - 2ab + b^2 - ac + bc$.

149. $x^4 - 2x^2(2x-1) + (2x-1)^2$.

150. $(x^4 - 5x^2)^2 + 8(x^4 - 5x^2) + 16$.

151. $(x^4 - 10x^2)^2 + 18(x^4 - 10x^2) + 81$.

152. $2a^3 - 2ac^2 - 3a^2b + 3bc^2 - a^2 + c^2$.

153. $(2x-1)^2-(3x+1)^2$.

154. $c^5-c^4+c^3-c^2$.

155. m^6-19m^3-216.

156. $5a-80a^5$.

157. $(2a+3)^2-(3a-4)^2$.

158. $x^2-y^2-4x+4y$.

159. $(2a+3x)^2-(3a-4x)^2$.

160. $2x^{16}y-2xy^{11}$.

161. $b^4+b^3+b^2+b$.

162. $1000x^3-1331y^6$.

163. $(5x+2y)^2-(4x-3y)^2$.

164. $a^2-b^2-2a+2b$.

165. $(7a-3c)^2-(2a+c)^2$.

166. $am-an+m^2-n^2$.

167. $\frac{1}{4}a^4+\frac{1}{3}a^3+\frac{1}{9}a^2$.

168. $4a^2+9b^2+12ab$.

169. $5a^2-8ab+3b^2-5a+3b$.

170. $(a+x)^2-1-2x(a+x-1)$.

171. $3x^2+6xy+3y^2-3xz-3yz$.

172. $(x^2-2xy)^2+2y^2(x^2-2xy)+y^4$.

173. $25+4a^2-4b^2+12bc-9c^2-20a$.

174. $(8a+5x)^2-(3a-2x)^2$.

175. $48x^2y-1875xy^5$.

176. $x^8-43x^5+42x^2$.

177. m^6+m^3-72.

178. $3c^3-c^2+3c-1$.

179. $x^{12}-y^9z^3$.

180. $(9a+3b)^2-(2a-5b)^2$.

181. $8a^3b^6+27x^9y^{12}$.

182. $9x^4-10x^2y^4+y^8$.

183. $16a^3-54b^{12}$.

184. $(x-6)^2-(x+y-6)^2$.

185. $32a^{12}-2x^8$.

186. $1-a+a^2-a^3$.

187. $5x^6y^2-5x^2y^6$.

188. $(x+y-z)^2-(x-y-z)^2$.

189. $\frac{4}{9}a^2+\frac{9}{16}-a$.

190. $ax+by-y^2-ay-bx+x^2$.

191. $a^2-b^2-c^2+2bc+a+b-c$.

192. $x^3-y^3-x(x^2-y^2)+y(x-y)^2$.

193. $(a-c)(c^3-x^3)-(c-x)(a^3-c^3)$.

194. $a^4-2a^2(3ac-2c^2)+(3ac-2c^2)^2$.

195. $16x^4 - 97x^2y^2 + 81y^4$.

196. $x^8 - 80x^4 - 81$.

197. $(x+y)^3 + 1$.

198. $x^{12} + x^2$.

199. $a^6 - 19a^3b^6 + 88b^{12}$.

200. $81a^4b - 375ab^7$.

201. $1 + (a+b)^3$.

202. $729 + 1000a^3$.

203. $(x-y)^3 - 1$.

204. $1 - (a-b)^3$.

205. $4x^4 - 9y^2 + 12xy - 4x^2$.

206. $x^4 + x^2 + 1$.

207. $9x^2 - 4y^2 - 3xz + 2yz$.

208. $8a^6 - b^9$.

209. $49a^4 - 65a^2x^2 + 16x^4$.

210. $(a+x)^4 - 1$.

211. $x^8 - 19x^4y^4 + 48y^8$.

212. $1 - (a-b)^4$.

213. $a^2 + a + 3x - 9x^2$.

214. $x^7 - x$.

215. $a^4 + a^2 - 36 - 2a^3$.

216. $1029a^3c - 3c^4$.

217. $36x^4 - 37x^2y^4 + y^8$.

218. $729x^6 + 125y^{12}z^3$.

219. $(a+x)^3 + (2a-x)^3$.

220. $343x^3 - 64y^{12}$.

221. $8ab - 4b^2 + 4 - 4a^2$.

222. $486x^6 + 2x$.

223. $1 - x^2 - y^2 - 2xy$.

224. $48m^4 - 1875n^4$.

225. $(2x-y)^3 - (x-y)^3$.

226. $c^2 - 7c^5 - 8c^8$.

227. $a^5c + 65a^3c^3 + 64ac^5$.

228. $8a^3 - (a-x)^3$.

229. $25b^4 - 45b^2c^2 + 4c^4$.

230. $8(a+c)^3 + x^3$.

231. $4a^2x^2 - (a^2 + x^2 - y^2)^2$.

232. $a^7b + ab^7$.

233. $a^4 - 2a^3 + a^2 - 4a + 4$.

234. $162a^5b - 512ab^5$.

235. $a^3 - x^3 - 3ax(a-x)$.

236. $x^4 - 4 - 2x^3 + x^2$.

237. $3ab(a+b) + a^3 + b^3$.

238. $a^2 - x^2 - 2a - 2x$.

239. $a^{2m} + b^{2n} - 2a^mb^n$.

240. $16a^{4n} - 81b^{4n}$.

241. $x^{2m} - x^my^n - 6y^{2n}$.

242. $8a^{3n} - 27b^{3n}$.

243. $4x^{4n} + 11x^{2n}y^{2n} + 9y^{4n}$.

244. $729a^{6n} + 1000b^{3n}$.

245. $4a^{2n} + 19a^nb^n - 5b^{2n}$.

246. $a^{2n} - x^{2n} - a^n - x^n$.

To the Teacher. — Teach 139 . . . . 143.

HIGHEST COMMON DIVISOR.

Find the highest common divisor of the following:

1. $4a^3b^2$, $6a^2bc$, $2a^4b^3c$, $8a^3b^4$, and $10a^2b^2c^2$.
2. $9x^2y^3$, $6xy^4z$, $12x^3y^3z$, $3x^4y^4$, and $15x^2y^3z^4$.
3. $8a^3c^3$, $4a^2bc^4$, $16a^4b^2c^2$, $12a^4c^3$, and $20a^3b^2c^3$.
4. $3a^2b^4c^3$, $3ab^3c^3$, $4a^3b^4c^2$, $5a^2b^3c^4$, and $7a^2b^4c^3d$.
5. $10x^3y^3z^2$, $5x^4y^3z$, $15x^5y^4z^3$, $5x^3y^3z^3$, and $25a^4y^3z$.
6. $12a^2b^3x^3$, $6a^3b^3x$, $18a^4b^2x^3$, $24ab^3x^3$, and $42a^2bx^4$.
7. $35a^3b^2y^3$, $49a^2b^3y^2$ $42b^2x^2y^3$, $77a^2by^3$, and $63ab^4y^4$.
8. $44a^2b^3x^3$, $77x^2y^2z^3$, $66b^3c^2x^3$, $88a^4xy^4$, and $99x^2y^3z^4$.
9. $72a^2c^2x^3$, $63ab^4c^3$, $54a^2c^2x^2y$, $45b^3cx^3$, and $27a^3b^3c^2$.
10. $49x^4y^3z^2$, $21x^3y^4z^3$, $35ac^2x^4y^3$, $56x^5y^4z^3$, and $42x^3y^2z^3$.
11. $35a^2b^3c^2$, $40a^3b^2x^3$, $30a^2x^3y^2z$, $45b^3x^2y^3$, and $21a^2b^2c^2x$.
12. $x+y$, x^4-y^4, and x^3+y^3.
13. $ac+ad-bc-bd$ and a^4-b^4.
14. x^2-6x+9 and $x^2-7x+12$.
15. x^2-4, x^2+6x+8, and x^6+8.
16. x^2-4x-5 and $3x^2-11x-20$.
17. $x^2+2x-15$, x^3-27, and x^2-6x+9.
18. $6ax^2-6ax$, $2abx-2ab$, and $2ax-2a$.
19. $18ax^2-2a$, $54ax^3+2a$, and $18ax+6a$.
20. $12x^3-18x$, $48x^3-108x$, and $24x^4-81x$.
21. $49a^2-16$, $16-56a+49a^2$, and $63a^2-36a$.
22. $2ab+2ay-3bx-3xy$ and $9x^2-12ax+4a^2$.

111. $16x^4 - 25x^2y^2 + 9y^4$.

112. $a^{15} + a$.

113. $9x^2 - 52xy - 12y^2$.

114. $14c^2 - 47c - 7$.

115. $(a-b)^2 - 12(a-b) + 36$.

116. $m^{16} - 625$.

117. $49a^4 - 53a^2x^2 + 4x^4$.

118. $a^2 - c^2 - a - c$.

119. $x^6 + 49x^3y^3 + 48y^4$.

120. $27x^{13}y + xy^4$.

121. $ax - ay + nx - ny$.

122. $15a^2 + 2a - 45$.

123. $a^4 - 23a^2c^2 + 112c^4$.

124. $27x^3 - 1331$.

125. $81x^4 - 214x^2y^2 + 121y^4$.

126. $3x^2 + 11x - 20$.

127. $3a + 3b + a^2 + 2ab + b^2$.

128. $ax + cy - x - ay - cx + y$.

129. $(x^2 - 3xy)^2 - 8y(x^2 - 3xy) - 48y^2$.

130. $4a^4 - 9d^2 + 12bd - 12a^2 - 4b^2 + 9$.

131. $(x^2 + 4xy)^2 - 24y(x^2 + 4xy) + 144y^2$.

132. $a^2 - b^2 - 2a + 1$.

133. $x^2y + xy^2$.

134. $a + m + a^2 - m^2$.

135. $a^{16} - c^4$.

136. $a^6c^2 + 15a^4c^4 - 16a^2c^6$.

137. $64a^{11}b + 2ab^6$.

138. $6a^3 - 4a^2 - 9a + 6$.

139. $c - 25c^5 + 24c^{11}$.

140. $(a-c)^2 + (a-c) - 6$.

141. $405x^2y^5 - 5xy$.

142. $2cx - x^2 - c^2 + 1$.

143. $8a^2 - 95a - 12$.

144. $100a^4 - 205a^2c^2 + 81c^4$.

145. $125a^{16}b + ab^4$.

146. $x + 62x^4 - 128x^7$.

147. $m^3 - 64n^6$.

148. $a^2 - 2ab + b^2 - ac + bc$.

149. $x^4 - 2x^2(2x-1) + (2x-1)^2$.

150. $(x^4 - 5x^2)^2 + 8(x^4 - 5x^2) + 16$.

151. $(x^4 - 10x^2)^2 + 18(x^4 - 10x^2) + 81$.

152. $2a^3 - 2ac^2 - 3a^2b + 3bc^2 - a^2 + c^2$.

153. $(2x-1)^2-(3x+1)^2$.

154. $c^5-c^4+c^3-c^2$.

155. m^6-19m^3-216.

156. $5a-80a^5$.

157. $(2a+3)^2-(3a-4)^2$.

158. $x^3-y^3-4x+4y$.

159. $(2a+3x)^2-(3a-4x)^2$.

160. $2x^{16}y-2xy^{11}$.

161. $b^4+b^3+b^2+b$.

162. $1000x^3-1331y^6$.

163. $(5x+2y)^2-(4x-3y)^2$.

164. $a^2-b^2-2a+2b$.

165. $(7a-3c)^2-(2a+c)^2$.

166. $am-an+m^2-n^2$.

167. $\frac{1}{4}a^4+\frac{1}{8}a^3+\frac{1}{8}a^2$.

168. $4a^2+9b^2+12ab$.

169. $5a^2-8ab+3b^2-5a+3b$.

170. $(a+x)^2-1-2x(a+x-1)$.

171. $3x^2+6xy+3y^2-3xz-3yz$.

172. $(x^2-2xy)^2+2y^2(x^2-2xy)+y^4$.

173. $25+4a^2-4b^2+12bc-9c^2-20a$.

174. $(8a+5x)^2-(3a-2x)^2$.

175. $48x^3y-1875xy^3$.

176. $x^8-43x^5+42x^2$.

177. m^6+m^3-72.

178. $3c^3-c^2+3c-1$.

179. $x^{12}-y^3z^3$.

180. $(9a+3b)^2-(2a-5b)^2$.

181. $8a^3b^6+27x^9y^{12}$.

182. $9x^4-10x^2y^4+y^6$.

183. $16a^3-54b^{12}$.

184. $(x-6)^2-(x+y-6)^2$.

185. $32a^{12}-2x^6$.

186. $1-a+a^2-a^3$.

187. $5x^5y^2-5x^2y^5$.

188. $(x+y-z)^2-(x-y-z)^2$.

189. $\frac{4}{9}a^2+\frac{3}{16}-a$.

190. $ax+by-y^2-ay-bx+x^2$.

191. $a^2-b^2-c^2+2bc+a+b-c$.

192. $x^3-y^3-x(x^2-y^2)+y(x-y)^2$.

193. $(a-c)(c^3-x^3)-(c-x)(a^2-c^2)$.

194. $a^4-2a^2(3ac-2c^2)+(3ac-2c^2)^2$.

23. $9x^2-16$, $3x^2-28x+32$, and $27x^3-64$.

24. $8x^3-125$, $4x^2-25$, and $4x^2-20x+25$.

25. $8ax^2+24ax$, x^3-9, x^4-81, and x^3+27.

26. $16-a^3$, $4-a^4$, $8-a^6$, and $16a^3bc-8a^5bc$.

27. a^4-27ab^3, $(a-3b)^2$, and $a^2+3ab-18b^2$.

28. $8a^3b-8ab$, $4-8a^2+4a^4$, and a^4+a^2-2.

29. a^3-8b^3, $a^2-ab-2b^2$, and $a^2-4ab+4b^2$.

30. $6x^2+10x-4$, $9ax^2-ax$, and $9x^2-6x+1$.

31. a^6-b^6, $b^4-2a^2b^2+a^4$, and $a^4-4a^2b^2+3b^4$.

32. $12xy-3x^2y$, x^3-4x+4, and $x^2-10x+16$.

33. $1-4x+4x^2$, $8x^3-2x$, and $24x^2+18x-15$.

34. $5a^2x^2+5a^2xy$, ax^2+ay^2+2axy, and ay^3+ax^3.

35. $x^3-2x-48$, $x^3-3x-54$, and $x^3+12x+36$.

36. $a^3-a^2c-ac^2+c^3$, $4a^2-4c^2$, and $a^2+2ac+c^2$.

37. $5(x-4)^2$, $x^3+2x-24$, and $64a-32ax+4ax^2$.

38. $a^2x-a^2y-abx+aby$, a^3-abx, and $5a^4+5ab^3$.

39. $(x-y)^4$, $x^4-2x^2y^2+y^4$, and $x^3-x^2y-xy^2+y^3$.

40. $a^3+a^2b-ab^2-b^3$, $b^2-2ab+a^2$, and $8a^3-8b^3$.

41. $ax-ay-cx+cy$, a^2-c^2, and $a^3+a^2c-ac^2-c^3$.

42. $2x-16x^2+32x^3$, $48x^3-3x$, and $16x^3+4x^2-2x$.

43. $9(a-x)^4$, $3a^2-6ax+3x^2$, and $3a^6-6a^4x^4+3a^2x^4$.

44. $4a^2+12ax+9x^2$, $8a^3+27x^3$, and $6a^2+13ax+6x^2$.

45. $6a^3b^2-6a^2b^3$, $3a^3c-6a^2bc+3ab^2c$, and $6a^3b-6ab^3$.

46. $2a^3+2a^2-4a$, $3a^4+6a^3-9a^2$, and $4a^5-20a^4+16a^3$.

47. $5a^3+10a^2c$, $2a^3+8a^2c+8ac^2$, and $a^3+5a^2c+6ac^2$.

48. $10ax-2a+15cx-3c$, $25x^2-1$, and $25x^2-10x+1$.

Find the highest common divisor of the following:

1. $3x^3+9x^2+9x+3$ and $9x^3+9x^2-9x-9$.
2. x^4+x^3+x-1 and $x^4-2x^3+2x^2-2x+1$.
3. $2x^3-3x^2-4x+4$ and $3x^4-4x^3-10x+4$.
4. $2x^4+5x^3-14x^2+7x-6$ and $6x^3-39x+45$.
5. $2x^4-4x^2+2$ and $3x^4-12x^3+18x^2-12x+3$.
6. $4x^4+8x^3+x^2-x+3$ and $6x^3-3x^2-6x+18$.
7. $2x^3-8x^2+8x-6$ and $x^4-4x^3+4x^2-4x+3$.
8. $3x^3-3x^2-33x+9$ and $6x^4+6x^3-42x^2-18x$.
9. $6x^3+10x^2-3x-5$ and $2x^4-2x^3-3x^2+x+1$.
10. $2a^5b-4a^4b^2+4a^3b^3-2a^2b^4$ and $a^4b-2a^3b^2+a^2b^3$.
11. $ax^6-ax^5-ax^4-ax^3+2ax^2$ and $a^2x^4-2a^2x^3+a^2x$.
12. $3x^4-4x^3+2x^2+x-2$ and $3x^4+8x^3-5x^2-6x$.
13. $2a^4-2a^3+4a^2+2a+6$ and $3a^4+6a^3-3a-6$.
14. $2a^4-3a^3+2a^2-2a-3$ and $4a^4+3a^2+4a-3$.
15. $2x^3+6x^2-7x-21$ and $2x^4+2x^3-9x^2-7x+7$.
16. $2x^4+8x^3+12x^2+8x+2$ and $7x^3-7x^2-7x+7$.
17. $8x^4-6x^3+3x^2-3x+1$ and $18x^3-3x^2-15x+6$.
18. $3a^4-2a^3b+2a^2b^2-2ab^3-b^4$ and $6a^3+5a^2b+ab^2$.
19. $3x^4-7x^3y+3x^2y^2-4y^4$ and $2x^3-5x^2y+3xy^2-2y^3$.
20. $4a^4-4a^3x-a^2x^2-3ax^3$ and $2a^3-5a^2x+5ax^2-3x^3$.
21. $3x^4+2x^3+4x^2+x+2$ and $2x^4y+5x^3y+5x^2y+3xy$.
22. $2a^4-7a^3+9a^2-8a-5$ and $6a^4-11a^3-16a^2+15a$.
23. $x^6+5x^5+10x^4+10x^3+5x^2+x$ and $x^4y+x^3y-x^2y-xy$.
24. $x^5-2x^4y+3x^3y^2-3x^2y^3+2xy^4-y^5$ and $x^4-x^3y^2+2xy^3-y^4$.
25. $x^6-x^5+x^4-5x^3-5x^2+6x$ and $x^5-x^4-x^3-3x^2+2x$.

26. $ax^5 + ax^4 - ax^3 - ax^2$ and $x^4y - 3x^2y - 2xy$.

27. $3x^4y - x^3y - 2x^2y$ and $3x^4 + 5x^3 + 2x^2 + 3x + 2$.

28. $3x^4 - 5x^3 + 7x^2 - 3x - 2$ and $6x^3 + 5x^2 - 8x - 3$.

29. $2x^4 - 3x^3 - 3x^2 - 3x - 5$ and $6x^3 - 17x^2 + x + 10$.

30. $3a^4 + 6a^3x + 6a^2x^2 + 3ax^3 - 6x^4$ and $a^3 - 2ax^2 + x^3$.

31. $2x^7 - 7x^6 + 2x^5 + 2x^4 + 3x^3$ and $x^5 - 2x^4 - 4x^3 + 3x^2$.

32. $4x^4 - 2x^3 - 4x^2 - 4x - 2$ and $15x^4 - 15x^3 - 30x - 15$.

33. $x^4 - x^3y - 3x^2y^2 + xy^3 + 2y^4$ and $x^3 - 3x^2y + 3xy^2 - 2y^3$.

34. $2x^5 + x^4y + x^3y^2 - xy^4 + y^5$ and $3x^3 + 3x^2y + 2xy^2 + 2y^3$.

35. $a^7 + 2a^6x + 2a^5x^2 + 2a^4x^3 + 2a^3x^4 + a^2x^5$ and $2a^4 - a^3x - 3a^2x^2$.

TO THE TEACHER. — Teach 149 . . . . 154.

LOWEST COMMON MULTIPLE.

Find the lowest common multiple of the following:

1. $2abc$, $6ab^2c$, $3a^2b^2c$, $4ab^3$, and a^2bc.
2. $3x^2yz$, $2xy^2z$, $6x^3yz^2$, $9xy$, and xy^2z.
3. $3a^3cx$, $4bcx$, $9a^2cx^2$, $2b^2c$, and ac^3x.
4. $4bcx^2$, $3cxy$, $8b^2x^3y$, $2b^2x^2$, and bcx.
5. $4abx^2$, $2b^2xy$, $5a^2x^2y$, $4ax^2$, and bxy.
6. $5ac^2x$, $2acx$, $3a^2c^2x$, $6c^3y^2$, and axy.
7. $6bc^2x^2$, $4c^2x^3y^2$, bc^3x^2, $16b^2x$, and $bcxy$.
8. $2bc^2x^3$, $4bc^3x^2$, $16b^2cx^3$, bx^4, and abc^2x.
9. $5cx^2y^2$, $3cx^2y^3$, $2b^3xy^2$, $6b^2c^2$, and $bcxy$.
10. $2a^2b^2c$, $5a^2b^3c$, $8a^3bc^2$, $4a^4b^2$, and $abcd$.
11. $4ab^2c^2$, $2bc^4x$, $8ab^3x^3$, $7ax^5$, and a^2b^2cx.

12. x^2-1 and x^3-1.

13. $1-x^3$ and $1+x^2$.

14. a^2-x^2 and a^2+ax.

15. x^2-1 and x^2-3x-4.

16. $4a^2-4b^2$ and $6a-6b$.

17. $5(x^4-y^4)$ and $2(x^2+y^2)$.

18. $x^2-10x+16$ and x^3-8.

19. x^2-y^2 and $x^2-2xy+y^2$.

20. $x^2+2xy+y^2$ and x^2-y^2.

21. x^2+1, x^3-1, and x^4-1.

22. x^3+x, x^2-1, and x^3+1.

23. x^2-16 and $x^2+14x+40$.

24. $2a(a^2-c^2)$ and $3c(a-c)^2$.

25. $6+2x$, x^2-9, and $27+x^3$.

26. x^2-x-20 and x^2+x-12.

27. a^2-2a+1 and a^2-5a+4.

28. a^2+5a+6 and a^2+6a+8.

29. $x^2-6x-16$ and $x^2-11x+24$.

30. $4x^2-16$, $6x-12$, and $8x^3-64$.

31. $x^2+8ax+15a^2$ and $x^2-5ax-24a^2$.

32. $ax-4a+bx-4b$ and $a^2-3ab-4b^2$.

33. $2a^2+2ab$, $3ab-3b^2$, and $4a^2c-4b^2c$.

34. x^2-16, $x^2-8x+16$, and $x^2+8x+16$.

35. $2x^2-2$, $4x-4$, $8x+8$, and $12x^2+12$.

36. x^3-2x^2+4x-8 and x^3+2x^2-4x-8.

37. $x^3+x^2y-xy^2-y^3$ and $x^3-x^2y+xy^2-y^3$.

38. x^2+4-5x and $3-4x+x^2$.
39. $12-7x+x^2$ and x^2+x-20.
40. c^2+4c+4, c^2-4, and c^4-16.
41. a^4+2a^3, a^2-4a+4, and a^2-4.
42. $1-2x+x^2$, $1+2x+x^2$, and x^2-1.
43. $2a(x^2+y^2)$, $3c(x+y)$, and $ac(x-y)$.
44. x^4+4x^2+4, $4-4x^2+x^4$, and x^4-4.
45. $5a^3(a^2-x)$, $3a^2(a^2+x)$, and $2a^4(a^4+x^2)$.
46. x^2+2x+1, $9-6x+x^2$, and x^2-2x-3.
47. a^2-4, a^3-a-6, and $a^3-3a^2-4a+12$.
48. $x^2+2x-15$, $x^2-9x+18$, and x^2-x-30.
49. $(a-b)(a-c)$, $(a-c)(b-c)$, and $(a-b)(b-c)$.
50. $a(a+b)(b+c)$, $b(a+c)(b+c)$, and $c(a+b)(a+c)$.

To the Teacher.—Teach 15 .

Find the lowest common multiple of the following:

1. x^3+3x^2+4x+2 and $2x^3+x^2+1$.
2. $2a^4-2a^3-2a-2$ and a^4-2a^3+a.
3. $x^3-4x^2+56x-15$ and $x^3-15x+4$.
4. $6x^3+10x^2+8x+4$ and $6x^3-2x^2-4$.
5. x^4-11x^2+25 and $x^4-x^3-6x^2+x+5$.
6. $6x^3-x^2-7x-2$ and $6x^3+5x^2-5x-2$.
7. $2x^3-5x^2-20x+9$ and $2x^3+x^2-43x-9$.
8. $2x^3+5x^2-8x-15$ and $4x^3-4x^2-9x+5$.
9. $x^3-10x^2+33x-36$ and $x^3-2x^2-23x+60$.
10. $6x^3-8x^2-17x-6$ and $12x^3-x^2-21x-10$.
11. $3x^3+17x^2-44x-28$ and $6x^3-5x^2-33x+28$.

To the Teacher.—Teach 1 . . . . 174.

FRACTIONS.

Reduce the following fractions to their lowest terms:

1. $\frac{4a^2b^2c}{8a^3b^2c}$.
2. $\frac{15b^3x^3y^3}{21b^3x^2y^3}$.
3. $\frac{3a^3b^3x^3y}{6a^4b^3x^2y}$.
4. $\frac{3a^3b^2c}{9a^2bc^2}$.
5. $\frac{18b^4x^2y^3}{27b^3x^3y^3}$.
6. $\frac{8a^2b^3xy^3}{4ab^2x^2y^2}$.
7. $\frac{6a^2b^4c}{8a^3b^2c}$.
8. $\frac{32x^2y^3z^3}{24b^2x^2y^3}$.
9. $\frac{4b^4x^2y^3z}{6ab^3x^3y^3}$.
10. $\frac{6a^3cx^2}{9a^2b^2x}$.
11. $\frac{23a^3x^4y^3}{46x^3y^3z^3}$.
12. $\frac{2a^3b^3xy^3}{3b^2xy^3z^3}$.
13. $\frac{2a^2b^2c}{4a^2b^3c}$.
14. $\frac{38b^3x^2y^4}{19b^4x^3y^3}$.
15. $\frac{4a^2b^3xy^3}{2a^3b^3x^2y}$.
16. $\frac{3a^3cx^3}{5a^2c^2x}$.
17. $\frac{35x^2y^3z^3}{42b^2x^3y^3}$.
18. $\frac{3a^3b^3x^3y}{9b^2x^4yz^2}$.
19. $\frac{3a^2b^2c}{9a^3bc^2}$.
20. $\frac{34x^2y^2z^3}{17x^2y^2z^4}$.
21. $\frac{6bx^2y^2z^3}{3a^2bx^2y^2}$.
22. $\frac{2ac^2x^3}{6a^2b^2c}$.
23. $\frac{25b^3x^3y^4}{30b^2x^2y^3}$.
24. $\frac{4a^3bx^2y^3}{7bx^3y^2z^3}$.
25. $\frac{8a^3b^2c}{2a^2cx^3}$.
26. $\frac{21a^2x^3y^3}{42x^2y^4z^3}$.
27. $\frac{9b^2x^4y^3z}{3ab^3x^3y^3}$.
28. $\frac{6a^2cx^3}{3a^3c^2x}$.
29. $\frac{46a^3b^2x^4}{23b^2x^3y^3}$.
30. $\frac{6a^3b^4x^2y}{8a^2b^3x^3y}$.
31. $\frac{9a^3b^5c}{6a^2b^2c}$.
32. $\frac{16a^2b^3x^2}{24b^2x^3y^3}$.
33. $\frac{6b^2x^3yz^3}{9b^3x^4yz^3}$.
34. $\frac{8ac^2x^3}{6a^2c^2x}$.
35. $\frac{54b^2x^3y^4}{27b^3x^3y^5}$.
36. $\frac{5x^2y^3z^2u}{7ax^2y^4z^3}$.
37. $\frac{4a^3bc^2}{3a^2bc^3}$.
38. $\frac{29b^2x^2y^3}{58b^3x^2y^4}$.
39. $\frac{4a^2b^4x^3y}{6b^3x^3yz^2}$.

40. $\frac{x-1}{x^2-1}$.

41. $\frac{a^2-1}{a^2-2a+1}$.

42. $\frac{x^2-1}{x^3-1}$.

43. $\frac{1-a^2}{a^2-2a+1}$.

44. $\frac{x^2+1}{x^4-1}$.

45. $\frac{a^2-1}{a^2+2a+1}$.

46. $\frac{x^2-1}{x^3+1}$.

47. $\frac{x^2-1}{x^2-3x+2}$.

48. $\frac{x^2-1}{x^6-1}$.

49. $\frac{a^2-4}{a^2+3a+2}$.

50. $\frac{x^2-y^2}{x^3-y^3}$.

51. $\frac{a^3-a}{a^2-2a+1}$.

52. $\frac{x^2+y^2}{x^4-y^4}$.

53. $\frac{3x^2-3}{x^2+2x+1}$.

54. $\frac{x^2-y^2}{x^3+y^3}$.

55. $\frac{3x^2+6x}{x^2+4x+4}$.

56. $\frac{x^2-y^2}{x^6-y^6}$.

57. $\frac{a^2-b^2}{a^2-2ab+b^2}$.

58. $\frac{x^2-y^2}{(x+y)^2}$.

59. $\frac{a^2-x^2}{a^2+2ax+x^2}$.

60. $\frac{x^3+y^3}{(x+y)^3}$.

61. $\frac{a^3-ax^2}{a^2-2ax+x^2}$.

62. $\frac{(x+y)^3}{(x^2-y^2)^2}$.

63. $\frac{x^3-y^3}{x^2-2xy+y^2}$.

64. $\frac{(x+y)^2}{(x^2-y^2)^2}$.

65. $\frac{x^3+y^3}{x^2+2xy+y^2}$.

66. $\frac{(x-y)^3}{(x^2-y^2)^2}$.

67. $\frac{x^3-a^2x}{a^2+2ax+x^2}$.

68. $\frac{x^2-1}{x^4-1}$.

69. $\frac{x^2+x-3}{x^2+6x+9}$.

70. $\frac{x^4-1}{x^6+1}$.

71. $\frac{x^2-4x+4}{x^2-5x+6}$.

72. $\frac{x^6-1}{x^3-1}$.

73. $\frac{x^2+2x+1}{x^2-3x-4}$.

74. $\frac{x^3+1}{x^9+1}$.

75. $\frac{a^2-16}{a^2-2a-8}$.

76. $\frac{a^4-x^4}{a^6-x^6}$.

77. $\frac{x^2+3x+2}{x^2+6x+5}$.

78. $\frac{a^6+x^6}{a^8-x^8}$.

79. $\frac{x^2+2x-3}{x^2+4x+3}$.

80. $\frac{a^3-x^3}{a^9-x^9}$.

81. $\frac{a^2+2a-3}{a^2+5a+6}$.

82. $\frac{2x^2}{6x^2-2x}$.

83. $\frac{x^2-5x+6}{x^2+x-12}$.

84. $\frac{2a+2}{3a^2+3a}$.

85. $\frac{x^2-5x}{x^2-4x-5}$.

86. $\frac{a^2-a}{5a^2-5a}$.

87. $\frac{x^3+27}{x^2-2x-15}$.

88. $\frac{2x+3}{8x^3+27}$.

89. $\frac{3x^3y-9x^2y}{x^2-8x+15}$.

90. $\frac{x^2-4y^2}{(x-2y)^2}$.

91. $\frac{ax^4+64ax}{x^2-4x+16}$.

92. $\frac{a^2-x^2}{3a^2+3ax}$.

93. $\frac{x^2+x-90}{x^2+8x-20}$.

94. $\frac{4a^2-9x^2}{4a^2+6ax}$.

95. $\frac{a^2+5a+6}{a^2-4a-12}$.

96. $\frac{a^2+a}{2b+2ab}$.

97. $\frac{a^2+a-12}{2a^2+3a-20}$.

98. $\frac{a^2-1}{2ax+2x}$.

99. $\frac{a^2-a-20}{2a^2-7a-15}$.

100. $\frac{3x^2-6x}{2xy-4y}$.

101. $\frac{6x^2-5x-6}{8x^2-2x-15}$.

102. $\frac{3a^4x^4+3}{4a^3x^3-4}$.

103. $\frac{x^2+2x-24}{x^2-12x+32}$.

104. $\frac{6a^3-4ab}{9a^2b-6b^2}$.

105. $\frac{a^2-3a-28}{a^2-11a+28}$.

106. $\frac{2x^3-4x^2y}{4xy^2-8y^3}$.

107. $\frac{x^2+x-72}{x^2-10x+16}$.

108. $\frac{6a^3+3a^2b}{3b^3+6ab^2}$.

109. $\frac{x^2-2x-15}{x^2+10x+21}$.

110. $\frac{2a^3+2a^2b}{6b^3+6ab^2}$.

111. $\frac{2a^2+a-15}{2a^2-19a+35}$.

112. $\frac{2x^3+6x^2y}{9y^4+3xy^3}$.

113. $\frac{3a^2+23a-36}{4a^2+33a-27}$.

114. $\frac{a^3-x^3}{7a^3+7a^2x}$.

115. $\frac{6x^2+x-15}{6x^2-11x-35}$.

116. $\frac{6xy^2+6y^3}{3y^4+3x^3y}$.

117. $\frac{6a^2-11a-10}{6a^2-19a+10}$.

118. $\frac{6x^4+12x^3}{16x^2+2x^4}$.

119. $\frac{a^2-8a+15}{2a^2-13a+21}$.

120. $\frac{4x^4y-4x^3y}{2x^5y-2x^3y}$.

121. $\frac{2x^2+17x+21}{3x^2+26x+35}$.

122. $\frac{x^4y+x^3y}{3x^3y+3x^5y}$.

123. $\frac{8x^3+27y^3}{9y^2+12xy+4x^2}$.

To the Teacher. — Teach 175.

Reduce these fractions to entire or mixed quantities:

1. $\frac{a^3}{a+x}$.
2. $\frac{x^4+1}{x+1}$.
3. $\frac{a^3+x^3}{a+x}$.
4. $\frac{3x^3}{x-y}$.
5. $\frac{a^5-1}{a-1}$.
6. $\frac{x^3+y^3}{x-y}$.
7. $\frac{a+x}{a-x}$.
8. $\frac{x^5-1}{x+1}$.
9. $\frac{a^3-x^3}{a-x}$.
10. $\frac{a-x}{a+x}$.
11. $\frac{x^5+1}{x-1}$.
12. $\frac{x^5-y^5}{x+y}$.
13. $\frac{x^3+1}{x}$.
14. $\frac{a^5+1}{a+1}$.
15. $\frac{x^4-y^4}{x-y}$.
16. $\frac{a^4-1}{a^2}$.
17. $\frac{x^2-4}{x-2}$.
18. $\frac{a^4-x^4}{a+x}$.
19. $\frac{x^3-1}{x-1}$.
20. $\frac{a^3-4}{a+2}$.
21. $\frac{x^4+y^4}{x-y}$.
22. $\frac{a^3-1}{a+1}$.
23. $\frac{x^3+4}{x-2}$.
24. $\frac{a^4+x^4}{a+x}$.
25. $\frac{x^3-1}{x-1}$.
26. $\frac{x^3+4}{x+2}$.
27. $\frac{a^5-x^5}{a-x}$.
28. $\frac{a^3+1}{a+1}$.
29. $\frac{x^3-8}{x-2}$.
30. $\frac{a^5-x^5}{a+x}$.
31. $\frac{x^5+1}{x-1}$.
32. $\frac{a^3-8}{a+2}$.
33. $\frac{x^5+y^5}{x-y}$.
34. $\frac{a^4-1}{a-1}$.
35. $\frac{x^3+8}{x-2}$.
36. $\frac{a^5+x^5}{a+x}$.
37. $\frac{x^4+1}{x-1}$.
38. $\frac{a^3+8}{a+2}$.
39. $\frac{x^9+y^3}{x^3-y}$.

40. $\frac{15a^2+5a-1}{5a}$.

41. $\frac{a^2+3a-10}{a+2}$.

42. $\frac{12x^3-4x+5}{2x}$.

43. $\frac{x^2-7x+12}{x-3}$.

44. $\frac{9a^2+15a-4}{3a}$.

45. $\frac{x^2-5x-11}{x+5}$.

46. $\frac{6x^3-18x-2}{6x}$.

47. $\frac{a^2-9a-10}{a+1}$.

48. $\frac{16a^3+8a-3}{4a}$.

49. $\frac{x^2-5x-12}{x-3}$.

50. $\frac{4x^4-16x+7}{2x}$.

51. $\frac{a^2+8a+15}{a+5}$.

52. $\frac{18a^2+6a-3}{3a}$.

53. $\frac{x^2+9x-20}{x+4}$.

54. $\frac{24x^3-6y-5}{6x}$.

55. $\frac{a^2-6a-27}{a+3}$.

56. $\frac{4a^3+20a+2}{4a}$.

57. $\frac{4x^2-3x+2}{x+2}$.

58. $\frac{5x^3-15y-4}{5x}$.

59. $\frac{2a^2-9a+9}{a-3}$.

60. $\frac{35a^2+7a-3}{7a}$.

61. $\frac{5x^2-9x-8}{x-4}$.

62. $\frac{24x^3-6y-2}{3x}$.

63. $\frac{4a^2+7a-2}{a+2}$.

64. $\frac{2a^2+18a+3}{2a}$.

65. $\frac{8x^2+2x-9}{4x-3}$.

66. $\frac{30x^3-5y-4}{5x}$.

67. $\frac{6a^2+9a+6}{2a+5}$.

To the Teacher. — Teach 176.

Reduce these mixed quantities to the fractional form:

1. $x+1+\frac{4x+3}{3x}$.
2. $\frac{4a-3x}{5}-3a+2x$.
3. $a-2-\frac{2a-3}{2a}$.
4. $\frac{2x+5y}{4}-2x-3y$.
5. $x-3+\frac{5x+2}{4x}$.
6. $3a-5b-\frac{6a-4b}{3}$.
7. $a-5-\frac{3a+4}{a-5}$.
8. $\frac{7x+4y}{2}-3x-4y$.
9. $x+2+\frac{4x-5}{x+2}$.
10. $\frac{2a-6x}{5}-3a+2x$.
11. $a-4-\frac{a-16}{a+4}$.
12. $5x+4y-\frac{8x+7y}{4}$.
13. $x+5-\frac{x-25}{x-5}$.
14. $\frac{9a-8b}{6}-3a-2b$.
15. $a+x-\frac{a^2+x^2}{a+x}$.
16. $\frac{8x^2+7y^2}{3x-2y}-2x-3y$.
17. $x-y+\frac{x^2+y^2}{x+y}$.
18. $2a-4x-\frac{6a^2-9x^2}{4a+3x}$.
19. $a-x-\frac{a^2+x^2}{a-x}$.
20. $\frac{6x^2-8y^2}{5x-3y}-3x-4y$.
21. $x+y+\frac{x^2+y^2}{x-y}$.
22. $\frac{4a^2+9x^2}{3a-2x}-4a+3x$.
23. $a+c-\frac{2ac+c^2}{a+c}$.
24. $3x-2y-\frac{8x^2-7y^2}{2x-3y}$.
25. $c+d^2-\frac{ac+bd^2}{a+b}$.
26. $\frac{8a^2-7b^2}{4a+6b}-3a-4b$.

To the Teacher. — Teach 177.

Reduce these fractions to a common denominator:

1. $\frac{2a^2}{3}, \frac{3ax}{2}, \frac{5xy}{6}.$

2. $\frac{a+1}{a}, \frac{a-1}{b}, \frac{a^2-1}{c}.$

3. $\frac{5ac}{b}, \frac{3x^2}{a}, \frac{4ax}{c}.$

4. $\frac{3}{x-1}, \frac{2}{x+1}, \frac{5}{x^2-1}.$

5. $\frac{4}{3ab}, \frac{3}{4a^2}, \frac{b}{6ax}.$

6. $\frac{x+2}{a}, \frac{x-5}{3}, \frac{x^2+4}{b}.$

7. $\frac{5a^2}{3}, \frac{4ab}{c}, \frac{2xy}{a}.$

8. $\frac{a}{x-3}, \frac{b}{x+3}, \frac{c}{x^2-9}.$

9. $\frac{a}{3bx}, \frac{4}{3a^2}, \frac{5ab}{2}.$

10. $\frac{4}{2-x}, \frac{5}{2+x}, \frac{3}{4-x^2}.$

11. $\frac{4ax}{3}, \frac{a}{3x^2}, \frac{4bx}{c}.$

12. $\frac{a-1}{a}, \frac{b-2}{b}, \frac{x^2-4}{c}.$

13. $\frac{5}{3ax}, \frac{a}{2b^2}, \frac{c}{6ab}.$

14. $\frac{x-2}{x+2}, \frac{x+2}{x-2}, \frac{x^2+4}{x^2-4}.$

15. $\frac{2x^2}{b}, \frac{3bx}{2}, \frac{4}{3bc}.$

16. $\frac{5}{a-x}, \frac{3ac}{a^2-x^2}, \frac{7}{a+x}.$

17. $\frac{3}{5ax}, \frac{b}{2x^2}, \frac{3bx}{5}.$

18. $\frac{x-y}{x+y}, \frac{x+y}{x-y}, \frac{x^2+y^2}{x^2-y^2}.$

19. $\frac{3xy}{2}, \frac{3}{2x^2}, \frac{a}{4xy}.$

20. $\frac{a+b}{3}, \frac{a^2-x^2}{2a}, \frac{a-4}{6}.$

21. $\frac{x}{3y^2}, \frac{4}{3xy}, \frac{b}{2ax}.$

22. $\frac{a}{x+2}, \frac{b}{2x+4}, \frac{c}{4x+8}.$

23. $\frac{5ax}{b}, \frac{3b^2}{a}, \frac{5}{3ax}.$

24. $\frac{x+2}{x^2-3x-4}, \frac{x-1}{x^2-2x-8}.$

25. $\frac{x}{3ab}, \frac{y}{4x^2}, \frac{z}{2bc}.$

26. $\frac{x+3}{x^2-5x+6}, \frac{x-2}{x^2-6x+9}.$

To the Teacher. — Teach 178 . . . . 184.

ADDITION AND SUBTRACTION OF FRACTIONS.

1. $\frac{a+x}{4}+\frac{a-x}{5}$.

2. $\frac{a+b}{2}+\frac{b+c}{3}+\frac{a+c}{2}$.

3. $\frac{a+b}{a}+\frac{b-x}{b}$.

4. $\frac{x+y}{4}+\frac{x-y}{2}-\frac{x+y}{8}$.

5. $\frac{a-x}{3}-\frac{a+x}{5}$.

6. $\frac{a+x}{3}+\frac{a-x}{6}+\frac{x-a}{2}$.

7. $\frac{1}{1+x}+\frac{1}{1-x}$.

8. $\frac{x-y}{2}-\frac{x-y}{3}-\frac{x+y}{4}$.

9. $\frac{1}{x+3}-\frac{1}{x+8}$.

10. $\frac{5}{a+x}+\frac{3}{a-x}+\frac{4}{a+x}$.

11. $\frac{a+4}{3}+\frac{5-a}{5}$.

12. $\frac{x}{x-y}+\frac{y}{x+y}+\frac{x-y}{x+y}$.

13. $\frac{x+3}{3}-\frac{4-x}{6}$.

14. $\frac{x+8}{2}-\frac{x+9}{3}-\frac{x+4}{4}$.

15. $\frac{a}{a+b}+\frac{b}{a-b}$.

16. $\frac{5}{a+x}+\frac{8}{a-x}-\frac{3x}{a^2-x^2}$.

17. $\frac{x+y}{4}-\frac{x+4}{5}$.

18. $\frac{x+y}{x-y}+\frac{x}{x+y}-\frac{2x^2}{x^2-y^2}$.

19. $\frac{1}{a-b}+\frac{1}{a+b}$.

20. $\frac{a+x}{a}+\frac{a-x}{x}+\frac{a^2-x^2}{ax}$.

21. $\frac{a+x}{a-x}-\frac{a-x}{a+x}$.

22. $\frac{x}{x-y}-\frac{y}{x+y}-\frac{y^2}{x^2-y^2}$.

23. $\frac{x-2}{x+2}-\frac{x+2}{x-2}$.

24. $\frac{4}{x+4}+\frac{4}{x-4}-\frac{8x}{x^2-16}$.

25. $\frac{a+b}{a-b}+\frac{a}{a+b}$.

26. $\frac{x+5}{x-5}-\frac{x-5}{x+5}-\frac{19x+5}{x^2-25}$.

To the Teacher. — Teach 185.

1. $\frac{4a^4}{a^2+x}+\frac{4a^2x}{a^2-x}$.

2. $\frac{a+x}{4}-\frac{a-x}{4}-\frac{2x^2}{a+x}$.

3. $\frac{1+x^2}{1-x^2}-\frac{1-x^2}{1+x^2}$.

4. $\frac{2}{x-1}-\frac{3}{x+1}-\frac{x^2-3}{1-x^2}$.

5. $\frac{x^2+1}{x^2-1}+\frac{4x^2}{1-x^4}$.

6. $\frac{5}{x-2}-\frac{4}{2+x}+\frac{16}{4-x^2}$.

7. $\frac{1}{x^2-3}-\frac{1}{x^2+2}$.

8. $\frac{x+3}{3-x}-\frac{x-3}{x+3}+\frac{8x}{x^2-9}$.

9. $\frac{3}{x^2-2}+\frac{3x^2}{4-x^4}$.

10. $\frac{5}{a+x}-\frac{8}{a-x}+\frac{3x}{x^2-a^2}$.

11. $\frac{x^2+4}{x^2+3}-\frac{x^2+3}{x^2+2}$.

12. $\frac{a+x}{a-x}-\frac{a}{a+x}+\frac{x^2}{x^2-a^2}$.

13. $\frac{2}{x^2+3x}+\frac{6x}{x^2-9}$.

14. $\frac{x}{x-y}-\frac{y}{x+y}-\frac{y^2}{y^2-x^2}$.

15. $\frac{1}{2x-x^2}-\frac{1}{x^2-4}$.

16. $\frac{3}{x+4}-\frac{3}{x-4}-\frac{24}{16-x^2}$.

17. $\frac{4x-1}{2x+2}+\frac{6x+2}{3+3x}$.

18. $\frac{x-5}{x+5}+\frac{x+5}{x-5}+\frac{x^2+45}{25-x^2}$.

19. $\frac{3x+1}{3x-3}+\frac{2x-3}{4-4x}$.

20. $\frac{x^2+x+1}{x+1}+\frac{x^2-x+1}{x-1}$.

21. $\frac{x}{6+3x}+\frac{2}{2x+4}$.

22. $\frac{x-1}{x^2-x+1}-\frac{1+x}{x^2+x+1}$.

23. $\frac{4x+3}{5-5x}-\frac{2x-1}{3x-3}$.

24. $\frac{1+x}{1+x+x^2}-\frac{1-x}{1-x+x^2}$.

25. $\frac{1}{a^2+ax}-\frac{1}{x^2+ax}$.

26. $\frac{a+3}{a^2+2a-8}-\frac{a-3}{a^2+a-6}$

27. $\frac{2}{a^2-ab}+\frac{3}{b^2-ab}$.

28. $\frac{5}{1-2x}-\frac{4}{2x+1}+\frac{16x}{4x^2-1}$.

29. $\frac{a+c}{c}-\frac{a}{a+c}-\frac{a(a^2-c)}{c(a^2-c^2)}$.

30. $\frac{y}{x+y}-\frac{y^2}{(x+y)^2}-\frac{x^2y}{(x+y)^3}$.

31. $\frac{4}{x-y}+\frac{4y}{(x-y)^2}+\frac{4xy}{(x-y)^3}$.

32. $\frac{2}{(a+x)^2}+\frac{2}{(a-x)^2}+\frac{4}{x^2-a^2}$.

33. $\frac{2}{x(x+2)}+\frac{3}{x(x-2)}-\frac{4}{x^2-4}$.

34. $\frac{2}{x-y}-\frac{x+y}{x^2+xy+y^2}+\frac{4xy}{y^3-x^3}$.

35. $\frac{x-2}{x+3}+\frac{x+4}{x-5}-\frac{x^2+31}{x^2-2x-15}$.

36. $\frac{a-x}{a^2-ax+x^2}-\frac{1}{a+x}+\frac{3a^2}{x^3+a^3}$.

37. $\frac{a-x}{a-b}+\frac{a-b}{a-x}-\frac{(a-b)^2}{(a-x)(a-b)}$.

38. $\frac{x+4}{x^2+x+1}-\frac{1}{x-1}-\frac{x^2+4x-2}{1-x^3}$.

39. $\frac{x^2+3x+5}{x^3+1}+\frac{2}{x+1}-\frac{3x+6}{x^3-x+1}$.

40. $\frac{x+4}{x^2-3x+9}+\frac{x^2-4x+9}{x^3+27}-\frac{2}{x+3}$.

41. $\frac{3}{x-2}-\frac{x+6}{x^2+2x+4}+\frac{x^2+2x+28}{8-x^3}$.

42. $\frac{1}{a+2x}+\frac{a-2x}{a^2-2ax+4x^2}+\frac{4ax}{a^3+8x^3}$.

43. $\dfrac{3(x+1)}{x^2-8x-20}-\dfrac{x-4}{x^2-18x+80}-\dfrac{x+5}{x^2-6x-16}$.

44. $\dfrac{2(x-3)}{x^2+6x-16}-\dfrac{x-2}{x^2+4x-12}+\dfrac{2}{x^2+14x+48}$.

45. $\dfrac{a^2+ac+c^2}{(a-b)(b-c)}-\dfrac{a^2+ab+b^2}{(a-c)(c-b)}-\dfrac{b^2+bc+c^2}{(a-c)(b-a)}$.

46. $\dfrac{1}{(a-b)(a-c)}+\dfrac{1}{(a-b)(b-c)}+\dfrac{1}{(a-c)(b-c)}$.

47. $\dfrac{3}{(a-b)(b-c)}-\dfrac{4}{(a-c)(a-b)}-\dfrac{3}{(c-a)(c-b)}$.

48. $\dfrac{a+c}{(a-3)(3-c)}-\dfrac{a+3}{(a-c)(c-3)}-\dfrac{c+3}{(a-c)(3-a)}$.

49. $\dfrac{a+1}{(a-b)(a-c)}+\dfrac{b+1.}{(b-a)(b-c)}-\dfrac{c+1}{(c-a)(b-c)}$.

50. $\dfrac{a}{(a-b)(a-c)}+\dfrac{b}{(b-c)(b-a)}+\dfrac{c}{(c-a)(c-b)}$.

51. $\dfrac{a^3}{(a-b)(a-c)}-\dfrac{b^3}{(b-a)(c-b)}-\dfrac{c^3}{(c-a)(b-c)}$.

52. $\dfrac{c}{(a-b)(a-c)}-\dfrac{c}{(c-b)(b-a)}-\dfrac{c}{(b-c)(c-a)}$.

53. $\dfrac{a+c}{(a-b)(b-c)}-\dfrac{b+c}{(c-a)(b-a)}-\dfrac{a+b}{(a-c)(c-b)}$.

54. $\dfrac{x^2-yz}{(x-y)(x-z)}+\dfrac{y^2+xz}{(y-x)(y+z)}+\dfrac{z^2+xy}{(z-x)(z+y)}$.

55. $\dfrac{x-2}{(x-1)(x-3)}+\dfrac{x-1}{(x-2)(3-x)}+\dfrac{x-3}{(1-x)(2-x)}$.

56. $\dfrac{1}{a(a-c)(a-x)}+\dfrac{1}{c(c-a)(c-x)}+\dfrac{1}{x(x-c)(x-a)}$.

To the Teacher. — Teach 186 and 187.

MULTIPLICATION OF FRACTIONS.

1. $\frac{4ab}{3cd} \times \frac{2ac}{5bd} \times \frac{5bd}{8ac}$.

2. $\frac{x-1}{a} \times \frac{x+1}{b}$.

3. $\frac{6ax}{5by} \times \frac{4cy}{3ad} \times \frac{5bd}{2cx}$.

4. $\frac{x+1}{4} \times \frac{8a}{x^2-1}$.

5. $\frac{9ac}{8bd} \times \frac{2by}{3cx} \times \frac{4bx}{3ay}$.

6. $\frac{1-x}{b} \times \frac{ac}{1-a^2}$.

7. $\frac{2bc}{3ax} \times \frac{9ax}{4bc} \times \frac{2xy}{5ab}$.

8. $\frac{1+x}{a-1} \times \frac{a-1}{1-x^2}$.

9. $\frac{3ab}{4xy} \times \frac{2bc}{5ax} \times \frac{6ad}{7bx}$.

10. $\frac{a+2}{a-1} \times \frac{a^2-1}{a^2-4}$.

11. $\frac{4xy}{3ab} \times \frac{9a^2}{8cy} \times \frac{2bc}{3ax}$.

12. $\frac{a+x}{a-c} \times \frac{a^2-c^2}{a^2+ax}$.

13. $\frac{4ab}{7x^2} \times \frac{5cy}{8bd} \times \frac{6x^3}{7ay}$.

14. $\frac{x+y}{x-y} \times \frac{x^2-y^2}{(x+y)^2}$.

15. $\frac{2c^3}{5ab} \times \frac{3bd}{4ac} \times \frac{5a^2}{6cd}$.

16. $\frac{x^2-9}{x^2+4x} \times \frac{x^2-16}{x^2-3x}$.

17. $\frac{5x^4}{3ad} \times \frac{9ab}{5x^2} \times \frac{7cy}{6bx}$.

18. $\frac{3x^2+x}{x-2} \times \frac{x-1}{2x+6x^2}$.

19. $\frac{2a^3}{7bx} \times \frac{3bx}{8ay} \times \frac{5cd}{6ax}$.

20. $\frac{a-x}{a^2+2ax} \times \frac{a^2-4x^2}{a^2-ax}$.

21. $\frac{4x^4}{9bc} \times \frac{2ac}{3x^2} \times \frac{6b^3}{5ax}$.

22. $\frac{a^2-b^2}{a^2-9x^2} \times \frac{3ax+a^2}{b^2+ab}$.

23. $\frac{8bx}{9a^2} \times \frac{3a^4}{2xy} \times \frac{3cy}{4a^2}$.

24. $\frac{(a+b)^2}{a+2b} \times \frac{a^3+8b^3}{(a^2-b^2)^2}$.

25. $\frac{2x^5}{3by} \times \frac{5ab}{7x^2} \times \frac{4y^3}{5ax}$.

26. $\frac{(a^2-16)}{xy^3+8x^4} \times \frac{2x+y}{(a+4)^2}$.

27. $\left(x-\frac{x^2}{a}\right)\times\left(\frac{1}{x}-\frac{1}{a}\right)$.

28. $\left(1+\frac{x}{1+x}\right)\left(1+\frac{3x}{1-x}\right)$.

29. $\left(a-\frac{x^2}{a}\right)\times\left(\frac{a}{x}+\frac{x}{a}\right)$.

30. $\left(1-\frac{2a}{1+a}\right)\left(1+\frac{2a}{1-a}\right)$.

31. $\left(b+\frac{a^2}{b}\right)\times\left(a-\frac{b^2}{a}\right)$.

32. $\left(1-\frac{a-b}{a+b}\right)\left(2+\frac{2b}{a-b}\right)$.

33. $\left(b+\frac{bx}{a}\right)\left(1-\frac{a}{a+x}\right)$.

34. $\left(a+\frac{ab}{a-b}\right)\left(b-\frac{ab}{a+b}\right)$.

35. $\left(4+\frac{2a}{3x}\right)\times\left(2-\frac{2a}{6x}\right)$.

36. $\left(1+\frac{a+x}{a-x}\right)\left(1-\frac{a-x}{a+x}\right)$.

37. $\left(\frac{a}{b}+\frac{b}{2a}\right)\times\frac{4ab}{b^2+2a^2}$.

38. $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\frac{a^2bc}{ac+ab+bc}$.

39. $\frac{3ax}{4by}\times\frac{a^2-x^2}{c^2-x^2}\times\frac{bx+bc}{ax+a^2}\times\frac{c-x}{a-x}$.

40. $\frac{3ax}{4by}\times\frac{2}{(a+x)^2}\times\frac{(a^2-x^2)^2}{6a^2}\times\frac{8b}{a-x}$.

41. $\frac{x-y}{(x+y)^2}\times\frac{x+y}{(x-y)^2}\times\frac{2ax}{y}\times\frac{xy-y^2}{x^2+xy}$.

42. $\frac{(x-y)^2}{2+x}\times\frac{x-3}{x^2-y^2}\times\frac{(y+x)^2}{x^3-27}\times\frac{x^3+8}{x^2-y^2}$.

43. $\frac{a^2-b^2}{ab^2x}\times\frac{ab-b^2}{(a+b)^2}\times\frac{ab+a^2}{(a-b)^2}\times\frac{x^2+4x}{a+x}$.

44. $\frac{x^2-6x+8}{x^2-4x+3}\times\frac{x^2-5x+6}{x^2-2x-8}\times\frac{2x+x^2}{x^2-4x+4}$.

45. $\frac{x^4-8x}{x^2-4x-5}\times\frac{x^2+2x+1}{x^3-x^2-2x}\times\frac{x-5}{x^2+2x+4}$.

46. $\frac{a^2-ax+x^2}{2a^3-3a^2x}\times\frac{4a^2-9x^2}{a^3+x^3}\times\frac{a^2+ax}{6x+4a}\times\frac{3ab}{2xy}$.

TO THE TEACHER. — Teach 188 . . . . 190.

DIVISION OF FRACTIONS.

1. $\frac{8a^2x}{5b^3y} \div \frac{4ax}{5by}$.

2. $\frac{a-1}{a} \div \frac{b}{a+1}$.

3. $\frac{9a^3}{8b^4x} \div \frac{3a^2}{2bx}$.

4. $\frac{a+1}{4} \div \frac{a^2-1}{8a}$.

5. $\frac{6x^2}{7by^2} \div \frac{3ax}{4by}$.

6. $\frac{1-a}{b} \div \frac{1-a^2}{ac}$.

7. $\frac{4a^2x}{3bc^2} \div \frac{8ax}{9bc}$.

8. $\frac{1+a}{x-1} \div \frac{1-a^2}{x-1}$.

9. $\frac{8ac^3}{9b^2x} \div \frac{4ac}{9bx}$.

10. $\frac{x+2}{x+1} \div \frac{x^2-4}{x^2-1}$.

11. $\frac{7a^3b}{12c^2} \div \frac{7ab}{4cx}$.

12. $\frac{a+x}{a-c} \div \frac{a^2+ax}{a^2-c^2}$.

13. $\frac{14x^2y}{15ab^3} \div \frac{7xy}{5ab}$.

14. $\frac{a+b}{a-b} \div \frac{(a+b)^2}{a^2-b^2}$.

15. $\frac{9ab}{28x^2y} \div \frac{3ac}{7xy}$.

16. $\frac{a^2-9}{a^2+4a} \div \frac{a^2-3a}{a^2-16}$.

17. $\frac{25xy^3}{27a^2b} \div \frac{5xy^2}{9ab}$.

18. $\frac{3x^2+x}{x-2} \div \frac{2x+6x^2}{x-1}$.

19. $\frac{35a^2x^3}{24b^2y^3} \div \frac{7a^2x}{8b^2y}$.

20. $\frac{a-x}{a^2+2ax} \div \frac{a^2-ax}{a^2-4x^2}$.

21. $\frac{8a^2}{a^2-x^2} \div \frac{4a}{a+x}$.

22. $\frac{x^2-y^2}{x^2-9c^2} \div \frac{y^2+xy}{3cx+x^2}$.

23. $\frac{12x^2y}{5x-10} \div \frac{6xy}{x^2-2x}$.

24. $\frac{(a+b)^2}{a+2b} \div \frac{(a^2-b^2)^2}{a^3+8b^3}$.

25. $\frac{15a^3b^2}{4x+8y} \div \frac{5a^2b^2}{x^2+2xy}$.

26. $\frac{(a^2-16)^2}{xy^3+8x^4} \div \frac{(a+4)^2}{2x+y}$.

27. $\left(\frac{a}{b}+\frac{1}{b}\right)\div\left(a-\frac{1}{a}\right)$.

28. $\frac{x^2-2x+4}{x-3}\div\frac{x^3+8}{x^2-9}$.

29. $\left(\frac{a}{b}-\frac{b}{a}\right)\div\left(1+\frac{b}{a}\right)$.

30. $\frac{a^2-x^2}{x^2-5x-6}\div\frac{a+x}{x^2+x}$.

31. $\left(\frac{1}{b}+\frac{1}{a}\right)\div\left(1+\frac{a}{b}\right)$.

32. $\frac{a^2-a-6}{a^2-a-2}\div\frac{2a+a^2}{a^2-2a}$.

33. $\left(1+\frac{2}{a}\right)\div\left(1+\frac{a}{2}\right)$.

34. $\frac{a^3-b^3}{x^2+6x+8}\div\frac{a-b}{8+2x}$.

35. $\left(a-\frac{b^2}{a}\right)\div\left(\frac{a}{b}+1\right)$.

36. $\frac{a^3+b^3}{a^3-b^3}\div\frac{a+b}{a^2+ab+b^2}$.

37. $\left(1-\frac{b^2}{a^2}\right)\div\left(1+\frac{a}{b}\right)$.

38. $\frac{a^2+2ab+b^2}{ab-b^2}\div\frac{a^2-b^2}{b^2}$.

39. $\left(a-\frac{c^2}{a}\right)\div\left(a+\frac{b}{a}\right)$.

40. $\frac{x^2-x-2}{x^2-9}\div\frac{x^2+x-6}{x-3}$.

41. $\left(\frac{1}{b^2}-\frac{1}{a^2}\right)\div\left(\frac{b}{a}+1\right)$.

42. $\frac{a^3+x^3}{a^2-2ax+x^2}\div\frac{ax+x^2}{a-x}$.

43. $\left(a^2+\frac{b^2}{a}\right)\div\left(1+\frac{a}{b}\right)$.

44. $\frac{x^2+x-6}{x^2-16}\div\frac{x^2-x-12}{4x+x^2}$.

45. $\left(a^2-\frac{b^2}{a}\right)\div\left(1-\frac{b}{a}\right)$.

46. $\frac{x^2+7x-8}{2a+x}\div\frac{x^2+x-56}{ax^2+2a^2x}$.

47. $\left(\frac{a}{b^2}+\frac{b}{a^2}\right)\div\left(\frac{b}{a}+1\right)$.

48. $\left(4-\frac{2}{a+1}\right)\div\left(8+\frac{6}{a^2-1}\right)$.

49. $\left(1+\frac{b^2}{a^2}\right)\div\left(\frac{a^4}{b^4}-1\right)$.

50. $\left(a+\frac{3a}{a^2-4}\right)\div\left(a+\frac{a}{a-2}\right)$.

51. $\left(\frac{1}{b^4}-\frac{1}{a^4}\right)\div\frac{c(a^2-b^2)}{a^3b^3}$.

52. $\left(3+\frac{5x}{a-x}\right)\div\left(9+\frac{5x^2}{a^2-x^2}\right)$.

53. $\left(a^2-\frac{b^4}{a}\right)\div\frac{c(a^2+b^2)}{a}$.

54. $\left(4+\frac{15}{x^2-2^2}\right)\div\left(2-\frac{3}{x+2}\right)$.

To the Teacher. — Teach 191.

COMPLEX FRACTIONS.

1. $\dfrac{a+\dfrac{b}{x}}{a-\dfrac{b}{c}}$.

2. $\dfrac{x-3+\dfrac{2}{x}}{x+1-\dfrac{6}{x}}$.

3. $\dfrac{\dfrac{1}{1-a}+\dfrac{1}{1+x}}{\dfrac{1}{1-a}-\dfrac{1}{1+x}}$.

4. $\dfrac{x+\dfrac{y}{2}}{a-\dfrac{x}{3}}$.

5. $\dfrac{a+2+\dfrac{1}{a}}{1+\dfrac{1}{a}}$.

6. $\dfrac{x-2-\dfrac{14}{x+3}}{x-1-\dfrac{21}{x+3}}$.

7. $\dfrac{a-\dfrac{1}{a}}{1+\dfrac{1}{a}}$.

8. $\dfrac{1-\dfrac{1}{a+1}}{1+\dfrac{1}{a-1}}$.

9. $\dfrac{\dfrac{a+b}{a-b}+\dfrac{a-b}{a+b}}{\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}}$.

10. $\dfrac{\dfrac{1}{a}+\dfrac{1}{b}}{\dfrac{b}{a}-\dfrac{a}{b}}$.

11. $\dfrac{\dfrac{a}{b}-\dfrac{b}{c}-\dfrac{c}{a}}{\dfrac{1}{c}-\dfrac{1}{a}-\dfrac{1}{b}}$.

12. $1-\dfrac{1}{1-\dfrac{1}{1-\dfrac{1}{a}}}$.

13. $\dfrac{\dfrac{a}{x}+\dfrac{y}{b}}{\dfrac{a}{x}-\dfrac{b}{c}}$.

14. $\dfrac{1+\dfrac{b}{a-b}}{1-\dfrac{b}{a+b}}$.

15. $\dfrac{1}{b+\dfrac{1}{1+\dfrac{b+1}{3-b}}}$.

16. $\dfrac{1+\dfrac{1}{a}}{1-\dfrac{1}{b}}$.

17. $\dfrac{1}{x+\dfrac{1}{x+\dfrac{1}{x}}}$.

18. $\dfrac{1-\dfrac{2y-2z}{x+y-z}}{1+\dfrac{2z}{x-y-z}}$.

19. $\dfrac{1+\dfrac{a}{b}}{1-\dfrac{b^2}{a^2}}$.

20. $\dfrac{a+\dfrac{3a}{a-4}}{a-\dfrac{3a}{a-4}}$.

21. $\dfrac{\dfrac{a+b}{c+d}+\dfrac{a-b}{c-d}}{\dfrac{a+b}{c-d}+\dfrac{a-b}{c+d}}$.

REVIEW OF FRACTIONS.

1. $\frac{a^6-x^6}{4a^4x^4}\left(\frac{a^3}{a^3+x^3}-1+\frac{x^3}{a^3-x^3}\right)$.

2. $\left(\frac{a}{b}-\frac{a+2b}{a+b}\right)\div\left(\frac{a}{b}-2+\frac{a}{a+b}\right)$.

3. $\frac{a^3+x^3}{a^3-x^3}\times\frac{a^2+ax+x^2}{a^2-ax+x^2}\div\frac{(a+x)^2}{(a-x)^2}$.

4. $\left(\frac{9+2x^2}{9-x^2}-\frac{3-x}{3+x}+1\right)\times\left(\frac{x+3}{2x+3}\right)$.

5. $\left(\frac{x}{x+2}+\frac{2-x}{x}\right)\div\left(\frac{x}{2+x}-\frac{2-x}{x}\right)$.

6. $\left(\frac{2}{a-x}-\frac{2}{a+3x}\right)\frac{1}{x}-\frac{6-3x-a}{a^2+2ax-3x^2}$.

7. $\frac{(a^2-2+a)3}{a^2-a-2}-\frac{(a^2-2-a)3}{a^2-2+a}-\frac{8a}{a^2-4}$.

8. $\frac{a^2+x^2-ax}{a^2+9x^2+3ax}\times\frac{a(a^3-x^3)}{a^3+x^3}\times\frac{a^3-27x^3}{a(a-x)}$.

9. $\left(\frac{12}{a-3}+1-\frac{4}{a-1}\right)\left(\frac{4}{a+1}+1-\frac{12}{a+3}\right)$.

10. $\frac{(x-y)y}{x^2+y^2+2xy}\times\frac{x(y+x)}{x^2+y^2-2xy}\times\frac{ac(x^2-y^2)}{bx^2y^2}$.

11. $\left(\frac{x-1}{x^2-x+1}+\frac{x^2-1}{x^3+1}\right)\div\left(\frac{x+1}{x^2-x+1}-\frac{x^2-1}{x^3+1}\right)$.

12. $\frac{2}{x}-\frac{x+y-z}{xy}+\frac{2}{y}-\frac{x+z-y}{xz}+\frac{2}{z}-\frac{y+z-x}{yz}$.

13. $\frac{2(b+c)}{(a-b)(a-c)}+\frac{2(a+c)}{(b-c)(b-a)}+\frac{2(a+b)}{(c-a)(c-b)}$.

14. $\left(a-x-\frac{a^2+x^2}{a+x}\right)\times\left(a+x-\frac{a^2+x^2}{a-x}\right)$.

15. $\left(\frac{a+x}{a-x}-\frac{a^3+x^3}{a^3-x^3}\right)\times\left(\frac{a+x}{a-x}+\frac{a^2+x^2}{a^2-x^2}\right)$.

16. $\frac{ax^4-8ax}{x^3-5x-6}\times\frac{x^2+1+2x}{x^3-x^2-2x}\div\frac{x^3+4+2x}{x^2-6x}$.

17. $\frac{(a+c)^2-x^2}{cx-ac+c^2}\times\frac{ac-c^2+cx}{a^2-ax+ac}\div\frac{(a+x)^2-c^2}{ac+ax-a^2}$.

18. $\left(2+\frac{a}{a-3}\right)\times\frac{9-a^2}{4-a^2}\times\frac{a+2}{a^2+a-6}-\frac{2}{a+2}$.

19. $\frac{(a+c)^2-x^2}{a^2+ac-ax}\times\frac{a}{(a+x)^2-c^2}\times\frac{(a-c)^2-x^2}{ac-c^2-cx}$.

20. $\left(\frac{x^2-14x+33}{y^2+10y-24}\div\frac{x^2+3x-18}{y^2+4y-12}\right)\times\frac{6y+xy}{6x+xy}$.

21. $\left(\frac{a-4}{a-3}+\frac{3-a}{9-a^2}-\frac{a}{a+3}\right)\times\left(\frac{a}{3}-1\right)\div\frac{5-a}{9-a^2}$.

22. $\frac{a}{a^3-x^3}+\frac{a^3+x^3}{x^4-a^4}-\frac{x}{a^3+x^3}+\frac{ax}{(x+a)(x^2+a^2)}$.

23. $\frac{4}{(x-1)(x-3)}+\frac{2}{(x-2)(3-x)}-\frac{2}{(2-x)(1-x)}$.

24. $\frac{a^2+2a-3}{a^2-a+30}\times\frac{a^2+6a+5}{a^3-a^2}\div\left(\frac{a^2+4a+3}{a^2-4a-12}\times\frac{2+a}{a^3}\right)$.

25. $\left(1+\frac{a-1}{a+1}\right)\div\left(1-\frac{a-1}{a+1}\right)+\left(1+\frac{a+1}{a-1}\right)\div\left(1-\frac{a+1}{a-1}\right)$.

26. $\frac{\frac{a}{b}+\frac{b}{a}}{\frac{a}{b}-\frac{b}{a}}\div\frac{\frac{1}{a}+\frac{1}{b}}{\frac{a}{b}-\frac{b}{a}}$.

27. $\frac{\frac{1}{x+2}-\frac{6}{(x+4)(x+3)}}{\frac{1}{x+4}-\frac{5}{(x+3)(x+2)}}$.

28. $\left(\frac{x^3+y^3}{2xy}-1\right)\frac{xy^3}{x^3+y^3}\div\frac{4x^2y-4xy^2}{x^2+y^2-xy}$.

29. $\left(\frac{x+1}{x^3+1}-\frac{x^3+1}{x^3+1}\right)\div\left(\frac{x^3+1}{x^3+1}-\frac{x^3+1}{x^4+1}\right)$.

30. $\left(\frac{a^2}{x}-\frac{x^2}{a}\right)\left(\frac{a}{x^2}+\frac{x}{a^2}\right)\div\left(\frac{a^2}{x}+\frac{x^2}{a}\right)\left(\frac{a}{x^2}-\frac{x}{a^2}\right)$.

31. $\frac{x^3+y^3+2xy-z^3}{x^3-2yz-y^3-z^3}\times\frac{x^2-2xz-y^2+z^2}{y^2-2yz-x^2+z^2}$.

32. $\left(\frac{2+n}{3+2n}-\frac{4n+5}{6+5n}\right)\div\left(\frac{3+2n}{4+3n}-\frac{4+3n}{4n+5}\right)$.

33. $\frac{(a+c)^2-x^2}{a^2-ax+ac}\times\frac{a}{(a+x)^2-c^2}\div\frac{ac-cx-c^2}{(a-c)^2-x^2}$.

34. $\frac{3x^2+x-24}{6xy-20y}\times\frac{6x^2}{x^2-9}\times\frac{x-3}{3x-8}\div\frac{3x^2+9x}{3x^2-x-30}$.

35. $\frac{x^2-81}{x^2+25x+136}\times\frac{x^3+9x^2-136x}{x^3-729}\div\frac{x^2+x-72}{x^2+9x+81}$.

36. $\frac{6x^2-22x+20}{3x^2+4x+1}\times\frac{4x^2-5x+1}{4x^2-10x+4}\times\frac{6x^2-x-1}{12x^2-23x+5}$.

37. $\frac{x^2+x-6}{x^2-x-30}\times\frac{x^2+6x+5}{x^2-4x+4}\div\left(\frac{x^2+4x+3}{x^2-4x-12}\times\frac{3+x}{x-2}\right)$.

38. $\dfrac{x+6+\frac{8}{x}}{1+\frac{2}{x}-\frac{8}{x^2}}$.

39. $\dfrac{x-3-\frac{1}{x-3}}{x-2-\frac{3}{x-4}}\times\dfrac{x-4-\frac{1}{x-4}}{x-5+\frac{3}{x-1}}$.

40. $\dfrac{\frac{a}{c^2}+\frac{c}{a^2}}{\frac{1}{a^2}-\frac{1}{ac}+\frac{1}{c^2}}$.

41. $\dfrac{\frac{a^3-x^3}{a^3+x^3}\times\frac{a+x}{(a-x)^2}\div\frac{a^2+x^2+ax}{a^2+x^2-ax}}{\frac{a^6+x^6}{a^6-x^6}\times\frac{a-x}{a^2+x^2}\div\frac{a^4+x^4-a^2x^2}{a^4+x^4+a^2x^2}}$.

To the Teacher. — Teach 192 . . . . 214.

SIMPLE EQUATIONS.

1. $6x-14=x+16$.
2. $12-4x=24-7x$.
3. $5x-11=13-x$.
4. $3x-24=9x-42$.
5. $8x-28=x+28$.
6. $11-2x=46-7x$.
7. $27-4x=12-x$.
8. $4x+15=29-3x$.
9. $5x+17=x+19$.
10. $17-7x=3x-13$.
11. $7x-36=36-x$.
12. $6x-12=16-8x$.
13. $15+8x=x+18$.
14. $16-5x=6x-28$.
15. $9x+26=36-x$.
16. $6x+17=22+4x$.
17. $24-2x=16-x$.
18. $4x+17=20-2x$.
19. $7x-24=16-x$.
20. $17-8x=5x-22$.
21. $28-3x=13-x$.
22. $4x+32=9x+17$.
23. $5x-14=2x+7$.
24. $7x+15=2x+35$.
25. $18-7x=3x-42$.
26. $8x+16=5x-11$.
27. $6x+13=2x+17$.
28. $3x-12=9x+18$.
29. $8x-15=3x+20$.
30. $24-2x=15-5x$.
31. $32-3x=5x-16$.
32. $4x+14=2x+15$.
33. $7x+13=17+9x$.
34. $7x-12=9x-18$.
35. $14+8x=3x+16$.
36. $15-4x=3x+22$.
37. $9x-19=44+2x$.
38. $18-3x=2x+15$.
39. $5x+32=14+2x$.
40. $7x-15=3x+13$.
41. $6x+16=3x+26$.
42. $10-5x=2x+38$.
43. $5x+34=7x+25$.
44. $6x-14=26-2x$.
45. $4x-13=9x+12$.
46. $4x+16=21-3x$.
47. $3x+43=13+8x$.
48. $4x+14=29+6x$.

49. $(x+7)(2x-5)=(x+3)(2x-1)$.

50. $(3x-5)(4+2x)=(2+2x)(3x-4)$.

51. $(7-4x)(4x+2)=(8x+1)(4-2x)$.

52. $8(x+4)-6(5-x)=38x-4(3-x)$.

53. $(x-3)(7+x)-(x-4)(2+x)-8=0$.

54. $(x-4)(4+x)-(x-5)(5+x)-x=0$.

55. $3(x-2)-2(x-3)+4(x-5)-3=7$.

56. $5(x+3)-4(x-2)-3(2+x)-7=0$.

57 $5x-3(x-6)=2(8-x)-4(9+x)+6$.

58. $x(x+6)-(x-2)(x-3)-8(x+2)+17=0$.

59. $(5x+2)^2-(4x+3)^2-(3x-3)^2-7x-3=0$.

60. $2(3x-2)^2-2(6+x)+8(2x-1)-2(1-3x)^2=0$.

61. $4(5+6x)-2(3x-2)(1-2x)-3(2x-3)^2-20=0$.

62. $2x+4a=x+6a$.

63. $3a-4x=3-7x$.

64. $3x-3a=x+5a$.

65. $5x-2a=3x-2$.

66. $4x-5a=x-4a$.

67. $3x+5a=2+4x$.

68. $7a-3x=x-5a$.

69. $5x-4a=x-4b$.

70. $6a-6x=4a-x$.

71. $7x-3a=4a+7$.

72. $3x+9a=x+3a$.

73. $6x+2a=5a-x$.

74. $2a-5x=x+8a$.

75. $7x-6a=4x-3$.

76. $ax-4a=x-4b$.

77. $3c+ax=2b+6x$.

78. $4a+ax=5a-x$.

79. $3x-3b=ax-ab$.

80. $2c-bx=3c-x$.

81. $ax-5b=5a-bx$.

82. $ax-3a=9-3x$.

83. $3a+ax=bx+5a$.

84. $b^2-bx=a^2-ax$.

85. $bx-3b=2a+cx$.

86. $ax-a^2=b^2-bx$.

87. $ax-18=2a-9x$.

1. $\frac{2x}{3}+2\frac{1}{2}+\frac{x}{2}=x+\frac{3x+5}{10}$.

2. $\frac{a}{3}+x-\frac{x}{4}+\frac{a}{2}=a-\frac{x}{3}$.

3. $\frac{x}{4}-1\frac{1}{4}+x=\frac{6x-9}{3}-\frac{2x}{3}$.

4. $\frac{x}{b}-2-\frac{b}{a}+\frac{x}{a}-\frac{a}{b}=0$.

5. $\frac{x}{3}+\frac{4x+5}{7}-\frac{2x}{10}=x-6\frac{2}{3}$.

6. $a+\frac{4}{x}+b-\frac{3}{x}=\frac{5}{x}-b$.

7. $\frac{2x}{5}-3\frac{1}{5}-x=\frac{3x+1}{2}-\frac{x}{4}$.

8. $\frac{a}{5}-x+\frac{x}{3}+\frac{a}{3}=a-\frac{x}{5}$.

9. $\frac{5x+9}{8}-x+\frac{3x}{4}-1\frac{2}{3}=\frac{x}{3}$.

10. $\frac{a}{x}-b-1+\frac{c}{x}=\frac{b}{x}+a$.

11. $x+\frac{4x}{5}-3\frac{3}{5}+\frac{x}{4}=\frac{9x+5}{5}$.

12. $1-\frac{x}{a}-\frac{b}{a}+\frac{x}{b}=\frac{a}{b}-1$.

13. $\frac{x}{2}+4\frac{1}{3}+\frac{8x-5}{6}-\frac{4x}{9}=x$.

14. $\frac{a}{3}-\frac{1}{x}+\frac{2}{3}-\frac{a}{x}=\frac{5}{3}-\frac{2}{x}$.

15. $\frac{4x+5}{9}-\frac{x-5}{6}-\frac{x-8}{3}=2\frac{1}{3}+\frac{3+2x}{9}$.

16. $\frac{4x-3}{2}-x+\frac{x}{3}-\frac{x+8}{4}=8\frac{1}{2}-\frac{2x+3}{2}$.

17. $\frac{2x-4}{5}-\frac{x-8}{4}+\frac{x+6}{2}=2x-\frac{5x-12}{4}$.

18. $\frac{3x+8}{3}-\frac{x-6}{8}+7\frac{2}{3}=\frac{5x+15}{2}-\frac{x-3}{3}$.

19. $6\frac{3}{4}+\frac{3(x-6)}{2}-2(x-3)=\frac{x+7}{2}-\frac{5(x-2)}{4}$.

20. $\frac{4(x-2)}{3}-\frac{3(x+2)}{2}+9\frac{2}{3}=\frac{3x+8}{2}-\frac{2(x-3)}{3}$.

21. $9\frac{1}{2}-\frac{(x-3)2}{5}-(x+4)3=\frac{8x+5}{2}-\frac{(x+2)4}{5}$.

22. $\frac{8}{x-1}-\frac{30}{x^2-1}=\frac{6}{1+x}$.

23. $\frac{6}{x+a}=\frac{5}{x-a}$.

24. $\frac{x+2}{x-2}=\frac{4x+8}{x^2-4}-\frac{x-2}{2+x}$.

25. $\frac{a}{a-b}=\frac{a+b}{a-x}$.

26. $\frac{1-x}{x+1}=\frac{3x+4}{1-x^2}-\frac{x+3}{1-x}$.

27. $\frac{a+c}{x+2}=\frac{a-c}{x-2}$.

28. $\frac{8x+3}{x^2-9}-\frac{x+2}{3+x}=\frac{x+3}{x-3}$.

29. $\frac{x+3}{x-3}=\frac{a+c}{a-c}$.

30. $\frac{2-x}{x+2}=\frac{2x^2+2x}{4-x^2}-\frac{x+2}{2-x}$.

31. $\frac{a-x}{a+x}=\frac{x^2}{a^2-x^2}$.

32. $\frac{2x+1}{2x-1}-\frac{3x^2}{4x^2-1}=\frac{2x-1}{1+2x}$.

33. $\frac{4x+a}{6x+c}=\frac{2x-c}{3x-a}$

34. $\frac{8}{x+1}-\frac{25}{(x+1)2}-5=\frac{x^2}{1+x}-x-\frac{5x}{x+1}$.

35. $\frac{3}{x-1}-4+\frac{4}{3(x-1)}=\frac{5}{2(x-1)}-x+\frac{x^2}{x-1}$.

36. $\frac{x-2}{3}-6-\frac{9}{3(x-3)}-x=\frac{x}{x-3}-\frac{2x^2}{3(x-3)}$.

37. $\frac{2+x}{x-2}-\frac{7x+86}{2(x^2-4)}-7=\frac{x-2}{2+x}-\frac{9x-1}{2(x-2)}-2\frac{1}{2}$.

38. $\frac{1+x}{x-1}-5-\frac{x^2+5}{3(x^2-1)}=\frac{x-1}{1+x}-3\frac{1}{3}-\frac{10x}{(x-1)2}$.

39. $\frac{2}{x-3}-3+\frac{17}{(x+2)12}=\frac{x}{x-3}-3\frac{1}{2}-\frac{2x+9}{4(x+2)}$.

40. $\frac{x}{x+1}-\frac{x}{x+2}+\frac{9}{4(x+1)}=\frac{11}{(x+2)2}-\frac{5}{(x+1)3}$.

41. $\frac{x}{a-c}-\frac{2}{a+c}-\frac{4ax-13a}{4(a^2-c^2)}=\frac{5}{(a-c)3}-\frac{x}{2(a+c)}$.

To the Teacher.—Teach 215 . . . . 218.

1. $\frac{8x+7}{12}-1\frac{1}{4}-\frac{8x-9}{2x+5}=\frac{2x-5}{3}-\frac{4x+2}{5+2x}$.

2. $\frac{9x+4}{15}-\frac{3x+2}{3x-4}+2\frac{1}{2}=\frac{3x+3}{5}-\frac{2x-5}{3x-4}$.

3. $\frac{8x+5}{4}-2\frac{1}{2}=\frac{5x+2}{2x+3}+\frac{6x-5}{3}-\frac{3x+6}{2x+3}$.

4. $\frac{6x-5}{8}-\frac{4x+5}{3x+2}+3\frac{1}{8}=\frac{3x+9}{4}-\frac{2x+7}{2+3x}$.

5. $\frac{4x+5}{5x-3}+\frac{3x-8}{3}-\frac{4x+9}{4}=\frac{7x+3}{5x-3}-5\frac{1}{2}$.

6. $\frac{6x+5}{4}-\frac{5x-3}{2x-1}-\frac{1}{2}x=\frac{3x+2}{3}-\frac{4x-4}{2x-1}$.

7. $\frac{5x+2}{5}-\frac{2}{3}x-\frac{3x+2}{2x+3}=\frac{2x-3}{6}-\frac{2x-5}{3+2x}$.

8. $\frac{3x+2}{4}+\frac{4+3x}{4-5x}-\frac{2}{3}x=\frac{2x+9}{24}-\frac{2-4x}{4-5x}$.

9. $\frac{8x+5}{9}-\frac{4}{5}x-\frac{3x-a}{2x-a}=\frac{4x-2}{45}-\frac{2x-2a}{2x-a}$.

10. $\frac{6x+3}{4}-\frac{2+8x}{3x-1}-\frac{3x-5}{3}=\frac{x+3}{2}-\frac{5x+6}{3x-1}$.

11. $\frac{3x-4}{6}-\frac{5x+7}{1-3x}+\frac{3x+2}{1-3x}=\frac{2x-5}{4}-\frac{4x+4}{1-3x}$.

12. $\frac{6x+7}{5a}-\frac{5x+8}{14x}-\frac{8x+7}{7a}=\frac{2x+9}{35a}-\frac{3x+4}{14x}$.

13. $\frac{5x-4}{3}-\frac{7x-2}{4x-5}-\frac{7x-6}{5}=\frac{4x-7}{15}-\frac{4x+7}{4x-5}$.

14. $\frac{2x-5}{2a}+\frac{2x-3}{3x}-\frac{2x-4}{3a}=\frac{4x-5}{12a}-\frac{5x-3}{6x}$.

To the Teacher. — Teach 219 . . . . 224, also 225 . . . . 236, as problems are found illustrating these suggestions.

PROBLEMS INVOLVING SIMPLE EQUATIONS.

1. The larger of two numbers is 4 times the smaller, and the sum of the numbers is 85. Find the numbers.

2. Seven times the smaller of two numbers equals 5 times the greater, and their sum is 84. Find the numbers.

3. The sum of two numbers is 48, and 5 times the less exceeds 3 times the greater by 32. Find the numbers.

4. A man bought two horses for three hundred sixty dollars, paying eighty dollars more for the better one than he paid for the other. Find the cost of each.

5. A man left one-half of his property to his wife, one-third to his daughter, and the remainder, which was $ 5500, to his son. How much did the wife receive?

6. A man paid two hundred twenty-five dollars for a horse and carriage, paying $ 85 less for the carriage than he paid for the horse. Find the cost of each.

7. Find two numbers which are to each other as three to seven, and whose sum is 140.

8. Find two numbers differing by thirty-six whose sum is equal to twice their difference.

9. A man has four children the sum of whose ages is 64 years, and the common difference of their ages is twice the age of the youngest. Find the age of the oldest.

10. A boy ate $\frac{1}{2}$ of his oranges and gave away $\frac{1}{3}$ of them. The difference between the number he ate and the number he gave away was four. How many did he have?

11. Divide 28 into two parts such that six times the less shall exceed three times the greater by 24.

12. A can build a wall in $4\frac{1}{2}$ days, and B can build it in 3 days. In how many days can both build it?

13. A can do a piece of work in m days, and B can do it in n days. In how many days can both do it?

14. A man saved \$ 2160 in two years. If he saved two and three-fifths times as much the second year as he saved the first, how much did he save each year?

15. A man bequeathed his property, amounting to \$ 22,600, to his wife, son, and daughter. The son received \$ 600 more than the daughter, and \$ 1000 less than the wife. How much did the wife receive?

16. A collection, amounting to \$ 6.40, consisted of dimes, 5-cent pieces, and 2-cent pieces. There were three times as many 2-cent pieces as 5-cent pieces and twice as many 5-cent pieces as dimes. How many coins were there?

17. A man paid \$ 9.55 for a hat and coat. The coat cost \$ 4.05 more than the hat. Find the cost of each.

18. Eight men agreed to share equally in buying a boat, but two being unable to pay their share, the others had each to pay \$ 20 more. How much did the boat cost?

19. A man left an estate of \$ 26,000 to be divided among his wife, two sons, and two daughters. The wife received twice as much as each daughter, and each son received $\frac{1}{2}$ as much as each daughter. How much did each receive?

20. A grocer mixed tea worth 60 cents a pound with tea worth 40 cents a pound in such proportions that the mixture, weighing 120 pounds, was worth fifty-six dollars. How many pounds of each kind did he take?

21. The sum of two numbers is 66, and their difference is 28. Find the numbers.

22. A man is three times as old as his son, but 5 years ago he was 4 times as old. Find the age of each.

23. A can do a piece of work in five days; B can do it in x days; and C can do it in ten days. In how many days can all do it, working together?

24. A and B can do a piece of work in 12 days, and A can do it in 20 days. In how many days can B do it?

25. A man spent one-sixth of his money for a suit of clothes and one-fourth of it for a watch, and had eighty-four dollars left. How much did he spend?

26. The sum of two numbers is fifty-eight; and if the greater be divided by the less, the quotient will be 4 and the remainder three. Find the numbers.

27. A's age is to B's as 4 to 5, and the sum of their ages is 117 years. Find the age of each.

28. A man bought land for $720. He gave his son twenty-four acres and sold one-half the remainder at cost for three hundred dollars. How many acres did he buy?

29. The value of a man's horse and carriage is $365; the value of his horse and harness is $280; the value of the carriage and harness is $195. Find the value of all.

30. Eight boys and 18 men earn two hundred ninety-four dollars a week. If each man earns five times as much as each boy, how much do the 8 boys earn per week.

31. The sum of two numbers is a, and their difference is b. What are the numbers?

32. Divide 32 into two such parts that the sum of twice the less and 5 times the greater shall be 118.

33. Four times a certain number is 54 more than twice the number. Find the number.

34. The sum of two numbers is a, and the greater is b times the less. Find the larger number.

35. If A can do one-half of a piece of work in six days and B can do the whole of it in eight days, in how many days can both do the work?

36. A man invested a certain sum at 5 per cent and twice as much at 6 per cent. His annual income from both investments was $765. How much did he invest?

37. A man left $\frac{2}{5}$ of his estate to his wife and $\frac{1}{4}$ of it to his daughter. The wife received $4200 more than the daughter. How much did each receive?

38. A boy has $9 in quarters and dimes, and he has 5 times as many dimes as quarters. How many coins has he?

39. The difference of two numbers is thirty-two; and if the greater be divided by the less, the quotient will be 5 and the remainder 4. Find the numbers.

40. A man divided $5000 among his five sons so that each one received two hundred dollars less than his next older brother. How much did the youngest son receive?

41. Three men earned a certain sum of money. A and B earned $140; A and C earned $125; and B and C earned $115. How much did they all earn?

42. A had nine acres of land more than B; but A sold B twenty-three acres, when he had only half as many acres as B. How many acres had each at first?

43. A can do a piece of work in seven and one-half days; B, in five days; and C, in three and one-third days. In how many days can all do the work?

44. At what rate per annum will $7500 yield $450 interest in one year and six months?

45. A man spends one-fifth of his annual salary for board, one-eighth for clothes, one-fourth for other expenses, and saves $850. How much are his annual expenses?

46. Divide the number two hundred ten into two parts such that the quotient of the greater divided by the less is four and the remainder ten.

47. A's age is to B's age as six to five, but twenty-eight years ago their ages were to each other as two to one. How old is each at the present time?

48. In a regiment of soldiers there is one officer to every thirty men; and if the whole number of men be divided by the number of officers less ten, the quotient will be 62. Find the number of men in the regiment.

49. A boy earns 75 cents a day less than his father, and in twelve days the father earns $12 more than the son earns in nine days. How much do both earn per day?

50. Divide ten dollars into two parts so that there may be twice as many dimes in the first part as there are five-cent pieces in the second part.

51. The difference between two numbers is eleven; and if three be added to the greater, the sum will equal three times the less. Find the numbers.

52. The sum of the third, fourth, and eighth parts of a number is thirty-four. Find the number.

53. A is three times as old as B, but in 16 years he will be only twice as old. Find the age of each.

54. A can do a piece of work in 12 days; A and C, in nine days; and A and B, in six days. In how many days can B and C together do the work?

55. What sum must be invested at $5\frac{1}{2}$ per cent to produce a quarterly income of $4125?

56. A man bought 25 sheep, paying $2.25 a head for some of them, and $3.50 a head for the others. The average cost was $2.80. How many did he buy at $2.25?

57. A and B are fifty-two miles apart. They travel toward each other, A four miles an hour and B two miles an hour, and B sets out two hours before A. How many miles will A have traveled when they meet?

58. A merchant mixes a pounds of tea worth d cents a pound, b pounds worth e cents a pound, and c pounds worth f cents a pound. Find the value of the mixture per pound.

59. There are 85 people in a railway car. There are five more men than women and children, and ten more women than children. How many are there of each?

60. A had three times as many sheep as B. Each sold to C half his flock, and A sold 40 to B, when A and B had the same number. How many did each have at first?

61. At what time between 4 and 5 o'clock are the hands of a clock together?

62. At what time between 2 and 3 o'clock are the hands of a clock at right angles to each other?

63. At what time between 3 and 4 o'clock are the hands of a clock opposite each other?

64. A can do a piece of work in 10 days, which B can do in 12 days, and with C's help they can do it in four days. In how many days can C do the work?

65. What is the distance between two cities, if an express train, running forty miles an hour, can go from one city to the other in two hours less time than a freight train, running twenty-four miles an hour?

66. What principal at r per cent interest will amount to a dollars in b years?

67. Two numbers are to each other as three to four; but if fifty be subtracted from each number, the remainders will be to each other as one to two. Find the numbers.

68. A man bought a horse, carriage, and harness for $390. He paid $60 more for the horse than for the other two, and the carriage cost $5 more than three times as much as the harness. Find the cost of the horse.

69. A man bought some cows at thirty dollars a head. If he had bought two more for the same money, each one would have cost five dollars less. How many did he buy?

70. A merchant divided a sum of money among his four clerks. He gave the first $\frac{1}{3}$ of the whole; the second, forty dollars; the third, $\frac{1}{5}$ of the whole; and the fourth, thirty dollars. How much did he give to all?

71. A, B, and C together earn $4200. A's salary is $\frac{2}{3}$ of B's and $350 less than C's. Find C's salary.

72. At what time between 8 and 9 o'clock are the hands of a clock together?

73. A is 48 years old, and B is $\frac{2}{3}$ as old. How many years have elapsed since B was half as old as A?

74. A and B can do a piece of work in eight days; A and C, in seven days; and A alone, in twelve days. In how many days can B and C do the work?

75. Three-fifths of a certain principal was invested at 4 %, and the remainder at 5 %. The annual income from both investments was $550. Find the sum invested.

76. A boy has ten dollars in half-dollars and five-cent pieces, there being in all fifty-six coins. How many coins has he of each kind?

77. A man going from a certain town traveled at the rate of 4 miles an hour. Five hours afterward a horseman, going at the rate of 6 miles an hour, was sent after him. How far did the latter travel to overtake the former?

78. A merchant sold silk for a dollars and gained b per cent. How much did he pay for it?

79. A lady bought 16 yards of silk; but if she had bought four yards more for the same money, it would have cost 25 cents a yard less. How much did it cost?

80. D is 4 years older than C; C is 3 years older than B; B is 2 years older than A; and in five years the sum of their ages will be 92 years. Find D's age.

81. One of two numbers is three times the other. If 18 be subtracted from the greater, and the less from 38, the remainders will be equal. Find the numbers.

82. The sum of two numbers is a, and m times the less equals n times the greater. Find the numbers.

83. A and B can do a piece of work in five days, which A and C can do in six days, and which B and C can do in $7\frac{1}{2}$ days. In how many days can all do the work?

84. At what time between 8 and 9 o'clock are the hands of a clock at right angles to each other?

85. At what rate per annum will c dollars yield a dollars interest in b years?

86. Four hours after a train left Albany, a second train set out to overtake the first in six hours. To accomplish this, it was necessary for the second train to run sixteen miles an hour faster than the first. How many miles per hour did the first train run?

87. A's money is to B's as three to seven, and together they have $\$15{,}000$. How much money has each?

88. A farmer bought sheep at three dollars a head and had $18 left; but if he had bought them at three dollars seventy-five cents a head, he would have needed 75 cents more to pay for them. How many sheep did he buy?

89. If it costs the same, at $2 a yard, to inclose a square court with a fence as to pave it at 40 cents a square yard, what are the dimensions of the court?

90. A man owed $92. He sold wheat at $.75 a bushel and corn at $.40, selling the same number of bushels of each. If he received just money enough to pay the debt, how many bushels of grain did he sell?

91. Find the number whose double diminished by seventeen is as much less than forty-eight as twenty-two is less than the number itself.

92. A father is forty-five years old, and his son is one-third as old. In how many years will the son be one-half as old as his father?

93. At what times between 4 and 5 o'clock are the hands of a clock at right angles to each other?

94. The length of a rectangle exceeds its breadth by nine inches. If the length were diminished four inches and the breadth increased three inches, the area would remain the same. Find the dimensions of the rectangle.

95. A man was hired for forty days at the rate of $2.75 a day and his board, and for every day he might be idle he was to pay $.75 for his board. At the end of the time he received $89. How many days did he work?

96. After A had been traveling four hours at the rate of seven miles in two hours, B set out at the rate of eleven miles in two hours to overtake A. In how many hours from the time B started did he overtake A?

97. A lady bought 42 yards of silk for \$48, paying \$1 a yard for part of it and \$1.25 a yard for the rest. How many yards of each kind of silk did she buy?

98. By selling tea at a cents a pound, a grocer lost b per cent. How much did he pay for the tea?

99. Two men engage in business. A invests $\frac{3}{4}$ as much as B. The first year, A gains \$1500, which he adds to his capital, and B loses $\frac{1}{8}$ of his money. The next year, A loses $\frac{1}{10}$ of his money, and B gains \$400, when they have equal amounts. How much had each at first?

100. A lady bought a hat and a pair of shoes, paying three-fourths as much for the hat as she paid for the shoes. If she had paid \$2 less for the shoes and \$1.50 more for the hat, the hat would have cost $\frac{2}{3}$ times as much as the shoes. How much did she pay for both?

101. The difference between the sixth and eighth parts of a number is six. Find the number.

102. Between what hours are the hands of a clock together? Form the equations for finding the times.

103. A man is 24 years older than his son, but two years ago he was 4 times as old. Find the father's age.

104. A can do a piece of work in 8 days, and B can do it in 12 days. If B works two days alone, in how many days can he complete it with A's assistance?

105. A boy bought a number of apples at the rate of three for five cents, and sold them at the rate of five cents for two, gaining \$1. How many did he buy?

106. A general arranged part of his men in a solid square and had 40 men left; but attempting to add one man to each side of the square, he found that he needed 41 men to complete the square. How many men had he?

107. A man invests one-third of his money in 3 per cent bonds, one-fourth in 4½ per cent bonds, and the remainder in 5 per cent bonds. His annual income from the whole investment is $1010. Find the whole sum invested.

108. A merchant has tea worth 30 cents a pound and some other worth 50 cents a pound. How many pounds of each must he take to mix 60 pounds worth $.46 a pound?

109. Divide a into two parts such that the quotient of the greater divided by the less is b and the remainder c.

110. A man bought two horses and a carriage, paying $175 for the carriage. The better horse and carriage cost $50 less than three times as much as the poorer horse, and the poorer horse and carriage cost twice as much as the better horse. Find the cost of the horses.

111. At what times between 5 and 6 o'clock are the hands of a clock twenty minutes apart?

112. The sum of the third and fourth parts of a certain number exceeds eight times the difference between the fifth and sixth parts by 38. Find the number.

113. A is fifty-five years old, and B is three-fifths as old. How many years have elapsed since A was two and four-sevenths times as old as B?

114. A can do half as much work as B; B can do ⅓ as much as C; and together they can complete a piece of work in 8 hours. In what time can each complete it?

115. What sum must be invested at a per cent to give a semi-annual income of c dollars?

116. A and B are eighty-two miles apart. They set out at the same time and travel toward each other. A travels at the rate of ten miles in three hours, and B at the rate of seven miles in two hours. How many miles will each have traveled when they meet?

117. A mason received $3.50 a day for his labor and paid $.75 a day for his board. At the end of thirty days he had saved $54.50. How many days did he work?

118. A man walking one and a half miles an hour set out from a certain town, and two hours after another man walking three miles an hour set out to overtake him. How many miles did the second walk to overtake the first?

119. A man bought sheep at $2.75 a head and had twenty-five dollars left; but if he had bought the same number at $3.25 a head, he would have had only five dollars left. How many sheep did he buy?

120. A young man spends one-fourth of the money he has in bank at the beginning of each year, and adds to it during each year an annuity of $6000. At the beginning of the fourth year he has $20,625 to his credit in the bank. How much did he have in the bank at first?

121. Divide 88 into two parts such that $\frac{2}{3}$ of the less shall be equal to three-sevenths of the greater.

122. The width of a room is $\frac{5}{7}$ of its length. If the length were 3 feet less and the width 3 feet more, the room would be square. Find the dimensions of the room.

123. A cistern can be filled by two pipes in four hours and six hours respectively, and can be emptied by a third in twelve hours. In what time will the cistern be filled, if all three pipes are running together?

124. At what times between 1 and 2 o'clock are the hands of a clock at right angles to each other?

125. A boy bought some peaches at the rate of two for a cent and as many more at the rate of two cents for three. He sold them all at the rate of four for five cents and gained $1.60. How many did he buy?

126. A man received an annual income equal to $\frac{1}{2}$ the sum invested. He invested the income, less \$4000 for annual expenses, at the same rate. At the end of 3 years he had \$21,500 invested. Find his original investment.

127. In how many years will a dollars amount to b dollars at c per cent interest?

128. A man bought a suit of clothes for \$24 and paid for it in two-dollar bills and fifty-cent pieces, giving twice as many coins as bills. How many bills did he give?

129. A and B have \$96. A buys sheep at \$4 a head and has \$12 left, and B buys the same number at \$5 a head and has the same sum left. How many does each buy?

130. A lady bought a cloak, a dress, and a hat, paying \$9 for the hat. The dress and hat cost $\frac{3}{4}$ as much as the cloak, and the cloak and hat cost \$7 more than twice as much as the dress. How much did she pay for all?

131. Divide 84 into two parts such that two-fifths of the greater shall exceed six-sevenths of the less by 16.

132. A's age is to B's age as 4 to 7, and the sum of their ages is 66 years. Find the age of each.

133. At what times between 6 and 7 o'clock are the hands of a clock at right angles to each other?

134. A can do a piece of work in 24 days, which B can do in 20 days. A begins the work, but after a time he is relieved by B, and the work is completed in $21\frac{1}{2}$ days from the beginning. How many days did each work?

135. If 1024 men be arranged in a hollow square 8 men deep, how many men will there be in the outside line?

136. Divide \$8000 into two parts such that the interest of the greater part for 4 years at 3% shall be the same as the interest of the smaller part for 5 years at 4%.

137. A man can row a miles an hour on still water. How far down a stream, whose current is b miles an hour, can he row so that he may go and return in c hours?

138. A wheelman who rides 32 miles an hour is 45 minutes in advance of a second, who rides 40 miles an hour. In how many hours can the second overtake the first?

139. The number of girls in a school is to the number of boys as five to three; but if there were fifteen more girls and fifteen less boys, the ratio would be as two to one. How many girls are there in the school?

140. A man bought two farms, one of which contained 40 acres less than 3 times as much as the other. He paid \$10 an acre for the smaller farm and \$4 an acre less for the other. He kept 10 acres of each farm and sold the remainder at \$1 an acre above cost, receiving for it \$100 more than the whole cost. How many acres did he buy?

141. Divide seventy-two into two parts such that the excess of the greater over thirty shall be four times the excess of thirty over the less.

142. Between what hours are the hands of a clock opposite each other? Form the equation for each solution.

143. The length of a rectangle is $3\frac{1}{2}$ times its width. If each dimension were two inches less, the area would be diminished 122 square inches. Find the length.

144. A, B, and C can together complete a piece of work in thirty days. A does three-fourths as much as B, and B does four-fifths as much as C. In how many days can each complete the work alone?

145. A man bought some eggs at thirty cents a dozen. He sold three-eighths of them at the rate of thirty cents for nine, and the remainder at the rate of fifteen for fifty cents. He gained \$2. How many eggs did he buy?

146. If a regiment of troops be arranged in a solid square with a certain number of men on a side, there will be 232 men left; but if four men be added to each side of the square, there will be just enough men to complete the square. How many men are there in the regiment?

147. A merchant starts in business with $8000 capital. His profits are equal to $\frac{1}{4}$ of his capital. If he adds his profits, less a fixed sum for annual expenses, to his capital, at the end of 3 years his capital will be increased to $11,050. How much are his annual expenses?

148. A man has $6.56 in dollars, dimes, and cents. He has $\frac{1}{8}$ as many cents as dimes and four times as many cents as dollars. How many pieces of money has he?

149. A has $92 more than B. A buys land at $8 an acre and has $180 left, and B buys 8 acres less at $12 an acre and has $24 left. How many acres does each buy?

150. A man earns three times as much per day as his older son and five times as much as his younger son. The father worked twenty-four days, the older son twenty days, and the younger son sixteen days, and all earned $127. How much do all earn per day?

151. A father is forty-two years old, and his son is three-sevenths as old. In how many years will the son be three-fifths as old as his father?

152. A tank can be filled by two pipes in 40 minutes and one hour 20 minutes respectively, and can be emptied by a third in two hours. In what time will the tank be filled, if all the pipes are running together?

153. A man was hired to do some work. He was to receive $3 a day for every day he worked and forfeit $.75 for every day he was idle. He worked 3 times as many days as he was idle and received $66. How many days did he work?

154. At what times between 12 and 1 o'clock are the hands of a clock at right angles to each other?

155. A fox is pursued by a hound. The fox makes 4 leaps while the hound makes 3, but 3 leaps of the hound are equal to 5 of the fox. The fox has a start of 30 of her own leaps. How many leaps must the hound make to catch the fox?

156. What sum must be invested at four per cent to give a semi-annual income of $ 1680?

157. A pedestrian walked a certain distance at the rate of $2\frac{1}{2}$ miles an hour. He rested an hour at the end of his journey and returned at the rate of 3 miles an hour. If he was out 12 hours, how far did he walk?

158. A and B are fifty-three miles apart. They travel towards each other, A $2\frac{1}{2}$ miles an hour, and B $1\frac{1}{2}$ miles an hour. If B starts two hours later than A, how far will each have traveled when they meet?

159. A lady bought silk at eighty cents a yard and had $ 2 left; but if she had bought it at ninety-five cents a yard, she would have needed twenty-five cents more to pay for it. How many yards of silk did she buy?

160. If 18 pounds of copper weighs 2 pounds less in water than in air, and 21 pounds of tin weighs 3 pounds less in water than in air, how many pounds of each are there in a mixture of the two metals, which weighs 450 pounds in air and 392 pounds in water?

161. The side of a square is four inches less than the length and three inches more than the side of an equivalent rectangle. Find the area of the rectangle.

162. A can do a piece of work in 24 days, which B can do in 20 days. B begins the work, but after a time he is relieved by A, who finishes the work, working two days more than B. How many days does each work?

163. A is 20 years older than B, but in two years A will be only twice as old as B. Find the age of each.

164. At what times between 10 and 11 o'clock are the hands of a clock at right angles to each other?

165. A man bought some oranges at the rate of two for five cents and twice as many more at the rate of five cents for three. He sold them all at thirty-five cents a dozen and gained $1.75. How many did he buy?

166. A merchant's profits are equal to $\frac{1}{5}$ of his capital. If he adds the profits, less $3000 for annual expenses, to his capital, at the end of three years his capital will be increased to $21,800. Find the original capital.

167. At what rate per annum will a dollars amount to b dollars in c years?

168. A merchant sold eighty pounds of tea, part of it at forty cents a pound and the rest at sixty cents a pound. The average price was forty-eight cents a pound. How many pounds did he sell at each price?

169. Two boys had equal sums of money. One bought some oranges at 5 cents apiece and had 3 cents left; the other bought $1\frac{1}{3}$ times as many oranges at $.36 a dozen and had 12 cents left. How much money had each?

170. A man spent half a dollar more than half his money. Then he spent half a dollar more than half of what remained. A third time he did the same, when he had nothing left. How much money had he at first?

171. A, B, and C can complete a piece of work in thirty hours, A doing half as much again as B, and B doing two-thirds as much again as C. In how many hours can each one complete the work alone?

172. At what times between 9 and 10 o'clock are the hands of a clock twenty minutes apart?

173. Eight years ago A was three times as old as B, but in eight years he will be only twice as old. Required the age of each at the present time.

174. A hare is pursued by a dog. The hare makes 5 leaps while the dog makes 4, but 5 leaps of the dog are equal to 7 of the hare. The hare has a start of 48 of her own leaps. How many leaps will the hare have made when caught?

175. An officer arranged part of his men in a solid square and had 80 men left; but upon attempting to form them into a column with 15 men more in front and 8 men less in depth, he found that he lacked 80 men to complete the column. How many men did he have?

176. A man invests $10,000, part of it at six per cent, and the rest at four per cent. The interest on the former for four years is the same as that on the latter for two years. Find the sum invested at four per cent.

177. A man can row 5 miles an hour on still water. How far down a river, whose current is 2 miles an hour, can he row so that he may go and return in ten hours?

178. A and B set out at the same time from two places and travel toward each other, A traveling $3\frac{1}{2}$ miles an hour and B $4\frac{1}{4}$ miles an hour. When they meet, B has traveled 12 miles more than A. How far apart were they?

179. A man bequeathed his property to his son and daughter, leaving the son fifteen hundred dollars more than $\frac{4}{9}$ of the whole. The son's share was to the daughter's share as 19 to 17. Find the amount of the estate.

180. A man paid thirty dollars for a hat and coat. The difference in the cost of the two was six times the cost of the hat. Find the cost of the coat.

181. Eighteen years ago a man was two-fifths as old as he will be in 12 years. What is his age at present?

182. At an election, two candidates together received 4229 votes, and the candidate elected received a majority of 521. How many votes did each receive?

183. A cask contains 12 gallons of wine mixed with 18 gallons of water, and another cask contains 18 gallons of wine mixed with six gallons of water. How many gallons must be drawn from each to make a mixture of fourteen gallons of wine and fourteen gallons of water?

184. At what times between 7 and 8 o'clock are the hands of a clock at right angles to each other?

185. The width of a rectangular piece of paper is 6 inches more than half its length; and if a strip 3 inches wide were cut off from the four sides, it would contain 360 sq. in. Find the dimensions of the paper.

186. A tank can be filled by two pipes in eighteen minutes. After the first has been running by itself for six minutes, the second is turned on, and the tank is filled in fourteen minutes more. In how many minutes can it be filled by each pipe separately?

187. I bought some oranges at the rate of five for three cents. I sold one-third of them at the rate of five cents for three, and the remainder at one cent apiece, gaining one dollar twelve cents. How many did I buy?

188. An officer can form his men into a hollow square six men deep, and also into a hollow square eight men deep. There are eight more men in the front of the first square than in the second. How many men has he?

189. A farmer sold wheat at 80 cents a bushel, rye at 75 cents a bushel, and corn at 60 cents a bushel, receiving for it all one hundred eighteen dollars. He sold ten bushels less of rye than of wheat, and twice as many bushels of corn as of rye. How many bushels of grain did he sell?

190. Of two consecutive numbers, $\frac{1}{4}$ of the less exceeds $\frac{1}{5}$ of the greater by two. Find the numbers.

191. A has twenty-five dollars less than B. A buys twenty-four sheep and has $18 left, and B buys twenty-one more than A at $1 less per head and has $25 left. At what price per head did each man buy his sheep?

192. A man invests $12,750. His income is equal to one-third of the investment. If he invests his income, less a fixed sum for annual expenses, at the same rate, at the end of three years his investment will amount to twenty-two thousand dollars. Find his annual expenses.

193. One boy takes from a basket one more than half the peaches in it, a second boy takes one more than half the remainder, and a third takes four more than half of those that still remained, and there were none left. How many peaches were there in the basket at first?

194. At what times between 11 and 12 o'clock are the hands of a clock twenty-five minutes apart?

195. B's age is to A's as 5 to 8, and the difference between their ages is 18 years. Find the age of each.

196. A and B can complete a piece of work in $2\frac{2}{3}$ days, A and C in three days, and B and C in $4\frac{4}{7}$ days. In how many days can each do the work alone?

197. A fox is pursued by a hound. The fox makes six leaps while the hound makes five, but two leaps of the hound are equal to three of the fox. The fox has a start equal to sixty of the dog's leaps. How many leaps will each have made when the fox is caught?

198. A farmer hired a laborer for a year, agreeing to pay him one hundred fourteen dollars and a suit of clothes. He worked only nine months and was entitled to receive $81 and the clothes. Find the value of the clothes.

199. A man had two farms of equal size. He sold 20 acres from one and 60 acres from the other, when he found that there remained $\frac{5}{7}$ as many acres in one as in the other. How many acres were there in each farm at first?

200. A man invested his fortune. The annual interest at 5 per cent on \$ 700 more than one-fourth of it exceeds by \$ 40 the interest at 4 per cent on \$ 500 less than one-third of it. Find the sum invested at 5 %.

201. A man set out at 6 A.M. and drove a certain distance at the rate of 5 miles an hour. He returned on foot, walking 2 miles an hour, and reached home at 5.30 P.M., having rested one hour at noon. How far did he walk?

202. A railway train leaves a station, and 40 minutes after, another train, which runs 30 miles an hour, starts out and overtakes the first in 2 hours and 40 minutes. How many miles per hour does the first train run?

203. A boy bought some oranges at a cents apiece and had b cents left; but if he had had c cents more, he could have bought the same number of better ones at d cents apiece. How many oranges did he buy?

204. If a pound of tin weighs $\frac{8}{7}$ of a pound in water, and a pound of lead weighs $\frac{21}{23}$ of a pound in water, how many pounds of each metal are there in a mixture of 120 pounds, if it weighs only 106 pounds in water?

205. A man bequeathed his entire estate to his three sons as follows: to his eldest son he leaves \$ 1000, together with $\frac{1}{4}$ of what remains; to the second son he leaves \$ 2000, together with $\frac{1}{4}$ of what remains after the eldest son's share and \$ 2000 have been deducted; to the third he leaves \$ 3000, together with $\frac{1}{4}$ of what remains after the portions of the two other sons and \$ 3000 have been deducted. Find the amount of the estate.

TO THE TEACHER. — Teach 237 . . . . 245.

SIMULTANEOUS EQUATIONS.

ELIMINATION BY ADDITION OR SUBTRACTION.

1. $\begin{cases} 2x+3y=7 \\ 3x-\ \ y=5 \end{cases}$

2. $\begin{cases} 3x-2y=-5 \\ 2x+\ \ y=-8 \end{cases}$

3. $\begin{cases} 4x-3y=7 \\ 2x-2y=2 \end{cases}$

4. $\begin{cases} 5x+4y=-7 \\ 2x+\ \ y=-1 \end{cases}$

5. $\begin{cases} 6x+3y=4 \\ 8x-6y=2 \end{cases}$

6. $\begin{cases} 2x+3y=-4 \\ 3x+7y=-1 \end{cases}$

7. $\begin{cases} 4x-3y=16 \\ 7x-5y=29 \end{cases}$

8. $\begin{cases} 3x-5y=-32 \\ 7x-9y=-40 \end{cases}$

9. $\begin{cases} 5x+3y=38 \\ 9y-3x=15 \end{cases}$

10. $\begin{cases} 6x+4y=-22 \\ 5y+7x=-20 \end{cases}$

11. $\begin{cases} 7x-3y=29 \\ 8x-5y=32 \end{cases}$

12. $\begin{cases} 5x+3y=-34 \\ 8y+9x=-30 \end{cases}$

13. $\begin{cases} \frac{6x}{5}+\frac{3y}{2}=24 \\ \frac{7x}{2}-\frac{5y}{4}=25 \end{cases}$

14. $\begin{cases} \frac{6x}{23}-\frac{5y}{2}=-13 \\ \frac{4y}{3}-3x=-15 \end{cases}$

15. $\begin{cases} \frac{5x}{4}+\frac{2y}{5}=26 \\ \frac{7y}{3}-\frac{3x}{2}=11 \end{cases}$

16. $\begin{cases} \frac{3x}{7}+\frac{5y}{3}=-21 \\ \frac{7y}{6}+\frac{4x}{12}=-14 \end{cases}$

17. $\begin{cases} \frac{6x}{5}-\frac{3y}{4}=30 \\ \frac{5y}{4}-\frac{5x}{10}=25 \end{cases}$

18. $\begin{cases} \frac{4x}{9}-\frac{5y}{7}=-45 \\ \frac{6y}{35}+\frac{9x}{5}=-75 \end{cases}$

To the Teacher. — Teach 246.

ELIMINATION BY SUBSTITUTION.

1. $\begin{cases} 4x-8y=8 \\ 3x+6y=9 \end{cases}$

2. $\begin{cases} 6x+4y=-4 \\ 3x+3y=-8 \end{cases}$

3. $\begin{cases} 3x-3y=6 \\ 4x-5y=3 \end{cases}$

4. $\begin{cases} 4x-5y=-4 \\ 3x-4y=-5 \end{cases}$

5. $\begin{cases} 4x+2y=4 \\ 8x+5x=3 \end{cases}$

6. $\begin{cases} 2x+2y=-3 \\ 6x+7y=-6 \end{cases}$

7. $\begin{cases} \frac{5x}{2}-\frac{3y}{2}=1 \\ \frac{3x}{4}+\frac{2y}{3}=7 \end{cases}$

8. $\begin{cases} \frac{3x}{7}-\frac{5y}{3}=-3 \\ \frac{9x}{14}-3y=-6 \end{cases}$

9. $\begin{cases} \frac{2x}{5}+\frac{2y}{7}=2 \\ \frac{5x}{3}+\frac{3y}{2}=4 \end{cases}$

10. $\begin{cases} \frac{3x}{4}+\frac{3y}{2}=-6 \\ \frac{5x}{6}+\frac{7y}{5}=-4 \end{cases}$

11. $\begin{cases} 9x-4y=22 \\ 6x+3y=43 \end{cases}$

12. $\begin{cases} 4x-7y=-35 \\ 3y-7x=-22 \end{cases}$

13. $\begin{cases} 5x-3y=48 \\ 4y-2x=20 \end{cases}$

14. $\begin{cases} 7x+6y=-70 \\ 4y+5x=-40 \end{cases}$

15. $\begin{cases} \frac{4x}{3}+\frac{4y}{3}=36 \\ \frac{6y}{5}-\frac{5x}{10}=12 \end{cases}$

16. $\begin{cases} \frac{3x}{8}+\frac{3y}{10}=-21 \\ \frac{8y}{5}-\frac{7x}{4}=-22 \end{cases}$

17. $\begin{cases} \frac{7x}{11}-\frac{2y}{13}=22 \\ \frac{4y}{3}-\frac{5x}{22}=42 \end{cases}$

18. $\begin{cases} \frac{4x}{9}+\frac{7y}{6}=-43 \\ \frac{5y}{3}+\frac{5x}{3}=-15 \end{cases}$

To the Teacher. — Teach 247 and 248.

ELIMINATION BY COMPARISON.

1. $\begin{cases} 2x+3y=38 \\ 3x+2y=37 \end{cases}$

2. $\begin{cases} 4x-5y=-27 \\ 3x-7y=-43 \end{cases}$

3. $\begin{cases} 4x+3y=20 \\ 8x+5y=34 \end{cases}$

4. $\begin{cases} 2x+5y=-32 \\ 2y-3x=-28 \end{cases}$

5. $\begin{cases} 7x-2y=13 \\ 2x+3y=43 \end{cases}$

6. $\begin{cases} 3x+2y=-33 \\ 5y-4x=-25 \end{cases}$

7. $\begin{cases} \frac{3x}{4}+\frac{3y}{2}=24 \\ \frac{7x}{3}-\frac{3y}{5}=22 \end{cases}$

8. $\begin{cases} \frac{4x}{7}+\frac{4y}{3}=-12 \\ \frac{2y}{5}-\frac{3x}{2}=-27 \end{cases}$

9. $\begin{cases} \frac{4x}{9}+\frac{5y}{7}=23 \\ \frac{7x}{2}-\frac{4y}{3}=35 \end{cases}$

10. $\begin{cases} \frac{4x}{3}-\frac{2y}{7}=-40 \\ \frac{5y}{4}+\frac{7x}{3}=-21 \end{cases}$

11. $\begin{cases} 8x-3y=17 \\ 6y+6x=43 \end{cases}$

12. $\begin{cases} 4x+9y=-22 \\ 6y-6x=-71 \end{cases}$

13. $\begin{cases} 7x-2y=11 \\ 7y-2x=29 \end{cases}$

14. $\begin{cases} 6x+8y=-14 \\ 4y-9x=-47 \end{cases}$

15. $\begin{cases} \frac{3x}{4}+\frac{2y}{5}=36 \\ \frac{7x}{4}-\frac{5y}{6}=31 \end{cases}$

16. $\begin{cases} \frac{7x}{2}+\frac{5y}{2}=-10 \\ \frac{4y}{3}-\frac{3x}{5}=-30 \end{cases}$

17. $\begin{cases} \frac{2x}{7}+\frac{5y}{6}=42 \\ \frac{3x}{2}-\frac{4y}{3}=15 \end{cases}$

18. $\begin{cases} \frac{3x}{7}+\frac{5y}{3}=-26 \\ \frac{9y}{2}-\frac{5x}{2}=-19 \end{cases}$

1. $\begin{cases} 5x+6y=17 \\ 6x+5y=16 \end{cases}$

2. $\begin{cases} 2x+3y=0 \\ 3x-4y=a \end{cases}$

3. $\begin{cases} \frac{2}{5}x+2y=90 \\ 2x-\frac{7}{5}y=62 \end{cases}$

4. $\begin{cases} 4x-3y=a \\ 3x-4y=a \end{cases}$

5. $\begin{cases} 4x+3y=23 \\ 7y+7x=21 \end{cases}$

6. $\begin{cases} ax+by=2 \\ cx-dy=a \end{cases}$

7. $\begin{cases} \frac{2}{5}x+3y=42 \\ 3x-\frac{4}{5}y=46 \end{cases}$

8. $\begin{cases} 2x-3y=a \\ 3x+5y=b \end{cases}$

9. $\begin{cases} \frac{3}{4}x-\frac{2}{5}y=20 \\ 3x+2y=24 \end{cases}$

10. $\begin{cases} ax-by=0 \\ bx-ay=c \end{cases}$

11. $\begin{cases} \frac{2}{5}x+4y=0 \\ 2y+\frac{7}{5}x=13 \end{cases}$

12. $\begin{cases} ax+by=c \\ a'x-b'y=c' \end{cases}$

13. $\begin{cases} \frac{2}{5}x-\frac{1}{5}y=17 \\ 5y-2x=15 \end{cases}$

14. $\begin{cases} ax-by=cd \\ bx+ay=ac \end{cases}$

15. $\begin{cases} cx+ay=2ac \\ ax+cy=a^2+c^2 \end{cases}$

16. $\begin{cases} x+y=a+c \\ ax-cy=c^2-a^2 \end{cases}$

17. $\begin{cases} ax+cy=a^2-c^2 \\ cx+ay=a^2-c^2 \end{cases}$

18. $\begin{cases} ax-cy=a^2-c^2 \\ ax+cy=(a+c)^2 \end{cases}$

19. $\begin{cases} a(x+y)+c(x-y)=3 \\ a(x-y)+c(x+y)=1 \end{cases}$

20. $\begin{cases} (a+c)x-(a-c)y=2 \\ (a-c)x+(a+c)y=2 \end{cases}$

21. $\begin{cases} (a+c)y-(a-c)x=ac \\ (a+c)x-(a-c)y=3ac \end{cases}$

To the Teacher. — Teach 249.

22. $\begin{cases} \frac{1}{x}+\frac{1}{y}=7 \\ \frac{1}{x}-\frac{1}{y}=1 \end{cases}$

23. $\begin{cases} \frac{5}{x}-\frac{4}{y}=12 \\ \frac{4}{x}+\frac{3}{y}=22 \end{cases}$

24. $\begin{cases} \frac{1}{x}+\frac{1}{y}=a \\ \frac{1}{x}-\frac{1}{y}=b \end{cases}$

25. $\begin{cases} \frac{3}{x}+\frac{2}{y}=22 \\ \frac{5}{x}-\frac{7}{y}=16 \end{cases}$

26. $\begin{cases} \frac{a}{x}+\frac{b}{y}=c \\ \frac{a}{x}-\frac{b}{y}=d \end{cases}$

27. $\begin{cases} \frac{4}{x}-\frac{3}{y}=11 \\ \frac{8}{y}+\frac{3}{x}=39 \end{cases}$

28. $\begin{cases} \frac{a}{x}+\frac{b}{y}=\frac{d}{c} \\ \frac{a}{x}-\frac{b}{y}=\frac{c}{d} \end{cases}$

29. $\begin{cases} \frac{3}{2x}+\frac{2}{3y}=5 \\ \frac{5}{3y}-\frac{6}{4x}=2 \end{cases}$

30. $\begin{cases} \frac{x}{a}+\frac{y}{b}=2 \\ \frac{x}{c}-\frac{y}{d}=3 \end{cases}$

31. $\begin{cases} \frac{3}{ax}+\frac{2}{by}=3 \\ \frac{2}{ax}-\frac{3}{by}=2 \end{cases}$

32. $\begin{cases} \frac{x}{a}-\frac{y}{b}=\frac{1}{2} \\ \frac{x}{c}+\frac{y}{d}=\frac{1}{3} \end{cases}$

33. $\begin{cases} \frac{a}{bx}+\frac{b}{ay}=c \\ \frac{c}{dx}+\frac{d}{cy}=a \end{cases}$

34. $\begin{cases} \frac{x}{a}+\frac{y}{b}=c \\ \frac{x}{b}-\frac{y}{a}=d \end{cases}$

35. $\begin{cases} \frac{a}{bx}+\frac{b}{ay}=\frac{c}{d} \\ \frac{c}{ax}-\frac{d}{by}=\frac{a}{b} \end{cases}$

36. $\begin{cases} \frac{x}{a}-\frac{y}{b}=\frac{c}{d} \\ \frac{x}{b}+\frac{y}{a}=\frac{d}{c} \end{cases}$

37. $\begin{cases} \frac{4x+5y+5}{7x-2y+9}=\frac{5}{4} \\ \frac{3x+3y+3}{4x-3y+4}=\frac{5}{2} \end{cases}$

38. $\begin{cases} \frac{a}{x}-\frac{b}{y}=1 \\ \frac{b}{x}+\frac{a}{y}=2 \end{cases}$

39. $\begin{cases} \frac{x+5}{2}-\frac{y+2}{3}=5 \\ \frac{y+8}{3}-\frac{x-3}{4}=3 \end{cases}$

40. $\begin{cases} \frac{a}{x}+\frac{b}{y}=\frac{2}{3} \\ \frac{b}{x}-\frac{a}{y}=\frac{3}{4} \end{cases}$

41. $\begin{cases} \frac{5}{x+4}+\frac{6}{y-3}=8 \\ \frac{9}{x+4}-\frac{4}{y-3}=7 \end{cases}$

42. $\begin{cases} \frac{a}{x}+\frac{b}{y}=c \\ \frac{b}{x}-\frac{a}{y}=d \end{cases}$

43. $\begin{cases} \frac{x+1}{y+1}-\frac{x-4}{y-1}=0 \\ \frac{y+4}{x-5}-\frac{y+2}{x-4}=0 \end{cases}$

44. $\begin{cases} \frac{a}{x}-\frac{b}{y}=\frac{c}{d} \\ \frac{b}{x}+\frac{a}{y}=\frac{d}{c} \end{cases}$

45. $\begin{cases} \frac{x}{a-c}+\frac{y}{a-d}=1 \\ \frac{x}{b-c}+\frac{y}{b-d}=1 \end{cases}$

46. $\begin{cases} \frac{a}{x}+\frac{2}{y}=c \\ \frac{b}{x}+\frac{3}{y}=d \end{cases}$

47. $\begin{cases} \frac{x}{a+c}+\frac{y}{a-c}=\frac{1}{a-c} \\ \frac{x}{a+c}-\frac{y}{a-c}=\frac{1}{a+c} \end{cases}$

48. $\begin{cases} \frac{3}{x}+\frac{2}{y}=a \\ \frac{2}{x}-\frac{3}{y}=b \end{cases}$

49. $\begin{cases} \frac{x}{a+c}+\frac{y}{a-c}=\frac{1}{a^2-c^2} \\ \frac{x}{a-c}+\frac{y}{a+c}=\frac{1}{a^2-c^2} \end{cases}$

To the Teacher. — Teach 250 . . . . 252.

1. $\begin{cases} 2x+2y-3z=3 \\ 4x-3y-2z=4 \\ 3x+3y-4z=6 \end{cases}$

2. $\begin{cases} x+y-15=0 \\ y+z-17=0 \\ x+z-16=0 \end{cases}$

3. $\begin{cases} 2x+3y-6z=4 \\ 3x-3y+2z=5 \\ 4x-6y+5z=4 \end{cases}$

4. $\begin{cases} x+y-15=14 \\ x+z-14=16 \\ y+z-18=13 \end{cases}$

5. $\begin{cases} 2x+3y=2+3z \\ 3x-5y=7-2z \\ 9x-4y=4+4z \end{cases}$

6. $\begin{cases} 2x-2y-4=0 \\ 4y-2z-4=0 \\ 5x-4z-3=0 \end{cases}$

7. $\begin{cases} 3x-4y+2z=15 \\ 2x+3y-3z=15 \\ 5y-5x+4z=22 \end{cases}$

8. $\begin{cases} 3x-2y-5=12 \\ 4x-2z-7=11 \\ 7y-3z-9=17 \end{cases}$

9. $\begin{cases} 5x+3y=3z+16 \\ 5y=33+3x-2z \\ 5z-4y-2x=11 \end{cases}$

10. $\begin{cases} 3x-3y-6=12 \\ 5x-6z-9=17 \\ 2y+2z-8=30 \end{cases}$

11. $\begin{cases} 7x-5y+3z=29 \\ 6y-4z+3x=30 \\ 5z-4x+2y=37 \end{cases}$

12. $\begin{cases} 5x-3z-10=0 \\ 4y-2z-22=0 \\ 2x+3y-61=0 \end{cases}$

13. $\begin{cases} x+y+z+u=30 \\ x-y+z+u=16 \\ x+y-z+u=12 \\ x+y+z-u=18 \end{cases}$

14. $\begin{cases} 3x+2y-4z=5 \\ 4x+3z-9u=6 \\ 5x-3y-4u=6 \\ 7y+3u-6z=5 \end{cases}$

15. $\begin{cases} x+y+z-u=12 \\ x+y-z+u=18 \\ x-y+z+u=14 \\ x+y+z+u=30 \end{cases}$

16. $\begin{cases} 5x+2y-4z=29 \\ 4x-3z+3u=46 \\ 3x+5y-5u=29 \\ 4y-4u+3z=34 \end{cases}$

17. $\begin{cases} x+y=6 \\ y+z=5 \\ z+u=4 \\ x-u=3 \end{cases}$

18. $\begin{cases} x+y+z=7 \\ y+z-u=3 \\ z+u+x=6 \\ u+x+y=8 \end{cases}$

19. $\begin{cases} x+y=a \\ y+z=b \\ x+z=c \end{cases}$

20. $\begin{cases} x+y+z=a \\ x+y-z=b \\ x-y+z=c \end{cases}$

21. $\begin{cases} \frac{1}{x}+\frac{1}{y}=7 \\ \frac{1}{y}+\frac{1}{z}=6 \\ \frac{1}{z}+\frac{1}{x}=9 \end{cases}$

22. $\begin{cases} \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=9 \\ \frac{1}{x}+\frac{1}{y}-\frac{1}{z}=5 \\ \frac{1}{y}+\frac{1}{z}-\frac{1}{x}=3 \end{cases}$

23. $\begin{cases} \frac{1}{x}+\frac{1}{y}=a \\ \frac{1}{y}+\frac{1}{z}=b \\ \frac{1}{z}+\frac{1}{x}=c \end{cases}$

24. $\begin{cases} \frac{a}{x}+\frac{b}{y}+\frac{c}{z}=a \\ \frac{a}{x}-\frac{b}{y}+\frac{c}{z}=b \\ \frac{b}{y}+\frac{c}{z}-\frac{a}{x}=c \end{cases}$

25. $\begin{cases} x+y=a \\ y+z=b \\ z+u=c \\ u-x=d \end{cases}$

26. $\begin{cases} x+y=a+b \\ y+z=a-c \\ z+u=a-b \\ u-x=c-a \end{cases}$

27. $\begin{cases} 2x+y=a \\ 2y+z=b \\ 2z+x=c \end{cases}$

28. $\begin{cases} 2x+y+z=a \\ 3y+z+x=b \\ 4z+x+y=c \end{cases}$

29. $\begin{cases} \frac{x}{3}+\frac{y}{7}-\frac{z}{4}=2 \\ \frac{x}{4}+\frac{y}{2}-\frac{z}{8}=8 \\ \frac{x}{2}-\frac{z}{2}+\frac{y}{2}=5 \end{cases}$

30. $\begin{cases} \frac{x}{5}+\frac{y}{7}-8=0 \\ \frac{y}{5}-\frac{z}{4}-4=0 \\ \frac{z}{3}+\frac{x}{3}-9=0 \end{cases}$

31. $\begin{cases} \frac{1}{x}+\frac{1}{y}-\frac{1}{z}=4 \\ \frac{1}{y}+\frac{1}{z}-\frac{1}{x}=6 \\ \frac{1}{z}+\frac{1}{x}-\frac{1}{y}=8 \end{cases}$

32. $\begin{cases} \frac{1}{x}+\frac{1}{y}-3=5 \\ \frac{1}{y}+\frac{1}{z}-9=8 \\ \frac{1}{z}+\frac{1}{x}-6=7 \end{cases}$

33. $\begin{cases} \frac{x}{a}+\frac{y}{b}-\frac{z}{c}=\frac{1}{2} \\ \frac{y}{b}+\frac{z}{c}-\frac{x}{a}=\frac{1}{2} \\ \frac{z}{c}+\frac{x}{a}-\frac{y}{b}=\frac{1}{2} \end{cases}$

34. $\begin{cases} \frac{x}{a}+\frac{y}{b}-3=0 \\ \frac{y}{b}+\frac{z}{c}-3=0 \\ \frac{z}{c}+\frac{x}{a}-3=0 \end{cases}$

35. $\begin{cases} \frac{a}{x}+\frac{b}{y}-\frac{c}{z}=ac \\ \frac{b}{y}+\frac{c}{z}-\frac{a}{x}=ab \\ \frac{c}{z}+\frac{a}{x}-\frac{b}{y}=bc \end{cases}$

36. $\begin{cases} \frac{a}{x}+\frac{b}{y}=a+4 \\ \frac{b}{y}+\frac{c}{z}=b+3 \\ \frac{c}{z}+\frac{a}{x}=a+2 \end{cases}$

37. $\begin{cases} \frac{1}{2}x+\frac{1}{6}y-\frac{1}{8}z=6 \\ \frac{1}{3}y+\frac{1}{4}z-\frac{1}{3}x=8 \\ \frac{1}{3}z+\frac{1}{6}x-\frac{1}{3}y=8 \end{cases}$

38. $\begin{cases} \frac{3}{5}x-\frac{3}{7}y-5=9 \\ \frac{5}{6}y-\frac{4}{5}z-8=7 \\ \frac{3}{5}z-\frac{1}{3}x-7=8 \end{cases}$

To THE TEACHER. — Show how problems involving simultaneous equations are solved. See 253 and examples following.

PROBLEMS INVOLVING SIMULTANEOUS EQUATIONS.

1. The sum of two numbers is sixty-eight, and their difference is thirty-four. Find the larger number.

2. The sum of two numbers is 72, and their difference exceeds $\frac{1}{8}$ of the smaller by nine. Find the numbers.

3. The sum of two numbers is 136, and two-thirds of the larger is equal to three-fourths of the smaller. How much does the larger number exceed the smaller?

4. The sum of the two digits of a number is eleven; and if twenty-seven be added to the number, the digits will be interchanged. Find the number.

5. If the greater of two numbers is divided by the less, the quotient is 5 and the remainder 2; but if 6 times the smaller is divided by the greater, the quotient is 1 and the remainder 7. Find the numbers.

6. If 3 be added to the numerator of a certain fraction, its value will be $\frac{1}{2}$; and if 1 be subtracted from the denominator, its value will be $\frac{1}{3}$. Find the fraction.

7. A farmer has two hundred twenty sheep in two fields. If thirty-five are taken out of the first field and put into the second, there will be the same number in each field. How many are there in each field?

8. Five times the greater of two numbers exceeds $\frac{1}{8}$ of the less by 97, and five times the less exceeds $\frac{1}{8}$ of the greater by 71. Find the sum of the numbers.

9. If eight be added to the sum of the two digits of a number, the result will be four times the left-hand digit; and if eighteen be added to the number, the digits will be interchanged. Find the number.

10. In an election one thousand four hundred ninety men voted for two candidates, and the candidate elected had a majority of two hundred forty-six. How many votes did each candidate receive?

11. A man sold 70 sheep for $240. He sold some of them at $3 a head and the rest at $4 a head. How many sheep did he sell at four dollars a head?

12. The sum of the ages of A and B is ninety-six years. If B were twice as old, his age would exceed A's age by eighteen years. Find the age of each.

13. If the sum of two numbers be divided by 6, the quotient will be 18 and the remainder 2; and if their difference be divided by 5, the quotient will be 15 and the remainder 3. Find the numbers.

14. The sum of the three digits of a number is 12. The digit in tens' place is $\frac{1}{2}$ the sum of the other two digits; and if 198 be added to the number, the first and last digits will be interchanged. Find the number.

15. Eight men and six boys earn seventeen dollars a day, and at the same wages, nine men and five boys earn $18.25. How much does each man earn per day?

16. When a man was married, his age was $\frac{4}{5}$ of his wife's age; eight years afterward his age was $\frac{5}{6}$ of his wife's age. How old were they when they were married?

17. If a rectangular plot of land were 10 feet longer and 20 feet wider, the area would be 3400 square feet more than it is; but if the length were 10 feet more and the width 20 feet less, the area would be 1800 square feet less than it is. Find the area of the plot.

18. If 2 be added to both terms of a certain fraction, its value will be $\frac{3}{4}$; and if 4 be subtracted from both terms, its value will be $\frac{1}{2}$. Find the fraction.

19. The sum of the digits of a number of two figures is 13; and if 45 be subtracted from the number, the digits will be interchanged. Find the number.

20. At an election A's majority was three hundred eighty-four, which was three-elevenths of the whole number of votes. How many votes did each receive?

21. A dealer bought 40 barrels of apples and 20 barrels of pears for $105. He sold the apples at a profit of 30%, and the pears at a profit of 20%, receiving for all $131. How much did he receive for the apples?

22. A and B can do a piece of work in 4 days; B and C can do it in 5 days; A and C can do it in $6\frac{2}{3}$ days. In how many days can each do the work?

23. Seven years ago B was three times as old as A, but in five years he will be only twice as old. Required the age of each seven years ago.

24. The sum of the two digits of a number is 12; and if the number be divided by the sum of its digits, the quotient will be seven. Find the number.

25. A, B, and C have $1380. If B gives A $70, A will have $80 more than B; but if B gets $60 from C, B and C will have the same sum. How much has each?

26. In three years a certain sum of money, at simple interest, will amount to $966, and in five years it will amount to $1050. Find the sum invested.

27. The average age of three persons is 60 years. The average age of the first and second is 52 years, and of the second and third is 70 years. Find the age of each.

28. A miller mixes corn worth $.60 a bushel with oats worth $.45 a bushel, making a mixture of 100 bushels, worth $.54 a bushel. How many bushels of corn does he use?

29. Two men are 12 miles apart. If they set out at the same time and travel in the same direction, they will be together in six hours; but if they travel in opposite directions toward each other, they will be together in two hours. At what rate do they travel?

30. A man can row 15 miles down a river in 3 hours, but it takes 5 times as long to row the same distance up the river. At what rate can he row on still water?

31. The sum of the first and last digits of a number exceeds six times the tens' digit by three. If the first and last digits be interchanged, the number will be diminished by 99; and if the digits in units' and tens' places be interchanged, the number will be increased by 5. Find the number.

32. A man paid \$11 for oranges, buying some of them at eighteen for 25 cents, and the rest at sixteen for 25 cents. He sold them all at 25 cents a dozen and gained \$4.50. How many oranges did he buy?

33. If the numerator of a fraction be doubled and 2 added to the denominator, its value will be $\frac{2}{5}$; and if the denominator be doubled and 3 added to the numerator, its value will be $\frac{3}{5}$. What is the fraction?

34. A and B together had \$99,000. After A had lost $\frac{2}{5}$ of his money in speculation and B $\frac{1}{3}$ of his, they together had \$61,000 left. How much had each at first?

35. A man invested \$20,000, partly in 4% bonds, and partly in 5% bonds. The annual income from the 4% bonds exceeds the annual income from the 5% bonds by \$125. How much did he invest at 5 per cent?

36. A and B can do a piece of work in a days; B and C can do it in b days; A and C can do it in c days. In how many days can each do the work?

37. Eight years ago a man was three times as old as his son, and five years hence four times the son's age will equal twice the father's age. Find the age of each.

38. A certain number exceeds five times the sum of its two digits by twenty; and if the number be divided by the left-hand digit, the quotient will be ten and the remainder five. Find the number.

39. After A had sold sixty-two sheep to B and eighteen to C, they each had the same number. Before A made these sales, he lacked only eight sheep of having as many as B and C together. How many have all?

40. If a rectangular piece of paper were three inches shorter and two inches wider, the area would be 11 square inches less; but if a strip two inches wide be cut off around the paper, the area will be diminished 124 square inches. Find the area of the paper.

41. A miller bought 40 bushels of corn and 30 bushels of oats for $45. At another time he bought at the same prices 43 bushels of oats and 55 bushels of corn for $62.75. Find the cost of four bushels of each.

42. A crew can row 12 miles down a river in 2 hours, but it takes them twice as long to row the same distance up the river. Find the rate of the current.

43. In nine months a certain sum of money, at simple interest, will amount to $996, and in sixteen months it will amount to $1024. Find the sum invested.

44. In 7 hours A walks 11 miles more than B does in 6 hours; and in 12 hours B walks 23 miles more than A does in 5 hours. How many miles does each walk per hour?

45. The sum of the two digits of a number is 15; and if the digits be interchanged, the resulting number will exceed the given number by 27. Find the number.

46. A man has silver dollars, half-dollars, and quarters, together worth $46. One-half of his dollars and one-fifth of his half-dollars are worth $12.50; one-seventh of his half-dollars and one-third of his quarters are worth six dollars. How many coins has he?

47. A and B can do a piece of work in $8\frac{4}{7}$ days; but if they work together six days, B can finish the work in six days more. In how many days can each do the work?

48. Two wheelmen race a mile, and A wins by 5 seconds. In the second trial B has a start of 56 yards and wins by 2 seconds. Assuming that each rides at a uniform rate, in how many seconds can each ride a mile?

49. A boy bought some peaches at the rate of three for five cents, and some at a cent apiece, paying $3 for all of them. He sold them all at two cents apiece and gained $1.40. How many peaches did he buy?

50. The numerator of the larger of two fractions is 5, and the numerator of the smaller fraction is 7. The sum of the fractions is $1\frac{5}{12}$; and if the numerators be interchanged, their sum will be $1\frac{7}{12}$. Find the fractions.

51. A man bought a piece of land. If he had bought 20 acres more for the same money, it would have cost $5 less per acre; but if he had bought ten acres less for the same money, it would have cost four dollars more per acre. How much did he pay for the land?

52. In m years a certain sum of money, at simple interest, will amount to a dollars, and in n years it will amount to b dollars. Find the sum invested.

53. A merchant sold thirty-six yards of silk, part of it at $1.25 a yard, and the rest at two dollars a yard, receiving fifty-seven dollars for the whole. How many yards of the better silk did he sell?

54. There are three numbers such that two-thirds of the first and three-fifths of the second together make eighty-five; two-thirds of the second and three-fifths of the third together make one hundred four; two-thirds of the third and three-fifths of the first together make ninety-six. Find the sum of the three numbers.

55. Two men race 440 yards. In the first trial A gives B a start of 13 seconds, and B wins by 20 yards; in the second trial A gives B a start of 88 yards, and the race is a tie. Assuming that each runs at a uniform rate, how many yards can each run in a minute?

56. The sum of the three digits of a number is 14. If the number, diminished by 10, be divided by the sum of its digits, the quotient will exceed 5 times the sum of the first and last digits by 8; and if 197 be subtracted from the number, all the digits will be the same as the tens' digit in the given number. Find the number.

57. A grocer has two measures. Twelve times the capacity of the larger or eighteen times that of the smaller will fill a certain vessel. If he fills the vessel, using both measures 16 times in all, how many times must he use each?

58. A man can row 3 miles with the current in 45 minutes, and 2 miles against the current in 1 hour and 20 minutes. At what rate can he row on still water?

59. If the length of a rectangle were five feet less and the width four feet more, the figure would be a square of the same area as the given rectangle. What is the area of the rectangle?

60. A tank has three pipes. A and B will empty it in 1 hour; B and C will empty it in 2 hours; A and C will empty it in 1 hour and 12 minutes. How many minutes will it take each pipe to empty the tank?

61. A pound of tea and 4 pounds of coffee cost \$2. At prices 30% higher, 2 pounds of tea and 6 pounds of coffee would cost \$4.42. Find the price of the tea.

62. A marketman bought eggs, some at 3 for 4 cents, and some at the rate of 5 cents for 4, paying \$4 for all of them. He found 22 that were unsalable, and he sold the remainder at 22 cents a dozen, making a profit of one dollar twenty-eight cents. How many did he buy?

63. A certain number is expressed by two figures. If the number be divided by the sum of its digits, the quotient will be seven and the remainder six. If the digits be interchanged and the resulting number divided by the sum of its digits, the quotient will be three and the remainder two. Find the number.

64. A man bought a certain number of sheep. If he had bought 5 more for the same money, they would have cost 50 cents less apiece; but if he had bought 5 less for the same money, they would have cost 75 cents more apiece. How much did he pay for the sheep?

65. A wife, son, and daughter inherit \$30,000. One-fifth of the wife's share added to one-third of the son's share and one-fourth of the daughter's share makes \$7500; and $\frac{1}{3}$ of the wife's share added to $\frac{1}{5}$ of the daughter's share exceeds $\frac{1}{2}$ of the son's share by \$1700. What part of the whole sum does the daughter receive?

66. If I invest my money at 5 per cent for a given time, I shall receive \$405 interest; but if I invest it at the same rate for $2\frac{1}{2}$ years longer, I shall receive \$1080 interest. How much money have I?

67. A and B can do a piece of work in a days; but if they work together b days, B can finish the work in c days more. In how many days can each do the work?

68. A boy bought at one time 4 apples and 5 pears for 16 cents; at another time, 3 pears and 6 peaches for 21 cents; at another time, 5 pears and 6 oranges for 28 cents; and at another time, 4 peaches and 5 oranges for 25 cents. Find the price of each kind of fruit.

69. Three boys played at marbles. B and C each won from A as many as they had at first; A and C each won from B as many as they then had; and A and B each won from C as many as they then had. Each boy then had 88 marbles. How many did A and B together have at first?

70. A boatman can row a miles down a river in m minutes and b miles up the river in n minutes. At what rate per hour can he row on still water?

71. A and B engaged in business, B having $2400 more capital than A. After A had lost one-fifth of his capital and B one-third of his, they found that they each had the same sum left. How much did both invest?

72. A merchant has tea worth 50 cents a pound, and another kind worth one dollar a pound. How many pounds of each kind must he take to make a mixture of twenty-five pounds, worth seventy-two cents a pound?

73. If a certain floor were 3 feet longer and one foot wider, it would require 7 square yards more of carpet to cover it; but if the length were 3 feet less and the width 2 feet less, it would require 8 square yards less of carpet to cover it. Find the area of the floor.

74. The hundreds' digit in a number exceeds the tens' digit by four times as much as the units' digit exceeds the tens'. If the number be divided by the sum of its digits, the quotient will be 70 and the remainder 3; and if 594 be subtracted from the number, the first and last digits will be interchanged. Find the number.

75. Half of A's money is \$ 1400 more than one-fifth of B's and C's together; five-ninths of B's is equal to the excess of A's over C's; and the sum of A's and C's lacks \$ 4000 of being 3 times B's. How much have all?

76. A merchant divided a sum of money among his four employees, giving A \$ 35 more than D. A received half as much as the other three; B received one-third as much as the other three; and C received one-fourth as much as the other three. How much did all receive?

77. In m months a certain sum of money, at simple interest, will amount to a dollars, and in n months it will amount to b dollars. Find the sum invested.

78. A boy bought a certain number of oranges. If he had bought 20 more for the same money, they would have cost half a cent apiece less; but if he had bought 20 less for the same money, they would have cost a cent apiece more. How much did he pay for the oranges?

79. A train, after running 1 hour from A toward B, met with an accident, which delayed it 30 minutes. The train then proceeded at $\frac{3}{4}$ of its former rate and arrived at B 90 minutes late. If the accident had happened 15 miles nearer A, the train would have been 100 minutes late. How far from B did the accident occur?

80. A boy expended one dollar for pears, buying some at two for a cent, and the rest at three for a cent. He sold them all at the rate of two cents for three and made fifty cents. How many pears did he buy?

81. A boatman can row down a river a distance of 15 miles, and back again, in 9 hours; and the current is such that he can row $3\frac{1}{2}$ miles up the stream in the same time that he can row 7 miles down the stream. How long does it take him to go down the fifteen miles?

82. A and B can do a piece of work in $6\frac{2}{3}$ days; B and C can do it in $8\frac{2}{11}$ days. If A can do $1\frac{1}{2}$ times as much as C, in how many days can each do the work?

83. A man has \$ 27,000 invested, partly at $3\frac{1}{2}$ per cent, partly at 4 per cent, and partly at 5 per cent, from which he receives an annual income of \$ 1126. The sum invested at $3\frac{1}{2}\%$ is equal to $\frac{4}{5}$ of the sum of the other two investments. Find the sum invested at $3\frac{1}{2}\%$.

84. Two athletes run half a mile. In the first trial A gives B a start of 88 yards and is beaten by 8 seconds; in the second trial B has a start of 36 seconds and is beaten by $7\frac{1}{3}$ yards. Assuming that each runs at a uniform rate, in how many minutes can each run a mile?

85. A certain number is expressed by two digits. If the number be divided by ten, the quotient will be seven and the remainder five; but if the number be divided by the sum of its digits, the quotient will be six and the remainder three. Find the number.

86. A, B, and C have farms adjoining. If A should buy 40 acres of B, A would have twice as many acres as B would have left; if B should buy 60 acres of C, C would have left $\frac{3}{5}$ as many acres as B would then have; if C should buy 125 acres of A, C would have 3 times as many acres as A would have left. How many acres have all?

87. A miller has rye worth 70 cents a bushel, corn worth 60 cents a bushel, and oats worth 40 cents a bushel. He wishes to make a mixture of 100 bushels of the three kinds of grain, worth 56 cents a bushel, and use twenty bushels of rye. How many bushels of corn must he use?

88. A dealer sold 50 barrels of flour at an average price of \$ 3.22, selling part of it at \$ 3 and the rest at \$ 3.50. How many barrels did he sell at \$ 3.50?

89. The difference between two fractions, which have the same denominator, is $\frac{2}{3}$. If 2 be added to the numerator of the smaller, its value will be $\frac{1}{2}$ of the larger; if 2 be subtracted from the numerator of the larger, its value will be 3 times the smaller. Find the fractions.

90. A grocer has three kinds of tea. A mixture of 4 pounds of the first and 6 pounds of the second is worth \$.52 a pound; a mixture of 6 pounds of the second and 4 pounds of the third is worth \$.68 a pound; a mixture of 5 pounds of the first and 5 pounds of the third is worth \$.60 a pound. Find the price of each kind of tea.

91. A railway train, after running 45 minutes from A toward B, met with an accident, which delayed it 20 minutes. The train then proceeded at $\frac{3}{4}$ of its former rate and arrived at B 65 minutes late. If the accident had happened 36 miles farther on, the train would have been only 50 minutes late. Find the distance from A to B.

92. A man made two investments, one at $3\frac{1}{2}$ per cent, and the other at $4\frac{1}{2}$ per cent. His income from both investments in two years was \$ 2600. If he had made the first investment at $4\frac{1}{2}$ per cent, and the second at $3\frac{1}{2}$ per cent, the income from both investments would have been \$ 2520. Find the whole sum invested.

93. A man bought 600 oranges, some at four for 10 cents, and the rest at six for 10 cents. He lost two dozen and sold the remainder at 35 cents a dozen, gaining \$ 3.80. How many of the better ones did he buy?

94. Two wheelmen race a mile. In the first trial A gives B a start of 176 yards and wins by 30 seconds; in the second trial A gives B a start of 1 minute and 9 seconds, and B wins by 66 yards. Assuming that each rides at a uniform rate, in what time can each ride a mile?

95. A cistern can be emptied by two pipes in 3 hours. If both pipes are open 2 hours, and then A is closed, it will take B 2 hours and 30 minutes longer to empty it. How long will it take each pipe to empty it?

96. The sum of the three digits of a number is ten, the tens' digit being zero. If the first and last digits be interchanged, the number will be increased by five hundred ninety-four; and if the number be divided by five times the sum of its digits, the quotient will be four and the remainder eight. Find the number.

97. A boatman rowed down a river, which flows at the rate of $1\frac{1}{2}$ miles an hour, going a certain distance in 1 hour and 12 minutes. It took him 3 hours and 36 minutes to return. How many miles did he row?

98. A man divided a sum of money equally among a certain number of boys. If there had been a more boys to receive the same sum, each one would have received m dollars less; but if there had been b less boys to receive the same sum, each one would have received n dollars more. Find the number of boys and each boy's share.

99. A, B, and C together had one hundred twenty dollars. A gave to B and C as much as each of them already had; B gave to A and C as much as each of them then had; C gave to A and B as much as each of them then had, and it was then found that they had equal amounts. How much did B and C together have at first?

100. A boy has fifteen dollars in 5-dollar bills, half-dollars, quarters, and dimes, and he has eight times as many coins as bills. If his half-dollars were quarters and his quarters half-dollars, he would have \$ 14.25; if his dimes were quarters and his quarters dimes, he would have \$ 15.15. How many pieces of money has he?

To the Teacher. — Teach 254 . . . . 263.

INVOLUTION.

1. $(a^2bc^3)^2$.
2. $(-xy^2z^3)^3$.
3. $(-a^4xy^3)^2$.
4. $(ax^2y^3)^2$.
5. $(-a^4cx^2)^4$.
6. $(-x^2y^2z)^3$.
7. $(2a^4x^3)^3$.
8. $(-b^3c^3d)^5$.
9. $(-a^4bc^3)^2$.
10. $(5a^2c^3)^2$.
11. $(-ax^2y^3)^4$.
12. $(-b^3c^3d)^6$.
13. $(a^2c^3x^4)^4$.
14. $(-3a^2x^4)^3$.
15. $(-x^5yz^4)^4$.
16. $(3ab^3c^3)^2$.
17. $(-2c^5d^3)^4$.
18. $(-ab^3c^3)^3$.
19. $(4c^3dx^3)^3$.
20. $(-4a^2x^3)^3$.
21. $(-b^2cd^3)^6$.
22. $(2a^3c^3x)^4$.
23. $(-5c^4x^3)^2$.
24. $(-c^3x^2y)^4$.
25. $(3ax^4y^3)^3$.
26. $(-6x^2z^3)^2$.
27. $(-a^3bc^3)^5$.
28. $(5b^3cd^2)^3$.
29. $(-a^3b^3c^4)^3$.
30. $(-x^4y^3z)^2$.
31. $(4a^2c^3x)^3$.
32. $(-x^2y^3z^2)^4$.
33. $(-3a^4c^3)^3$.
34. $(2ac^3d^4)^5$.
35. $(-b^3c^4d^3)^3$.
36. $(-2x^3y^3)^6$.
37. $(6b^3c^3x)^2$.
38. $(-a^2c^3x^3)^3$.
39. $(-4b^4c^3)^3$.
40. $(3a^4cd^3)^4$.
41. $(-b^3c^nd^2)^5$.
42. $(-5a^2x^3)^2$.
43. $(5x^2y^4z)^2$.
44. $(-a^nx^4y^3)^4$.
45. $(-x^3y^2z^4)^3$.
46. $(2bc^3d^4)^3$.
47. $(-c^3d^2x^n)^5$.
48. $(-a^2b^4c^3)^4$.
49. $(4a^5dx^3)^3$.
50. $(-x^ny^4z^3)^2$.
51. $(-b^3c^nx^2)^3$.
52. $(3ac^4x^3)^3$.
53. $(-a^3b^nc^3)^3$.
54. $(-c^nd^4x^3)^4$.
55. $(2a^4b^3c^3)^4$.
56. $(-b^4c^3d^n)^4$.
57. $(-x^2y^nz^3)^5$.
58. $(5a^nb^4c^3)^3$.
59. $(-x^ny^2z^4)^3$.
60. $(-a^4c^3x^n)^2$.
61. $(4c^3d^nx^3)^4$.
62. $(-a^5c^nx^3)^4$.
63. $(-b^nx^2y^4)^5$.
64. $(2a^4c^3x^n)^5$.
65. $(-b^4d^2y^n)^3$.
66. $(-x^5y^5z^n)^4$.
67. $(a^3b^nc^2d^4)^2$.
68. $(-x^ny^5z^3)^4$.
69. $(-c^2d^ny^3)^3$.
70. $(a^nb^3x^4y^5)^3$.
71. $(-a^6b^nc^4)^5$.
72. $(-x^5y^4z^n)^6$.

73. $\left(\frac{a^2bc^3}{xy^3z^2}\right)^2$.

74. $\left(\frac{cd^3x^2}{a^4by^3}\right)^3$.

75. $\left(-\frac{a^4b^2}{x^2y^5}\right)^2$.

76. $\left(\frac{c^3d^4x}{a^2by^5}\right)^3$.

77. $\left(\frac{2\,a^2x^3}{3\,c^4y^5}\right)^2$.

78. $\left(-\frac{c^3d^5}{x^4y^2}\right)^4$.

79. $\left(\frac{b^3c^5d}{ax^4y^3}\right)^2$.

80. $\left(\frac{4\,b^3c^3}{5\,a^3x^2}\right)^3$.

81. $\left(-\frac{a^4y^n}{b^3c^2}\right)^3$.

82. $\left(\frac{cd^2x^4}{a^2yz^5}\right)^4$.

83. $\left(\frac{5\,c^3d^4}{3\,x^5y^3}\right)^2$.

84. $\left(-\frac{b^4c^5}{x^ny^2}\right)^5$.

85. $\left(\frac{a^2bc^3}{d^4x^5y}\right)^n$.

86. $\left(\frac{2\,x^ny^2}{4\,a^4x^3}\right)^3$.

87. $\left(-\frac{a^nb^2}{c^4x^n}\right)^3$.

88. $\left(\frac{b^5c^3d}{ax^4y^2}\right)^n$.

89. $\left(\frac{8\,a^4c^5}{7\,x^3y^3}\right)^2$.

90. $\left(-\frac{a^5x^n}{b^nc^3}\right)^4$.

91. $\left(\frac{3\,a^3x^5}{2\,b^4y^5}\right)^4$.

92. $\left(\frac{5\,a^2b^3}{3\,c^4x^2}\right)^n$.

93. $\left(-\frac{x^ny^5}{a^2x^4}\right)^2$.

94. $\left(\frac{2\,c^nd^3}{5\,a^2x^n}\right)^3$.

95. $\left(\frac{3\,c^3d^4}{2\,x^2y^5}\right)^n$.

96. $\left(-\frac{a^4c^3}{b^nx^n}\right)^5$.

97. $\left(\frac{4\,a^5b^n}{3\,c^ny^3}\right)^2$.

98. $\left(\frac{8\,a^6y^5}{9\,b^4x^3}\right)^n$.

99. $\left(-\frac{b^nc^n}{a^3y^2}\right)^3$.

100. $\left(\frac{2\,x^ny^n}{3\,a^4c^3}\right)^4$.

101. $\left(\frac{6\,b^3c^4}{7\,a^2x^5}\right)^n$.

102. $\left(-\frac{x^4y^5}{a^nc^2}\right)^4$.

103. $\left(\frac{a^3c^nx^4}{b^nd^2y^n}\right)^3$.

104. $\left(\frac{3\,a^5x^5}{2\,b^4c^3}\right)^n$.

105. $\left(-\frac{a^2y^n}{c^nx^3}\right)^3$.

106. $\left(\frac{a^2b^4c^n}{x^3y^nz^5}\right)^4$.

107. $\left(\frac{c^3x^2y^4}{a^5b^6d^7}\right)^n$.

108. $\left(-\frac{b^nx^4}{a^5x^n}\right)^5$.

To the Teacher. — Teach 264 . . . . 270.

Powers of Binomials.

1. $(x+y)^3$.	**2.** $(a+x)^4$.	**3.** $(b-c)^4$.
4. $(a+b)^5$.	**5.** $(x-a)^3$.	**6.** $(c-b)^5$.
7. $(x+y)^6$.	**8.** $(b+x)^5$.	**9.** $(a-c)^6$.
10. $(b+x)^4$.	**11.** $(y-x)^5$.	**12.** $(a-c)^7$.
13. $(a-b)^6$.	**14.** $(b-a)^4$.	**15.** $(x-c)^5$.
16. $(x+y)^6$.	**17.** $(b-x)^5$.	**18.** $(a-x)^7$.
19. $(x-y)^5$.	**20.** $(a+x)^6$.	**21.** $(b-x)^7$.
22. $(x+b)^4$.	**23.** $(a-x)^5$.	**24.** $(a-y)^6$.
25. $(x+1)^5$.	**26.** $(1-a)^4$.	**27.** $(b-1)^6$.
28. $(1+x)^7$.	**29.** $(a-1)^6$.	**30.** $(1-x)^5$.
31. $(x-1)^5$.	**32.** $(1+y)^4$.	**33.** $(1-a)^6$.
34. $(b-2)^6$.	**35.** $(3-x)^5$.	**36.** $(a-4)^4$.
37. $(2-x)^5$.	**38.** $(a-3)^4$.	**39.** $(b-2)^5$.
40. $(x-2)^4$.	**41.** $(4+x)^4$.	**42.** $(3-y)^6$.
43. $(x^2+x)^4$.	**44.** $(a+a^3)^3$.	**45.** $(b^2-b)^5$.
46. $(a-a^2)^4$.	**47.** $(x^3-x)^5$.	**48.** $(b-b^3)^4$.
49. $(x^2-1)^5$.	**50.** $(2-a^3)^4$.	**51.** $(b^2-5)^4$.
52. $(\frac{2}{3}+x^2)^5$.	**53.** $(a^3-\frac{1}{2})^5$.	**54.** $(x^2-\frac{3}{4})^4$.
55. $(a^2+x^3)^3$.	**56.** $(x^3-y^2)^4$.	**57.** $(a^3-c^2)^5$.
58. $(\frac{1}{2}x-y^2)^5$.	**59.** $(a^3-\frac{1}{3}x)^4$.	**60.** $(\frac{1}{4}b-c^3)^3$.
61. $(2x+2y)^5$.	**62.** $(ab^2-2c)^5$.	**63.** $(2b-3c)^4$.
64. $(\frac{1}{2}x^2-2x)^5$.	**65.** $(3a-3x)^3$.	**66.** $(xy^2-3y)^5$.
67. $(2x^3+3y^2)^4$.	**68.** $(4a^3+\frac{1}{4}a)^4$.	**69.** $(\frac{1}{3}a^2-3a)^5$.
70. $(\frac{1}{2}a^2x-\frac{1}{3}x^2)^5$.	**71.** $(\frac{2}{3}xy^2+\frac{1}{2}x^3)^4$.	**72.** $(2a^3-\frac{1}{2}a^2c)^6$.

To the Teacher.—Teach 271.

Square of Polynomials.

1. $(a+b+c)^2$.
2. $(a-b+x-y)^2$.
3. $(x-y+z)^2$.
4. $(b+c-x-y)^2$.
5. $(a-x-2)^2$.
6. $(x-y-z+3)^2$.
7. $(3a+b-c)^2$.
8. $(2b+c-d+x)^2$.
9. $(b-2c-d)^2$.
10. $(a-2b-x-y)^2$.
11. $(2a-3b+1)^2$.
12. $(3x+2y-z-1)^2$.
13. $(2b-3+2c)^2$.
14. $(2a-1-x+2y)^2$.
15. $(1-2x-3y)^2$.
16. $(1+3c-2x-y)^2$.
17. $(a-2+b-x)^2$.
18. $(2x-1+3y-2z)^2$.
19. $(2a+3b-2c)^2$.
20. $(3a-2b+3x-y)^2$.
21. $(a+b-c-1)^2$.
22. $(1-2a+3c-2x)^2$.
23. $(1-x-y-z)^2$.
24. $(3x+2-3y-2z)^2$.

To the Teacher.—Teach 272 . . . . 283.

Evolution.

1. $\sqrt{a^6b^2c^{10}}$.
2. $\sqrt[3]{27x^3y^{12}z^6}$.
3. $\sqrt[5]{-32b^{10}c^5x^{15}}$.
4. $\sqrt[3]{x^3y^{15}z^9}$.
5. $\sqrt[4]{16a^8b^4c^{12}}$.
6. $\sqrt[3]{-27x^{12}y^6z^{15}}$.
7. $\sqrt[4]{\dfrac{b^4c^8x^{12}}{a^{16}d^4y^4}}$.
8. $\sqrt{\dfrac{25a^4b^{10}c^2}{36x^{12}y^6z^4}}$.
9. $\sqrt[3]{-\dfrac{8b^{12}c^3d^{15}}{27a^{18}x^{21}y^3}}$.
10. $\sqrt[4]{b^8c^{12}d^4}$.
11. $\sqrt{81c^4x^{10}y^2}$.
12. $\sqrt[5]{-32a^5b^{20}c^{15}}$.
13. $\sqrt[3]{\dfrac{a^6x^{12}y^9}{b^3c^6d^{15}}}$.
14. $\sqrt[4]{\dfrac{16c^8x^{12}y^4}{81a^4b^8d^{16}}}$.
15. $\sqrt[3]{-\dfrac{27a^6b^{18}x^{15}}{64c^{12}d^9y^{12}}}$.
16. $\sqrt{a^4b^{14}x^8}$.
17. $\sqrt[3]{64x^3y^{12}z^6}$.
18. $\sqrt[3]{-64x^{12}y^3z^{15}}$.
19. $\sqrt[n]{x^ny^{2n}z^n}$.
20. $\sqrt[n]{a^nb^{2n}x^{4n}y^{5n}}$.
21. $\sqrt[3]{-125b^3c^{3n}y^6}$.

22. $\sqrt{a^4b^{10}x^8}$.
23. $\sqrt[3]{27\,x^6y^3z^{12}}$.
24. $\sqrt[3]{-64\,b^{15}c^9d^3}$.
25. $\sqrt[4]{x^8y^4z^{12}}$.
26. $\sqrt[5]{32\,a^5x^{10}y^5}$.
27. $\sqrt[3]{-27\,a^6b^{12}c^3}$.
28. $\sqrt[3]{\dfrac{b^{21}c^3d^9}{a^6x^{12}y^3}}$.
29. $\sqrt[4]{\dfrac{16\,a^{16}b^4x^8}{81\,c^{20}d^8y^4}}$.
30. $\sqrt[5]{-\dfrac{a^5c^{15}d^5y^5}{32\,b^5x^{10}z^5}}$.
31. $\sqrt[n]{\dfrac{a^{2n}x^ny^n}{b^nc^{3n}d^n}}$.
32. $\sqrt{\dfrac{49\,x^{12}y^4z^2}{81\,a^6b^{10}d^4}}$.
33. $\sqrt[3]{-\dfrac{b^9c^{6n}d^3y^6}{64\,a^6x^{3n}z^3}}$.
34. $\sqrt[5]{x^{15}y^5z^{5n}}$.
35. $\sqrt[4]{81\,a^{16}b^8c^4}$.
36. $\sqrt[3]{-125\,b^6c^3d^{3n}}$.
37. $\sqrt[3]{b^{12}c^6d^{18}}$.
38. $\sqrt{49\,x^{2n}y^2z^4}$.
39. $\sqrt[7]{-128\,a^7b^{14}c^7}$.
40. $\sqrt[n]{a^{2n}c^nx^{4n}}$.
41. $\sqrt[3]{16\,a^6x^{3n}y^9}$.
42. $\sqrt[5]{-243\,x^5y^{20}z^5}$.
43. $\sqrt[4]{\dfrac{b^8x^{16}y^{4n}}{a^{12}c^4d^{4n}}}$.
44. $\sqrt[3]{\dfrac{a^nb^{3n}x^n}{27\,c^3d^3y^{3n}}}$.
45. $\sqrt[3]{-\dfrac{8\,b^6c^{3n}d^9}{216\,a^{15}x^9y^3}}$.
46. $\sqrt[n]{\dfrac{c^{2n}d^ny^{3n}}{a^{4n}b^{2n}x^n}}$.
47. $\sqrt[5]{\dfrac{b^5d^{5n}y^{15}}{32\,a^5c^{5n}x^5}}$.
48. $\sqrt[3]{-\dfrac{a^3b^{3n}y^9}{729\,c^3d^{3n}x^6}}$.
49. $\sqrt[4]{a^{12}b^4c^{4n}}$.
50. $\sqrt[5]{243\,x^{5n}y^5z^5}$.
51. $\sqrt[5]{-243\,x^{15}y^5z^{10}}$.
52. $\sqrt{x^{4n}y^2z^{2n}}$.
53. $\sqrt[3]{125\,a^{3n}b^6c^3}$.
54. $\sqrt[3]{-216\,a^{6n}b^3c^{3n}}$.
55. $\sqrt[n]{b^{4n}c^nd^{2n}}$.
56. $\sqrt[4]{256\,x^4y^{12}z^8}$.
57. $\sqrt[3]{-343\,a^{18}b^9c^{15}}$.
58. $\sqrt{\dfrac{a^{2n}x^4y^{10}}{b^2c^{4n}d^{12}}}$.
59. $\sqrt[3]{\dfrac{a^{6n}b^nc^{15}}{216\,x^6y^{3n}z^3}}$.
60. $\sqrt[3]{-\dfrac{27\,a^9x^{3n}y^3}{512\,b^{6n}c^6d^{3n}}}$.
61. $\sqrt[n]{\dfrac{5^na^{5n}b^{2n}}{c^{3n}d^nx^{4n}}}$.
62. $\sqrt[4]{\dfrac{256\,x^8y^{16}z^4}{625\,a^4b^{4n}c^8}}$.
63. $\sqrt[5]{-\dfrac{32\,a^{10}c^{5n}d^5}{243\,b^{20}x^5y^{5n}}}$.
64. $\sqrt[n]{x^{4n}y^nz^{6n}}$.
65. $\sqrt{144\,b^6c^{2n}d^2}$.
66. $\sqrt[3]{-512\,x^{21}y^9z^{3n}}$.
67. $\sqrt[n]{2^nx^{2n}y^{3n}}$.
68. $\sqrt[4]{256\,x^{8n}y^4z^8}$.
69. $\sqrt[5]{-1024\,x^5y^{5n}z^{\frac{1}{2}}}$

To the Teacher. — Teach 284 . . . . 287.

Square Root of Polynomials.

1. $x^4 - 2x + 2x^3 - x^2 + 1.$
2. $a^4 - 2a^3 - a^2 + 2a + 1.$
3. $x^4 + 6x^2 - 4x^3 + 1 - 4x.$
4. $a^4 - 6a + 7a^2 + 6a^3 + 1.$
5. $a^4 - 8a - 8a^3 + 18a^2 + 1.$
6. $x^4 + 5x^3 + 12x - 6x^2 + 4.$
7. $4x^4 - 7x^2 - 4x + 4x^3 + 4.$
8. $a^4 - 8a^3 + 24a + 10a^2 + 9.$
9. $x^4 - 16x + 12x^2 + 8x^3 + 4.$
10. $9x^4 + 13x^2 - 4x - 6x^3 + 4.$
11. $9a^4 + 4a + 12a^3 + 10a^2 + 1.$
12. $x^6 + 9x^3 + 24x^5 - 8x^5 + 10x^4.$
13. $48x - 24x^3 - 20x^2 + 9x^4 + 36.$
14. $x^2 + y^2 + z^2 + 2xy - 2xz - 2yz.$
15. $4a^4 - 16a^3 + 36a^2 - 40a + 25.$
16. $a^2 + b^2 + c^2 - 2ab + 2ac - 2bc.$
17. $81x^4 + 193x^2 - 88x - 198x^3 + 16.$
18. $36a^4 + 144 - 63a^2 + 216a - 108a^3.$
19. $4x^4 + 33x^2y^2 + 36y^4 - 36xy^3 - 12x^3y.$
20. $25a^4 - 31a^2x^2 + 30a^3x + 16x^4 - 24ax^3.$
21. $4x^6 - 10x^3 + 2x - 5x^2 + 12x^5 + 5x^4 + 1.$
22. $16x^2 - 40xy + 48xz + 25y^2 + 36z^2 - 60yz.$
23. $36a^4 - 60a^2c^2 - 36a^2x^2 + 25c^4 + 9x^4 + 30c^2x^2.$
24. $4a^{2n} - 12a^nb^n - 16a^nc^n + 9b^{2n} + 16c^{2n} + 24b^nc^n.$
25. $9x^{4n} + 24x^{2n}z^{2n} - 12x^{2n}y^n + 4y^{2n} + 16z^{4n} - 16y^nz^{3n}.$

26. $\frac{a^4}{x^4} - \frac{2a^3}{x^3} - \frac{a^2}{x^2} + \frac{2a}{x} + 1.$

27. $\frac{4x^4}{y^4} - \frac{4x^3}{y^3} - \frac{3x^2}{y^2} + \frac{2x}{y} + 1.$

28. $\frac{x^6}{y^6} - \frac{2x^4}{y^4} - \frac{4x^3}{y^3} + \frac{x^2}{y^2} + \frac{4x}{y} + 4.$

29. $\frac{x^4}{y^4} - \frac{4x^3}{3y^3} + \frac{2ax^2}{cy^2} + \frac{a^2}{c^2} - \frac{4a}{3c} + \frac{4}{9}.$

30. $\frac{x^4}{4} - x^2y^2 - \frac{5x^2}{2} + y^4 + 5y^2 + \frac{25}{4}.$

31. $25a^4 + 1\frac{5}{2}a^2x^2 - 10a^2 + \frac{9}{16}x^4 + \frac{3}{2}x^2 + 1.$

To the Teacher. — Teach 288 . . . . 296.

Square Root of Numbers.

Find the square root of the following numbers:

1. 18769.	2. 13734436.	3. 14722569.
4. 94249.	5. 33860761.	6. 16752649.
7. 210681.	8. 16224784.	9. 37810201.
10. 811801.	11. 24403600.	12. 15429184.
13. 419904.	14. 49984900.	15. 40883236.
16. 258064.	17. 35892081.	18. 63329764.

Find the square root to three decimal places:

1. 138.7.	2. $4\frac{2}{5}$.	3. .1.	4. $\frac{2}{3}$.
5. 64.05.	6. $7\frac{3}{4}$.	7. .3.	8. $\frac{5}{7}$.
9. 1.008.	10. $1\frac{1}{7}$.	11. .4.	12. $\frac{3}{4}$.
13. 904.2.	14. $6\frac{2}{3}$.	15. .7.	16. $\frac{5}{6}$.
17. 80.07.	18. $9\frac{5}{6}$.	19. .9.	20. $\frac{4}{5}$.
21. 3.504.	22. $3\frac{1}{3}$.	23. .8.	24. $\frac{2}{7}$.

To the Teacher. — Teach 297 . . . . 300.

Cube Root of Polynomials.

1. $8x^3 + 12x^2 + 6x + 1$.
2. $27x^3 - 27x^2 + 9x - 1$.
3. $27x^3 - 54x^2 + 36x - 8$.
4. $125x^3 + 75x^2 + 15x + 1$.
5. $64x^3 - 144x^2 + 108x - 27$.
6. $343a^3 - 294a^2b + 84ab^2 - 8b^3$.
7. $27a^3 - 108a^2xy + 144ax^2y^2 - 64x^3y^3$.
8. $x^6 - 3x^5 + 6x^4 - 7x^3 + 6x^2 - 3x + 1$.
9. $24x - 16x^3 + 6x^4 - 12x^2 + 6x^5 + x^6 - 8$.
10. $12x^4 - 6x^2 + 24x^5 - 16x^3 + 6x + 8x^6 - 1$.
11. $63x^2 - 54x - 44x^3 - 6x^5 + x^6 + 21x^4 + 27$.
12. $8x^6 + 66x^4 + 33x^2 - 9x - 63x^3 - 36x^5 + 1$.
13. $17a^2 - 3a - 18a^4 - 27a^5 + 6a^3 + 27a^6 - 1$.
14. $30x^4 - 25x^3 + 8x^6 - 12x + 30x^2 - 12x^5 + 8$.
15. $63a^4 - 6a - 44a^3 - 54a^5 + 27a^6 + 21a^2 + 1$.
16. $48a^5 - 80a^3 - 60a^4 - 8a^6 + 90a^2 + 108a + 27$.
17. $59x^3 - 75x + 8x^6 - 54x^4 - 12x^5 + 135x^2 - 125$.
18. $6a^2x^4 - 3a^4x^3 + a^6 - 11a^3x^3 + 3a^5x + 12ax^5 - 8x^6$.
19. $28x^3y^3 - 54x^5y + 27x^6 - 3x^2y^4 + 9x^4y^2 - 6xy^5 - y^6$.
20. $8a^6 + 42a^4x^2 - 9a^3x^3 + 36a^5x + 9ax^5 - 21a^2x^4 - x^6$.
21. $102a^2 - 144a^5 + 64a^6 - 36a + 204a^4 - 171a^3 + 8$.
22. $216x^6 - 432x^5 + 368x^3 + 18x^2 - 36x^4 - 27 - 108x$.
23. $440x^3 - 45x^4 - 108x^5 + 27x^6 - 588x + 105x^2 - 343$.
24. $48x^5y - 80x^3y^3 + 108xy^5 + 8x^6 + 60x^4y^2 - 90x^2y^4 - 27y^6$.
25. $240a^2x^4 - 192ax^5 + a^6 - 160a^3x^3 - 12a^5x + 60a^4x^2 + 64x^6$.

To the Teacher. — Teach 301 . . . . 307.

Cube Root of Numbers.

Find the cube root of the following numbers:

1. 59319.	2. 14348907.	3. 303464448.
4. 74088.	5. 44361864.	6. 131872229.
7. 140608.	8. 68417929.	9. 315821241.
10. 103823.	11. 62570773.	12. 354894912.
13. 195112.	14. 201230056.	15. 521660125.
16. 474552.	17. 743677416.	18. 714516984.

Find the cube root to three decimal places:

1. 4.3953.	2. .27.	3. $\frac{2}{5}$.	4. $7\frac{2}{3}$.
5. 66.973.	6. .64.	7. $\frac{1}{3}$.	8. $1\frac{1}{5}$.
9. 205.59.	10. .08.	11. $\frac{4}{7}$.	12. $5\frac{2}{7}$.
13. 9.1819.	14. .01.	15. $\frac{5}{6}$.	16. $8\frac{1}{3}$.
17. 55.132.	18. .47.	19. $\frac{7}{8}$.	20. $4\frac{2}{3}$.
21. 971.19.	22. .09.	23. $\frac{5}{9}$.	24. $3\frac{3}{4}$.

To the Teacher. — Teach 313 . . . . 319.

SIGNIFICATION OF EXPONENTS.

Express the following without fractional exponents:

1. $ax^{\frac{1}{2}}$.	2. $a^{\frac{1}{2}}x^{\frac{1}{2}}$.	3. $ab^{\frac{1}{2}}x^{\frac{1}{2}}$.	4. $ab^{\frac{1}{2}}xy^{\frac{1}{2}}$.
5. $ax^{\frac{2}{3}}$.	6. $a^{\frac{2}{3}}b^{\frac{1}{2}}$.	7. $a^{\frac{2}{3}}xy^{\frac{1}{2}}$.	8. $a^{\frac{1}{2}}b^{\frac{2}{3}}xy$.
9. $a^{\frac{2}{3}}x$.	10. $a^{\frac{1}{2}}x^{\frac{2}{3}}$.	11. $b^{\frac{2}{3}}c^{\frac{2}{3}}x$.	12. $bc^{\frac{2}{3}}x^{\frac{1}{2}}y$.
13. $a^{\frac{1}{n}}b$.	14. $x^{\frac{2}{3}}y^{\frac{2}{3}}$.	15. $ac^{\frac{1}{2}}x^{\frac{2}{3}}$.	16. $a^{\frac{1}{n}}bcx^{\frac{2}{n}}$.
17. $3x^{\frac{2}{n}}$.	18. $a^{\frac{1}{n}}x^{\frac{2}{n}}$.	19. $b^{\frac{2}{3}}x^{\frac{4}{3}}y$.	20. $a^{\frac{2}{3}}cx^{\frac{2}{3}}y$.
21. $2^{\frac{1}{3}}a$.	22. $a^{\frac{2}{n}}b^{\frac{n}{n}}$.	23. $a^{\frac{2}{n}}bx^{\frac{1}{2}}$.	24. $b^3c^{\frac{n}{3}}x^{\frac{1}{2}}y$.
25. $3^{\frac{n}{2}}x$.	26. $x^{\frac{n}{3}}y^{\frac{1}{3}}$.	27. $a^{\frac{1}{2}}b^{\frac{1}{n}}x$.	28. $a^{\frac{2}{3}}bx^3y^{\frac{2}{3}}$.
29. $2x^{\frac{2}{3}}$.	30. $3^{\frac{2}{3}}x^{\frac{2}{n}}$.	31. $xy^{\frac{n}{4}}z^{\frac{1}{2}}$.	32. $b^3c^{\frac{1}{n}}xy^{\frac{1}{3}}$.
33. $4^{\frac{2}{3}}a$.	34. $2^{\frac{n}{2}}a^{\frac{m}{n}}$.	35. $4^{\frac{2}{n}}xy^{\frac{1}{n}}$.	36. $3^{\frac{n}{3}}a^2x^{\frac{n}{3}}y$.

Express the following without the radical sign:

1. $\sqrt{a}$.	2. $2\sqrt[3]{x^3}\sqrt[4]{y^3}$.	3. $\sqrt[3]{ax^2}$.	4. $x\sqrt{a}\sqrt[3]{b^2}$.
5. $\sqrt[3]{x}$.	6. $3\sqrt{a^3}\sqrt[3]{x^2}$.	7. $\sqrt[4]{x^3y}$.	8. $y\sqrt[3]{c}\sqrt{x^3}$.
9. $\sqrt[n]{c}$.	10. $a\sqrt[5]{x^3}\sqrt[4]{y^3}$.	11. $\sqrt[n]{bc^3}$.	12. $2\sqrt[n]{x}\sqrt[4]{y^3}$.
13. $\sqrt[3]{a^2}$.	14. $x\sqrt[n]{a^2}\sqrt{c^n}$.	15. $\sqrt[5]{x^4y}$.	16. $3\sqrt{a}\sqrt[3]{x^n}$.
17. $\sqrt{x^3}$.	18. $4\sqrt[3]{x^n}\sqrt[4]{y^n}$.	19. $\sqrt[n]{ax^3}$.	20. $x\sqrt[5]{b}\sqrt{c^5}$.
21. $\sqrt[n]{c^3}$.	22. $a\sqrt{x^3}\sqrt[5]{y^3}$.	23. $\sqrt[4]{x^ny}$.	24. $y\sqrt[n]{a}\sqrt{x^n}$.
25. $\sqrt[3]{x^n}$.	26. $x\sqrt[n]{b^3}\sqrt{c^n}$.	27. $\sqrt[7]{ac^4}$.	28. $4\sqrt[4]{x}\sqrt[n]{y^4}$.

Express the following without negative exponents:

1. $2ax^{-2}$.	2. $x^{-2}y^{-3}$.	3. $3a^{-1}x^3$.	4. $ab^{-2}c^3d^{-1}$.
5. $3b^{-1}c$.	6. $a^{-3}x^{-1}$.	7. $2b^3c^{-3}$.	8. $a^{-1}b^3x^{-3}y$.
9. $5xy^{-3}$.	10. $b^{-2}c^{-1}$.	11. $4x^{-n}y^4$.	12. $ac^{-1}x^4y^{-2}$.
13. $4a^{-n}x$.	14. $x^{-1}y^{-5}$.	15. $5a^nx^{-3}$.	16. $b^3d^{-1}xy^{-3}$.
17. $3by^{-1}$.	18. $a^{-2}x^{-4}$.	19. $7b^{-1}a^4$.	20. $c^{-n}d^3x^{-1}y$.
21. $2x^{-2}y$.	22. $b^{-n}x^{-1}$.	23. $2x^2y^{-1}$.	24. $a^{-3}cd^nx^{-1}$.
25. $4ay^{-4}$.	26. $x^{-3}y^{-n}$.	27. $3a^{-4}x^3$.	28. $bc^{-1}x^3y^{-2}$.

Find the value of the following:

1. $4^{\frac{1}{2}}$.	2. $(-27)^{\frac{1}{3}}$.	3. $36^{-\frac{1}{2}}$.	4. $125^{\frac{2}{3}}$.
5. $9^{\frac{1}{2}}$.	6. $(-32)^{\frac{1}{5}}$.	7. $27^{-\frac{1}{3}}$.	8. $243^{\frac{2}{5}}$.
9. $8^{\frac{2}{3}}$.	10. $(-64)^{\frac{1}{3}}$.	11. $49^{-\frac{1}{2}}$.	12. $256^{\frac{3}{4}}$.
13. $27^{\frac{2}{3}}$.	14. $(-27)^{\frac{4}{3}}$.	15. $16^{-\frac{1}{4}}$.	16. $144^{\frac{1}{2}}$.
17. $25^{\frac{1}{2}}$.	18. $(-32)^{\frac{4}{5}}$.	19. $64^{-\frac{1}{3}}$.	20. $729^{\frac{1}{3}}$.
21. $64^{\frac{1}{3}}$.	22. $(-64)^{\frac{4}{3}}$.	23. $64^{-\frac{1}{3}}$.	24. $512^{\frac{2}{3}}$.
25. $81^{\frac{1}{4}}$.	26. $(-27)^{\frac{5}{3}}$.	27. $32^{-\frac{1}{5}}$.	28. $100^{\frac{1}{2}}$.

Find the numerical value of the following expressions when $a = 64$, $b = 8$, $c = 9$, $d = 3$, $n = 4$, $x = 2$, $y = 1$:

1. $a^{\frac{2}{3}}n^{-2}$. 2. $a^{\frac{1}{2}}n^{-\frac{1}{2}}x^2$. 3. $c(nx)^{\frac{1}{2}}$. 4. $2(ny^2)^{-\frac{1}{2}}$.
5. $c^{-\frac{2}{n}}x^3$. 6. $b^{\frac{2}{3}}c^{\frac{3}{2}}d^{-3}$. 7. $b(cd)^{\frac{2}{3}}$. 8. $3(cx^2)^{-\frac{1}{2}}$.
9. $b^{\frac{4}{3}}y^{-4}$. 10. $a^{-\frac{1}{2}}n^{\frac{5}{2}}y^3$. 11. $a(bx)^{\frac{4}{3}}$. 12. $d(ax^3)^{-\frac{2}{3}}$.
13. $a^{-\frac{5}{6}}x^n$. 14. $c^{\frac{3}{2}}d^{-1}y^{\frac{4}{3}}$. 15. $c(bn)^{\frac{3}{4}}$. 16. $9(c^2d)^{-\frac{1}{3}}$.
17. $b^{\frac{2}{3}}y^{-n}$. 18. $a^{\frac{1}{6}}c^{\frac{6}{n}}d^{-2}$. 19. $b(ab)^{\frac{3}{2}}$. 20. $c(n^3x)^{-\frac{2}{7}}$.
21. $a^{-\frac{1}{2}}n^2 + b^{\frac{4}{3}}x^{-2} \times c^2d^{-1}n^{\frac{1}{2}}$. 22. $b^{\frac{1}{3}}y^{-\frac{1}{2}} - d(a^{\frac{3}{2}}x^{-1} - b^{-\frac{3}{2}}nx)$.
23. $c^{\frac{3}{2}}d^{-1} + a^{-\frac{1}{3}}b^2 \div a^{\frac{1}{2}}x^{-2}y^3$. 24. $a^{\frac{2}{3}}b^{-\frac{1}{3}} \times x(c^{-\frac{1}{2}}d^3 + bn^{-\frac{1}{2}}y)$.
25. $x^5y^{-\frac{3}{4}} + c^{\frac{3}{2}}d^{-2} \times c^{-\frac{1}{2}}d^4y^3$. 26. $a^{-\frac{1}{3}}b^{\frac{1}{3}} - d(b^{\frac{2}{3}}x^{-2} - an^{-\frac{1}{2}}x)$.
27. $b^{\frac{4}{3}}y^{-4} - a^{\frac{4}{3}}x^{-2} \div b^{\frac{3}{2}}n^{-1}x^4$. 28. $c^{\frac{3}{2}}y^{-\frac{2}{3}} \times x(b^{-\frac{1}{2}}x^3 - c^{-\frac{1}{2}}dn)$.
29. $a^{-\frac{1}{3}}n^3 + c^{-\frac{1}{2}}d^2 \times a^{-\frac{2}{3}}b^2y^4$. 30. $b^{-\frac{2}{3}}n^{\frac{1}{2}} - 2(a^{\frac{2}{3}}y^{-2} - bn^{-\frac{2}{3}}y)$.
31. $c^{\frac{5}{2}}d^{-3} - b^{\frac{2}{3}}x^{-2} \times a^{\frac{1}{2}}d^2y^{-1}$. 32. $a^{\frac{2}{3}}n^{-\frac{1}{2}} - 3(b^{-\frac{1}{2}}n^3 - ab^{-\frac{1}{3}}x)$.

Perform the following indicated multiplications:

1. $3a \times 2a^{-2}$. 2. $x^2y^{-3} \times x^{-1}y$. 3. $b^{-\frac{2}{3}} \times a\sqrt{b^3}$.
4. $4a^{-1} \times 3a$. 5. $a^{-4}x^{-2} \times a^3x$. 6. $y^{-\frac{4}{3}} \times x\sqrt[3]{y^4}$.
7. $5x \times 2x^{-3}$. 8. $b^{-2}c \times b^3c^{-3}$. 9. $y^{-\frac{1}{2}} \times x\sqrt[3]{y^2}$.
10. $3a^3 \times 2a^{-1}$. 11. $ab^{-2} \times a^{-\frac{1}{2}}b^{\frac{1}{3}}$. 12. $a^{-2}x^{\frac{3}{5}} \times a\sqrt[5]{x^2}$.
13. $4x^{-5} \times 3x^2$. 14. $b^{-\frac{2}{3}}c^{\frac{1}{2}} \times bc^{-1}$. 15. $x^{-1}y^{\frac{1}{2}} \times x\sqrt[3]{y^2}$.
16. $3a^{-n} \times 2a^3$. 17. $x^{-1}y \times x^{\frac{2}{3}}y^{-\frac{1}{2}}$. 18. $a^{-\frac{1}{2}}x^{\frac{1}{2}} \times a\sqrt[3]{x^2}$.
19. $4x^{-1} \times 2x^n$. 20. $b^{-2}c \times b^{\frac{3}{2}}c^{-\frac{1}{2}}$. 21. $x^{\frac{1}{3}}y^{-\frac{1}{n}} \times x\sqrt[n]{y^2}$.
22. $a^{\frac{1}{3}}x^{-\frac{2}{3}} \times ax^{-1}$. 23. $x^{-\frac{4}{3}}y^{\frac{3}{4}} \times xy^{-1}$. 24. $a^{-\frac{5}{3}}b^{\frac{n}{2}} \times a\sqrt{b^n}$.
25. $a^{-3}b \times a^{\frac{7}{6}}b^{-\frac{5}{6}}$. 26. $b^{-\frac{1}{3}}c^{\frac{3}{2}} \times b^{-2}c$. 27. $x^{\frac{1}{n}}y^{-\frac{2}{n}} \times x\sqrt{y^n}$.
28. $a^{-\frac{3}{4}}x^{\frac{4}{5}} \times ax^{-1}$. 29. $x^{-1}y \times x^{\frac{3}{2}}y^{-\frac{5}{6}}$. 30. $a^{-\frac{2}{n}}x^{\frac{1}{2}} \times a\sqrt[n]{x^3}$.
31. $b^{-\frac{1}{2}}c^{\frac{3}{4}} \times b^{-1}c$. 32. $a^{-2}x \times a^{\frac{3}{4}}x^{-\frac{1}{2}}$. 33. $b^{\frac{4}{n}}c^{-\frac{3}{n}} \times b\sqrt[n]{c^4}$.

Multiply:

1. $a^{\frac{3}{4}}+b^{\frac{1}{4}}$ by $a^{\frac{1}{4}}+b^{\frac{3}{4}}$.
2. $x^{\frac{3}{2}}+y^{-\frac{1}{2}}$ by $x^{-\frac{1}{2}}-y^{\frac{3}{2}}$.
3. $a^{\frac{1}{2}}-x^{\frac{1}{3}}$ by $a^{\frac{1}{2}}-x^{\frac{1}{3}}$.
4. $x^{-\frac{2}{3}}-y^{\frac{2}{3}}$ by $x^{-\frac{1}{3}}+y^{\frac{2}{3}}$.
5. $a^{\frac{1}{3}}+b^{\frac{1}{2}}$ by $a^{\frac{1}{3}}+b^{\frac{1}{2}}$.
6. $x^{\frac{2}{3}}+y^{-\frac{2}{3}}$ by $x^{\frac{1}{3}}-y^{-\frac{1}{3}}$.
7. $a^{\frac{2}{3}}-x^{\frac{2}{5}}$ by $a^{\frac{1}{3}}-x^{\frac{2}{5}}$.
8. $x^{-\frac{1}{2}}-y^{\frac{1}{3}}$ by $x^{\frac{2}{3}}+y^{-\frac{1}{3}}$.
9. $a^{-2}-2a^{-1}+1-2a$ by $a^{-3}+2a^{-2}$.
10. $2a^{\frac{3}{4}}-3a^{\frac{1}{4}}-4+a^{-\frac{1}{4}}$ by $3a^{\frac{1}{4}}+a-2a^{\frac{3}{4}}$.
11. $3x^{-1}-x^{-2}y^{-1}+x^{-3}y^{-2}$ by $6x^3y^2+2x^2y+2x$.
12. $x^{\frac{3}{2}}y^{-\frac{1}{2}}+2x^{\frac{1}{2}}-3y^{\frac{1}{2}}$ by $2y^{-1}-4x^{-\frac{1}{2}}-6x^{-\frac{3}{2}}y^{\frac{1}{2}}$.
13. $3x^{\frac{3}{4}}y^{-1}+x^{\frac{1}{4}}-2x^{-\frac{1}{4}}y$ by $6x^{\frac{1}{4}}y^{-1}-2x^{-\frac{1}{4}}-3x^{-\frac{3}{4}}y$.
14. $x^{\frac{3}{2}}y^{-\frac{3}{2}}+2+x^{-\frac{3}{2}}y^{\frac{3}{2}}$ by $2x^{-\frac{3}{2}}y^{\frac{3}{2}}-4x^{-\frac{4}{2}}y^{\frac{4}{2}}+2x^{-\frac{5}{2}}y^{\frac{5}{2}}$.
15. $a^{-3}x^3-a^{-2}x-2a^{-1}$ by $2a^2x^{-1}+2a^3x^{-2}-4a^4x^{-3}$.

Write the following indicated products by inspection:

1. $(a^{\frac{1}{3}}+x^{\frac{1}{3}})(a^{\frac{1}{3}}+x^{\frac{1}{3}})$.
2. $(x^{\frac{1}{2}}+y^{\frac{1}{2}})(x^{\frac{1}{2}}-y^{\frac{1}{2}})$.
3. $(a^{\frac{2}{3}}-b^{\frac{2}{3}})(a^{\frac{2}{3}}-b^{\frac{2}{3}})$.
4. $(x^{\frac{2}{5}}-y^{\frac{2}{5}})(x^{\frac{2}{5}}+y^{\frac{2}{5}})$.
5. $(a^{\frac{1}{4}}+b^{\frac{1}{4}})(a^{\frac{1}{4}}+b^{\frac{1}{4}})$.
6. $(x^{\frac{2}{5}}+y^{\frac{2}{5}})(x^{\frac{2}{5}}-y^{\frac{2}{5}})$.
7. $(a^{\frac{3}{4}}-a^{\frac{1}{4}})(a^{\frac{3}{4}}-a^{\frac{1}{4}})$.
8. $(x^{\frac{2}{3}}+2y)(x^{\frac{2}{3}}+3y)$.
9. $(a^{\frac{3}{2}}+b^{-\frac{1}{2}})(a^{\frac{3}{2}}+b^{-\frac{1}{2}})$.
10. $(x^{\frac{1}{2}}+4y)(x^{\frac{1}{2}}-3y)$.
11. $(a^{\frac{2}{3}}-x^{-\frac{2}{3}})(a^{\frac{2}{3}}-x^{-\frac{2}{3}})$.
12. $(x^{\frac{2}{3}}-5a)(x^{\frac{2}{3}}+4a)$.
13. $(a^{\frac{1}{2}}+a^{-\frac{1}{2}})(a^{\frac{1}{2}}+a^{-\frac{1}{2}})$.
14. $(a^{\frac{1}{3}}+3x)(a^{\frac{1}{3}}+5x)$.
15. $(a^{\frac{7}{4}}-a^{-1})(a^{\frac{7}{4}}-a^{-1})$.
16. $(x^{\frac{5}{3}}-2a)(x^{\frac{5}{3}}-3a)$.
17. $(a^{\frac{3}{5}}+a^{-2})(a^{\frac{3}{5}}+a^{-2})$.
18. $(a^{\frac{3}{5}}+7a)(a^{\frac{3}{5}}-6a)$.
19. $(a^{\frac{1}{4}}-a^{-\frac{1}{4}})(a^{\frac{1}{4}}-a^{-\frac{1}{4}})$.
20. $(x^{\frac{1}{3}}-5x)(x^{\frac{1}{3}}-2x)$.
21. $(a^{\frac{3}{4}}+a^{-\frac{1}{2}})(a^{\frac{3}{4}}+a^{-\frac{1}{2}})$.
22. $(x^{\frac{3}{2}}-4x)(x^{\frac{3}{2}}+3x)$.

Perform the following indicated divisions:

1. $a^2b \div a^{-1}b^3$.
2. $3x \div x^{\frac{1}{2}}$.
3. $a^{\frac{1}{2}}b^{-1} \div a\sqrt[3]{b}$.
4. $ab^3 \div a^{-2}b^2$.
5. $2a \div b^{\frac{1}{2}}$.
6. $a^{-2}b^{\frac{1}{2}} \div a\sqrt{b}$.
7. $a^{-2}b^4 \div ab^5$.
8. $4x^{\frac{3}{2}} \div x$.
9. $x^{\frac{3}{2}}y^{-1} \div x\sqrt{y}$.
10. $ab^2 \div a^{-1}b^3$.
11. $5x \div a^{\frac{3}{2}}$.
12. $a^{\frac{3}{2}}b^{-2} \div a\sqrt[3]{b}$.
13. $a^3x \div a^4x^{-1}$.
14. $a \div 2a^{\frac{1}{2}}$.
15. $x^{-\frac{3}{2}}y^2 \div a\sqrt[3]{x}$.
16. $a^{\frac{3}{2}}b^{\frac{1}{2}} \div a^{-\frac{1}{2}}b^2$.
17. $a^{\frac{1}{2}} \div 3a^{\frac{1}{2}}$.
18. $a^{-\frac{1}{2}}b^3 \div a\sqrt[3]{b^2}$.
19. $x^{\frac{1}{2}}y^2 \div x^{\frac{3}{2}}y^{-\frac{1}{2}}$.
20. $x^{\frac{3}{4}} \div ax^{-\frac{1}{4}}$.
21. $x^{-2}y^{\frac{1}{2}} \div x\sqrt[4]{y^3}$.
22. $a^3b^{\frac{1}{3}} \div a^{\frac{1}{3}}b^{-\frac{1}{3}}$.
23. $a^{-\frac{2}{3}} \div ab^{-2}$.
24. $a^3b^{-\frac{1}{3}} \div b\sqrt[3]{x^2}$.
25. $x^2y^{\frac{1}{2}} \div x^{\frac{3}{2}}y^{-\frac{3}{2}}$.
26. $xy \div x^{-\frac{1}{2}}y^3$.
27. $x^{\frac{1}{2}}y^{-1} \div a\sqrt[3]{x^{-2}}$.
28. $a^{\frac{1}{2}}b^{\frac{5}{2}} \div a^{-\frac{3}{2}}b^3$.
29. $a^{\frac{3}{2}}b \div ab^{-\frac{3}{2}}$.
30. $a^{-2}b^{\frac{1}{2}} \div a^2\sqrt[4]{b^{-3}}$.
31. $x^{-1}y^{\frac{3}{2}} \div x^{\frac{3}{2}}y^{\frac{1}{2}}$.
32. $x^3y \div x^{-\frac{1}{2}}y^{\frac{3}{2}}$.
33. $x^{-\frac{1}{2}}y^{\frac{3}{2}} \div x^3\sqrt{y^{-3}}$.

Divide:

1. $x - y$ by $x^{\frac{1}{2}} + y^{\frac{1}{2}}$.
2. $a - b$ by $a^{\frac{1}{2}} - b^{\frac{1}{2}}$.
3. $x + y$ by $x^{\frac{1}{2}} + y^{\frac{1}{2}}$.
4. $a - b$ by $a^{\frac{1}{4}} - b^{\frac{1}{4}}$.
5. $x - y$ by $x^{\frac{1}{3}} - y^{\frac{1}{3}}$.
6. $a + b$ by $a^{\frac{1}{7}} + b^{\frac{1}{7}}$.
7. $x^{-2}y^2 + x^{-1}y - xy^{-1} - x^2y^{-2}$ by $x^2 + y^2$.
8. $a + b + c - 3a^{\frac{1}{3}}b^{\frac{1}{3}}c^{\frac{1}{3}}$ by $a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}}$.
9. $a^{-3} + a^{-2}b^{-1} + a^{-1}b^{-2} + b^{-3}$ by $a^{-1} + b^{-1}$.
10. $a^{-6} - 3a^{-4}x^{-2} + 3a^{-2}x^{-4} - x^{-6}$ by $a^{-2} - x^{-2}$.
11. $x^{-\frac{4}{3}} + x^{-\frac{2}{3}}y^{-2} + y^{-4}$ by $x^{-1} + x^{-\frac{2}{3}}y^{-1} + x^{-\frac{1}{3}}y^{-2}$.
12. $9a - 12a^{\frac{1}{2}} - 2 + 4a^{-\frac{1}{2}} + a^{-1}$ by $3a^{\frac{1}{2}} - 2 - a^{-\frac{1}{2}}$.
13. $2a^{-2} + 6a^{-1}x^{-1} - 16a^2x^{-4}$ by $2a + 2a^2x^{-1} + 4a^3x^{-2}$.
14. $a^{-3}x^{-5} - 3a^{-5}x^{-7} + a^{-7}x^{-9}$ by $a^{-2}x^{-3} + a^{-3}x^{-4} - a^{-4}x^{-5}$.

Simplify the following:

1. $(a^{\frac{1}{2}})^4$.
2. $(x^{-\frac{1}{2}})^3$.
3. $(2a^{\frac{1}{2}})^{-2}$.
4. $(5x^{-\frac{1}{2}})^{-2}$.
5. $(a^0)^{\frac{1}{2}}$.
6. $(x^{-0})^{\frac{1}{2}}$.
7. $(8a^2)^{-\frac{1}{3}}$.
8. $(4x^{-4})^{-\frac{1}{2}}$.
9. $(a^{\frac{3}{4}})^6$.
10. $(x^{-\frac{1}{2}})^4$.
11. $(3a^{\frac{1}{2}})^{-2}$.
12. $(2x^{-\frac{1}{4}})^{-4}$.
13. $(a^4)^{\frac{1}{2}}$.
14. $(x^{-0})^{\frac{3}{4}}$.
15. $(4a^4)^{-\frac{1}{2}}$.
16. $(8x^{-2})^{-\frac{1}{3}}$.
17. $(a^{\frac{3}{4}})^{\frac{1}{2}}$.
18. $(x^{-\frac{1}{2}})^{\frac{2}{3}}$.
19. $(8a^{\frac{6}{5}})^{-\frac{1}{3}}$.
20. $(9x^{-\frac{1}{3}})^{-\frac{1}{2}}$.
21. $(a^{\frac{4}{3}})^{\frac{3}{4}}$.
22. $(x^{-\frac{1}{4}})^{\frac{2}{3}}$.
23. $(9a^{\frac{2}{3}})^{-\frac{1}{4}}$.
24. $(8x^{-\frac{3}{5}})^{-\frac{1}{3}}$.

Expand the following to four terms:

1. $(1+x)^{\frac{1}{2}}$.
2. $(2a-1)^{-2}$.
3. $(a^{\frac{n}{2}}+x^{-\frac{1}{2}})^{-\frac{1}{n}}$.
4. $(x-1)^{\frac{1}{3}}$.
5. $(1+2a)^{-3}$.
6. $(x^{\frac{3}{5}}+y^{-\frac{1}{3}})^{-\frac{3}{4}}$.
7. $(2+x)^{\frac{3}{2}}$.
8. $(2a-3)^{-4}$.
9. $(a^{\frac{1}{2}}-x^{-\frac{2}{3}})^{-\frac{3}{4}}$.
10. $(x-3)^{\frac{2}{3}}$.
11. $(2+5a)^{-3}$.
12. $(\frac{1}{3}a^2+2x)^{-\frac{1}{2}}$.
13. $(1+x)^n$.
14. $(3a-1)^{-n}$.
15. $(\frac{1}{2}a^3-x^{-\frac{1}{4}})^{-\frac{1}{3}}$.

Extract the square root of the following:

1. $x^3y^{-\frac{3}{2}}-4x^{\frac{3}{2}}y^{-\frac{1}{2}}+6-4x^{-\frac{3}{2}}y^{\frac{1}{2}}+x^{-3}y^{\frac{3}{2}}$.
2. $a^{-\frac{2}{3}}+4a^{\frac{1}{3}}-10a^{-\frac{1}{3}}+4a^{\frac{4}{3}}-20a^{\frac{2}{3}}+25$.
3. $4x+12x^{\frac{1}{2}}y^{\frac{1}{2}}+4x^{\frac{1}{2}}z+9y^{\frac{2}{3}}+6y^{\frac{1}{3}}z+z^2$.
4. $25a^2x^{-2}+\frac{1}{4}x^2a^{-2}-20ax^{-1}-2xa^{-1}+9$.
5. $4a^{\frac{1}{2}}-20a^{\frac{3}{4}}b^{\frac{1}{3}}+37a^{\frac{1}{2}}b^{\frac{1}{2}}-30a^{\frac{1}{4}}b^{\frac{2}{3}}+9b^{\frac{1}{2}}$.
6. $4x^2y^{-2}+12xy^{-1}+25+24x^{-1}y+16x^{-2}y^2$.
7. $x^{\frac{8}{5}}-2a^{-\frac{8}{5}}x^{\frac{11}{5}}+2a^{\frac{4}{5}}x^{\frac{4}{5}}+a^{-\frac{6}{5}}x^{\frac{14}{5}}-2a^{\frac{1}{5}}x^{\frac{7}{5}}+a^{\frac{6}{5}}$.
8. $1+4x^{-\frac{1}{3}}-2x^{-\frac{2}{3}}-4x^{-1}+25x^{-\frac{4}{3}}-24x^{-\frac{5}{3}}+16x^{-2}$.

TO THE TEACHER. — Teach 325 . . . . 336.

RADICALS.

Reduce the following radicals to their simplest form:

1. $\sqrt[4]{9}$.
2. $3\sqrt{8}$.
3. $\sqrt[3]{8}$.
4. $a\sqrt{a^3}$.
5. $\sqrt[6]{8}$.
6. $2\sqrt{9}$.
7. $\sqrt{8}$.
8. $a\sqrt{x^5}$.
9. $\sqrt[4]{4}$.
10. $2\sqrt[3]{8}$.
11. $\sqrt[9]{8}$.
12. $x\sqrt{x^4}$.
13. $\sqrt{\frac{3}{4}}$.
14. $3\sqrt{\frac{4}{9}}$.
15. $\sqrt{\frac{8}{9}}$.
16. $\frac{a}{b}\sqrt{\frac{a^3}{b^2}}$.
17. $\sqrt{\frac{2}{3}}$.
18. $2\sqrt[3]{\frac{3}{8}}$.
19. $\sqrt[3]{\frac{8}{9}}$.
20. $\frac{b}{a}\sqrt{\frac{a^3}{b^3}}$.
21. $\sqrt[6]{\frac{1}{8}}$.
22. $2\sqrt[3]{\frac{1}{8}}$.
23. $\sqrt[4]{\frac{5}{8}}$.
24. $\frac{a}{x}\sqrt[3]{\frac{a^2}{x^2}}$.
25. $\sqrt[4]{49}$.
26. $4\sqrt{50}$.
27. $\sqrt[3]{24}$.
28. $a\sqrt{ab^3}$.
29. $\sqrt[3]{72}$.
30. $5\sqrt[3]{27}$.
31. $\sqrt[6]{27}$.
32. $x\sqrt{x^4y}$.
33. $\sqrt[9]{64}$.
34. $3\sqrt{75}$.
35. $\sqrt{98}$.
36. $b\sqrt[3]{a^4b}$.
37. $\sqrt{\frac{32}{81}}$.
38. $4\sqrt{\frac{27}{32}}$.
39. $\sqrt[6]{\frac{25}{36}}$.
40. $\frac{x}{y}\sqrt{\frac{a^2b}{xy^2}}$.
41. $\sqrt[3]{\frac{11}{25}}$.
42. $7\sqrt{\frac{54}{49}}$.
43. $\sqrt[5]{\frac{13}{16}}$.
44. $\frac{a}{b}\sqrt{\frac{a^3b}{xy^3}}$.
45. $\sqrt[4]{\frac{16}{27}}$.
46. $3\sqrt[6]{\frac{27}{64}}$.
47. $\sqrt{\frac{11}{12}}$.
48. $\frac{x}{y}\sqrt[4]{\frac{x^2y}{ab^3}}$.
49. $\sqrt{108}$.
50. $4\sqrt{121}$.
51. $\sqrt[6]{216}$.
52. $5x\sqrt{a^3x^5}$.
53. $\sqrt[3]{162}$.
54. $5\sqrt[9]{729}$.
55. $\sqrt[3]{243}$.
56. $a\sqrt{9\,a^2b^6}$.
57. $\sqrt[6]{144}$.
58. $2\sqrt{288}$.
59. $\sqrt{175}$.
60. $2a\sqrt[4]{a^4x^3}$.
61. $\sqrt{128}$.
62. $3\sqrt[3]{375}$.
63. $\sqrt[4]{800}$.
64. $2\sqrt[3]{8\,a^4x^7}$.
65. $\sqrt[3]{625}$.
66. $6\sqrt[4]{405}$.
67. $\sqrt[3]{540}$.
68. $3x\sqrt{x^5y^3}$.

69. $\sqrt{55\,a^3x^3}$.
70. $2\sqrt[4]{64\,x^2y^4}$.
71. $ab\sqrt{98\,a^3b^5}$.
72. $\sqrt[3]{64\,a^3b^3}$.
73. $3\sqrt[5]{96\,a^5x^3}$.
74. $xy\sqrt[3]{56\,x^2y^4}$.
75. $\sqrt[4]{81\,a^4x^6}$.
76. $2\sqrt[3]{27\,x^5y^3}$.
77. $ax\sqrt{23\,a^3x^3}$.
78. $\sqrt{\frac{11\,a^3b}{12\,c^3x^3}}$.
79. $3\sqrt[4]{\frac{16\,a^3x^3}{27\,b^4y^4}}$.
80. $ac\sqrt[5]{\frac{53\,a^5b^6}{16\,c^4d^4}}$.
81. $\sqrt[3]{\frac{11\,a^2c^3}{25\,b^3d^3}}$.
82. $4\sqrt{\frac{13\,a^2x^2}{18\,b^3y^3}}$.
83. $cd\sqrt[3]{\frac{64\,a^3b^6}{27\,c^3d^3}}$.
84. $\sqrt[4]{\frac{16\,a^4x^4}{81\,b^8y^8}}$.
85. $8\sqrt[3]{\frac{27\,a^3b^3}{32\,x^3y^3}}$.
86. $ac\sqrt[6]{\frac{27\,a^3b^3}{64\,c^3d^3}}$.
87. $\sqrt[3]{192\,a^4b^3}$.
88. $2\sqrt[4]{176\,a^5c^4}$.
89. $ax\sqrt[6]{512\,a^3x^6}$.
90. $\sqrt[4]{100\,x^2y^3}$.
91. $3\sqrt{396\,a^3b^5}$.
92. $ab\sqrt[3]{750\,a^5b^4}$.
93. $\sqrt[4]{243\,a^5x^3}$.
94. $4\sqrt[5]{352\,x^2y^3}$.
95. $ax\sqrt[4]{225\,a^2x^4}$.
96. $\sqrt{\frac{343\,a^5b^4}{144\,c^3x^4}}$.
97. $5\sqrt{\frac{448\,a^3b^3}{225\,x^2y^2}}$.
98. $ax\sqrt[3]{\frac{189\,b^3c^3}{100\,a^2x^2}}$.
99. $\sqrt[6]{\frac{125\,a^3x^3}{343\,b^3y^3}}$.
100. $6\sqrt{\frac{121\,b^2c^3}{144\,x^2y^3}}$.
101. $xy\sqrt{\frac{243\,a^3b^3}{256\,x^3y^3}}$.
102. $\sqrt[4]{\frac{256\,a^3x^3}{625\,b^6y^6}}$.
103. $5\sqrt{\frac{539\,a^3c^3}{400\,x^2y^3}}$.
104. $xy\sqrt[6]{\frac{125\,c^3d^3}{729\,x^6y^6}}$.
105. $\sqrt[3]{222\,a^4bc^3}$.
106. $2\sqrt{450\,a^3bc^3}$.
107. $3\,a\sqrt[3]{1000\,a^3x^6}$.
108. $\sqrt[3]{1029\,a^4x^3}$.
109. $2\sqrt[4]{3125\,a^5b^3}$.
110. $2\,a\sqrt[6]{1600\,a^6b^6}$.
111. $\sqrt[4]{1250\,x^4y^5}$.
112. $3\sqrt[3]{2187\,x^2y^3}$.
113. $3\,x\sqrt[4]{2500\,x^4y^5}$.
114. $\sqrt[3]{\frac{297\,a^2b^3c^4}{512\,x^3y^2z^3}}$.
115. $7\sqrt[4]{\frac{160\,a^5c^3x^4}{243\,b^4d^8y^4}}$.
116. $\frac{by}{ax}\sqrt[3]{\frac{125\,a^6c^3x^6}{343\,b^6d^3y^6}}$.
117. $\sqrt{\frac{110\,a^2b^3c^4}{144\,x^2y^4z^3}}$.
118. $9\sqrt[3]{\frac{176\,a^3b^3c^4}{243\,x^2y^3z^3}}$.
119. $\frac{dy}{cx}\sqrt[4]{\frac{121\,a^2c^4x^3}{256\,b^4d^2y^4}}$.

120. $\sqrt[6]{3^3a^3 \times 2^6a^6 \times 3^6a^3}$.

121. $(x+1)\sqrt{(x^2-1)(x-1)}$.

122. $\sqrt[4]{2^3x^3 \times 2^2x^4 \times 2^4x^3}$.

123. $(1-x)\sqrt{(1-x)(1-x^2)}$.

124. $\sqrt[3]{3^3a^3 \times 9^2a^4 \times 2^3a^3}$.

125. $(a+2)\sqrt{(a^2-4)(a-2)}$.

126. $\sqrt[5]{2^3x^3 \times 8^3x^2 \times 4^2x^3}$.

127. $(a-x)\sqrt{(a^2-x^2)(a-x)}$.

128. $\sqrt[6]{3^4a^2 \times 3^6a^4 \times 3^2a^5}$.

129. $(a+x)\sqrt{(a-x)(a^2-x^2)}$.

130. $\sqrt[7]{5^4x^3 \times 5^6x^5 \times 5^5x^3}$.

131. $(a-3)\sqrt{(a^2-16)(a+4)}$.

132. $\sqrt[5]{8^4a^3 \times 4^3a^4 \times 2^4a^5}$.

133. $(a+4)\sqrt{(a-5)(a^2-25)}$.

134. $2a\sqrt{a^3-2a^2b+ab^2}$.

135. $(a+3)\sqrt{(a-3)(a^2-27)}$.

136. $2\sqrt{2x^3+18x+12x^2}$.

137. $(2-b)\sqrt{(4+b)(64+b^3)}$.

To the Teacher. — Teach 337 and 338.

Express the following as radicals of the second degree:

1. $2a^2$.
2. $3x^2y$.
3. $5a^2x^3$.
4. $5x^{\frac{1}{2}}y^n$.
5. $6a^3$.
6. $7x^4z$.
7. $3a^5x^4$.
8. $5x^{\frac{1}{2}}y^n$.
9. $9a^4$.
10. $8x^3y$.
11. $4a^3x^3$.
12. $7x^{\frac{1}{2}}y^n$.

Express the following as radicals of the third degree:

1. $3a^3$.
2. $2xy^3$.
3. $4a^2x^3$.
4. $5x^{\frac{1}{3}}y^n$.
5. $6a^3$.
6. $8xy^3$.
7. $7a^5x^4$.
8. $3x^{\frac{1}{3}}y^n$.
9. $2a^5$.
10. $4xy^4$.
11. $5a^3x^3$.
12. $6x^{\frac{1}{3}}y^n$.

Express the following as entire surds:

1. $2\sqrt{3}$.
2. $-2\sqrt[3]{3a}$.
3. $a\sqrt[4]{5a^3}$.
4. $-3a\sqrt{a^3x}$.
5. $3\sqrt[3]{2}$.
6. $-3\sqrt{5x}$.
7. $x\sqrt[6]{3x^3}$.
8. $-2x\sqrt[5]{x^3y}$.
9. $2\sqrt[5]{2}$.
10. $-2\sqrt[5]{3a}$.
11. $a\sqrt[3]{4a^4}$.
12. $-5a\sqrt[4]{a^4x}$.
13. $3\sqrt[4]{2}$.
14. $-3\sqrt[4]{2x}$.
15. $x\sqrt[7]{2x^3}$.
16. $-2x\sqrt[7]{x^2y}$.

17. $\frac{2}{3}\sqrt{18}$. 18. $-2\sqrt[5]{\frac{3}{4}}$. 19. $2a\sqrt[3]{3a}$. 20. $-1\frac{1}{2}\sqrt[4]{8a}$.

21. $\frac{1}{2}\sqrt[4]{32}$. 22. $-3\sqrt{\frac{2}{3}}$. 23. $3x\sqrt[4]{2x}$. 24. $-2\frac{1}{3}\sqrt[3]{9x}$.

25. $\frac{3}{4}\sqrt[3]{32}$. 26. $-5\sqrt[3]{\frac{4}{5}}$. 27. $7a\sqrt{5a}$. 28. $-3\frac{1}{3}\sqrt{6a}$.

29. $\frac{2}{3}\sqrt[4]{\frac{27}{64}}$. 30. $-\frac{3}{2}\sqrt[4]{\frac{4}{9}}$. 31. $\frac{2a}{3b}\sqrt[3]{\frac{9b}{8a}}$. 32. $-\frac{2\frac{1}{2}}{3\frac{1}{3}}\sqrt[3]{\frac{8a}{9x}}$.

33. $\frac{3}{5}\sqrt{\frac{50}{27}}$. 34. $-\frac{4}{5}\sqrt[3]{\frac{5}{8}}$. 35. $\frac{a^2x}{b^2y}\sqrt[4]{\frac{by}{ax}}$. 36. $-\frac{2\frac{1}{2}}{1\frac{1}{3}}\sqrt[4]{\frac{5a}{8b}}$.

37. $\frac{a+b}{a-b}\sqrt{\frac{a-b}{a+b}}$. 38. $(x-2)\sqrt{\frac{x+2}{x-2}}$. 39. $\frac{a-x}{a+x}\sqrt{\frac{a-x}{a+x}}$.

To the Teacher. — Teach 339.

Reduce the following to radicals of the same degree:

1. $\sqrt{2a}$ and $\sqrt[3]{3a^2b}$.
2. $\sqrt{a^2x}$, $\sqrt[3]{xy^3}$, and $\sqrt{2a^4b}$.
3. $\sqrt[3]{2x}$ and $\sqrt[4]{3xy^3}$.
4. $\sqrt[6]{ax^3}$, $\sqrt[3]{x^2y}$, and $\sqrt{3b^2c}$.
5. $\sqrt[6]{9a}$ and $\sqrt{2a^3b}$.
6. $\sqrt[3]{a^2x}$, $\sqrt[6]{xy^3}$, and $\sqrt[4]{2x^2y}$.
7. $\sqrt[5]{3a}$ and $\sqrt[3]{2xy^3}$.
8. $\sqrt[3]{ax^4}$, $\sqrt{b^3y}$, and $\sqrt[3]{6ax^3}$.
9. $\sqrt[6]{7a}$ and $\sqrt[4]{5a^3x}$.
10. $\sqrt[4]{a^3b}$, $\sqrt{a^2x}$, and $\sqrt[8]{9x^4y}$.
11. $\sqrt{2a}$ and $\sqrt[5]{6ab^2}$.
12. $\sqrt[6]{ac^4}$, $\sqrt[3]{a^2b}$, and $\sqrt[4]{4xy^3}$.
13. $\sqrt[3]{4x}$ and $\sqrt[9]{8x^2y}$.
14. $\sqrt{a^3b}$, $\sqrt{xy^3}$, and $\sqrt[3]{7a^4x}$.
15. $\sqrt[8]{7a}$ and $\sqrt{3ab^3}$.
16. $\sqrt{ax^3}$, $\sqrt[4]{x^4y}$, and $\sqrt[3]{9ab^4}$.
17. $\sqrt[4]{6x}$ and $\sqrt[3]{3x^2y}$.
18. $\sqrt[4]{a^3b}$, $\sqrt[3]{ax^3}$, and $\sqrt[6]{9x^5y}$.
19. $\sqrt{4a}$ and $\sqrt[n]{5ab^2}$.
20. $\sqrt[3]{ab^2}$, $\sqrt[n]{a^2c}$, and $\sqrt{4ax^3}$.
21. $\sqrt[n]{7x}$ and $\sqrt[m]{4x^2y}$.
22. $\sqrt[n]{a^3x}$, $\sqrt[m]{xy^2}$, and $\sqrt[n]{3x^2y}$.

To the Teacher. — Teach 340.

Addition and Subtraction of Radicals.

1. $2\sqrt{13}+\sqrt{52}+3\sqrt{13}$.
2. $\sqrt{16\,a}+\sqrt{49\,a}-\sqrt{25\,a}$.
3. $2\sqrt{27}+\sqrt{48}+2\sqrt{75}$.
4. $\sqrt{18\,a}-\sqrt{32\,a}+\sqrt{72\,a}$.
5. $5\sqrt{10}-\sqrt{90}+4\sqrt{40}$.
6. $\sqrt{44\,a}+\sqrt{99\,a}-\sqrt{11\,a}$.
7. $3\sqrt{18}+\sqrt{98}-2\sqrt{72}$.
8. $\sqrt[3]{64\,a}-\sqrt[3]{24\,a}+\sqrt[3]{27\,a}$.
9. $4\sqrt{56}-\sqrt{14}+3\sqrt{14}$.
10. $\sqrt{36\,a^3}-\sqrt{16\,a^3}+\sqrt{81\,a^3}$.
11. $7\sqrt{24}+\sqrt{54}-4\sqrt{96}$.
12. $\sqrt[3]{27\,a^4}+\sqrt[3]{16\,a^4}+\sqrt[3]{64\,a^4}$.
13. $5\sqrt{99}-\sqrt{11}-2\sqrt{44}$.
14. $\sqrt{45\,a^5}-\sqrt{20\,a^5}+\sqrt{80\,a^5}$.
15. $8\sqrt{80}+\sqrt{20}-7\sqrt{45}$.
16. $\sqrt[4]{16\,a^5}+12\sqrt[4]{a^5}-\sqrt[4]{81\,a^5}$.
17. $7\sqrt{60}-\sqrt{15}-\sqrt{135}$.
18. $\sqrt{28\,a^3}+\sqrt{63\,a^3}-2\sqrt{7\,a^3}$.
19. $9\sqrt[3]{16}-\sqrt[3]{54}-\sqrt[3]{128}$.
20. $\sqrt{96\,a^5}-\sqrt{24\,a^5}+\sqrt{54\,a^5}$.
21. $5\sqrt{63}-\sqrt{28}-\sqrt{112}$.
22. $\sqrt[3]{16\,a^5}-\sqrt[3]{64\,a^4}+\sqrt[3]{54\,a^5}$.
23. $4\sqrt[3]{81}+\sqrt[3]{24}-\sqrt[3]{192}$.
24. $\sqrt{75\,a^2}+\sqrt{48\,a^2}-\sqrt{27\,a^2}$.
25. $3\sqrt{68}+\sqrt{17}-\sqrt{153}$.
26. $\sqrt[3]{16\,a^3}-\sqrt[3]{80\,a^3}+\sqrt[3]{54\,a^3}$.
27. $6\sqrt{48}-\sqrt{12}-\sqrt{108}$.
28. $\sqrt[4]{16\,a^5}-\sqrt[4]{80\,a^5}+\sqrt[4]{81\,a^5}$.
29. $3\sqrt{45}+\frac{1}{4}\sqrt{48}-3\sqrt{27}+4\sqrt{20}-6\sqrt{12}$.
30. $4\sqrt{63}+5\sqrt{45}-2\sqrt{28}-\frac{3}{4}\sqrt{80}+5\sqrt{112}$.
31. $7\sqrt{40}-2\sqrt{10}+\frac{3}{5}\sqrt{90}+4\sqrt{160}-3\sqrt{250}$.
32. $3\sqrt[3]{24}-2\sqrt[3]{81}+\frac{3}{4}\sqrt[3]{192}-4\sqrt[3]{375}+8\sqrt[3]{648}$.
33. $14\sqrt{\frac{5}{7}}+\frac{2}{3}\sqrt{189}-5\sqrt{\frac{21}{25}}+3\sqrt{84}-\sqrt{21}-15\sqrt{\frac{7}{5}}$.
34. $5\sqrt{8\,a^3}+7\sqrt{2\,a^3}-\frac{2}{3}\sqrt{18\,a^3}+3\sqrt{72\,a^3}-\sqrt{162\,a^3}$.
35. $7\sqrt{60}-12\sqrt{\frac{5}{3}}+20\sqrt{\frac{3}{5}}-\frac{4}{3}\sqrt{135}+8\sqrt{\frac{15}{16}}-\sqrt{15}$.
36. $\frac{1}{3}\sqrt{315}-\sqrt{35}+21\sqrt{\frac{5}{7}}+7\sqrt{\frac{35}{49}}-2\sqrt{140}+20\sqrt{\frac{7}{5}}$.

37. $4\sqrt{\frac{1}{4}} - \sqrt{\frac{1}{3}} + \frac{1}{3}\sqrt{3}$.

38. $\frac{1}{2}\sqrt{\frac{1}{2}} + \frac{3}{4}\sqrt{8} - 2\sqrt{\frac{1}{8}}$.

39. $6\sqrt{\frac{4}{5}} + \sqrt{\frac{1}{5}} - \frac{1}{5}\sqrt{5}$.

40. $\frac{4}{5}\sqrt{\frac{3}{5}} - \frac{4}{5}\sqrt{6} + 3\sqrt{\frac{2}{3}}$.

41. $4\sqrt[3]{\frac{2}{3}} - \sqrt[3]{\frac{1}{3}} + \frac{1}{3}\sqrt[3]{9}$.

42. $\frac{2}{3}\sqrt{\frac{3}{4}} + \frac{1}{3}\sqrt{3} - 6\sqrt{\frac{1}{3}}$.

43. $3\sqrt{\frac{7}{9}} + \sqrt{\frac{1}{7}} - \frac{5}{7}\sqrt{7}$.

44. $\frac{3}{4}\sqrt{\frac{2}{3}} + \frac{1}{2}\sqrt{8} - 3\sqrt{\frac{3}{8}}$.

45. $2\sqrt{1\frac{1}{3}} - \sqrt{3} + \frac{1}{2}\sqrt{1\frac{2}{25}}$.

46. $\frac{3}{4}\sqrt{\frac{5}{3}} + \frac{1}{3}\sqrt{60} - 5\sqrt{\frac{3}{5}}$.

47. $3\sqrt{4\frac{1}{2}} - \sqrt{2} + \frac{1}{3}\sqrt{1\frac{1}{25}}$.

48. $\frac{3}{4}\sqrt{\frac{7}{3}} - \frac{1}{3}\sqrt{84} + 7\sqrt{\frac{3}{7}}$.

To the Teacher. — Teach 341.

Multiplication of Radicals.

1. $3\sqrt{3} \times 2\sqrt{8}$.

2. $5\sqrt{5} \times \sqrt{20}$.

3. $2\sqrt{6} \times 4\sqrt{3}$.

4. $4\sqrt{3} \times \sqrt{27}$.

5. $3\sqrt{2} \times 2\sqrt{8}$.

6. $7\sqrt{3} \times \sqrt{15}$.

7. $4\sqrt{3} \times 2\sqrt{2}$.

8. $6\sqrt{2} \times \sqrt{18}$.

9. $2\sqrt{8} \times 5\sqrt{6}$.

10. $4\sqrt[3]{2} \times \sqrt[3]{32}$.

11. $2\sqrt[3]{3} \times 3\sqrt[3]{9}$.

12. $5\sqrt[3]{3} \times \sqrt[3]{45}$.

13. $3\sqrt{5} \times 2\sqrt{5}$.

14. $2\sqrt[5]{9} \times \sqrt[5]{27}$.

15. $2\sqrt[4]{6} \times 4\sqrt[4]{8}$.

16. $4\sqrt[3]{3} \times \sqrt[3]{18}$.

17. $4\sqrt[5]{4} \times 2\sqrt[5]{8}$.

18. $5\sqrt[4]{3} \times \sqrt[4]{27}$.

19. $3\sqrt{3} \times 3\sqrt[3]{2}$.

20. $2\sqrt{5} \times \sqrt{28}$.

21. $2\sqrt[3]{3} \times 3\sqrt{2}$.

22. $3\sqrt[5]{9} \times \sqrt[5]{54}$.

23. $5\sqrt{2} \times 2\sqrt[4]{8}$.

24. $2\sqrt[3]{9} \times \sqrt[6]{27}$.

25. $4\sqrt{5} \times 5\sqrt[3]{2}$.

26. $4\sqrt{3} \times \sqrt[3]{81}$.

27. $3\sqrt[4]{8} \times 4\sqrt[3]{8}$.

28. $2\sqrt[3]{9} \times \sqrt{90}$.

29. $3\sqrt{\frac{2}{3}} \times \sqrt{\frac{5}{7}}$.

30. $\frac{1}{2}\sqrt{\frac{5}{6}} \times \frac{2}{3}\sqrt{\frac{3}{4}} \times 1\frac{1}{2}\sqrt{\frac{2}{3}}$.

31. $2\sqrt[3]{\frac{1}{3}} \times \sqrt[6]{\frac{1}{3}}$.

32. $\frac{1}{3}\sqrt{\frac{2}{3}} \times \frac{2}{3}\sqrt{\frac{4}{5}} \times 1\frac{1}{3}\sqrt{\frac{5}{6}}$.

33. $4\sqrt{\frac{3}{4}} \times \sqrt[3]{\frac{7}{8}}$.

34. $\frac{2}{5}\sqrt{\frac{5}{7}} \times \frac{3}{5}\sqrt{\frac{3}{5}} \times 2\frac{1}{2}\sqrt{\frac{7}{9}}$.

35. $7\sqrt[3]{\frac{3}{5}} \times \sqrt{\frac{4}{7}}$.

36. $\frac{2}{3}\sqrt[3]{\frac{2}{3}} \times \frac{3}{4}\sqrt{\frac{1}{2}} \times 2\frac{2}{3}\sqrt[3]{\frac{3}{4}}$.

37. $6\sqrt{\frac{5}{6}} \times \sqrt[3]{\frac{3}{5}}$.

38. $\frac{1}{4}\sqrt[3]{\frac{1}{3}} \times \frac{4}{7}\sqrt{\frac{3}{4}} \times 3\frac{1}{2}\sqrt[3]{4}$.

39. $4\sqrt[4]{\frac{2}{3}} \times \sqrt[3]{\frac{1}{3}}$.

40. $\frac{1}{2}\sqrt[3]{\frac{3}{5}} \times \frac{3}{5}\sqrt{\frac{5}{6}} \times 1\frac{2}{3}\sqrt[3]{3}$.

41. $3\sqrt[3]{\frac{5}{6}} \times \sqrt{\frac{3}{5}}$.

42. $\frac{5}{6}\sqrt[3]{\frac{5}{6}} \times \frac{6}{7}\sqrt[3]{\frac{4}{3}} \times 1\frac{2}{3}\sqrt{5}$.

43. $\sqrt{2a} \times \sqrt{3a^3}$.

44. $\sqrt{3a} \times \sqrt{6a^3} \times \sqrt{8ax^3}$.

45. $\sqrt{3x} \times \sqrt{6x^3}$.

46. $\sqrt{2a} \times \sqrt{5a^3} \times \sqrt{6ax^4}$.

47. $\sqrt{2x} \times \sqrt{8x^5}$.

48. $\sqrt[3]{4a} \times \sqrt{2a^3} \times \sqrt[3]{6ax^3}$.

49. $\sqrt{8a} \times \sqrt{6a^3}$.

50. $\sqrt{3x} \times \sqrt[3]{6x^3} \times \sqrt[6]{9ax^3}$.

51. $\sqrt[3]{4a} \times \sqrt[6]{8a^3}$.

52. $\sqrt[3]{4a} \times \sqrt[3]{6x^2} \times \sqrt[3]{9ax^4}$.

53. $\sqrt[3]{9x} \times \sqrt{3x^5}$.

54. $\sqrt[4]{2x} \times \sqrt[3]{4a^3} \times \sqrt[6]{2ax^5}$.

55. $\sqrt[4]{8a} \times \sqrt[3]{2a^4}$.

56. $\sqrt{3a} \times \sqrt[3]{9x^3} \times \sqrt{5ax^3}$.

57. $\sqrt[3]{9x} \times \sqrt[4]{9x^3}$.

58. $\sqrt[4]{9x} \times \sqrt{3a^3} \times \sqrt[3]{6ax^4}$.

59. $(5\sqrt{50} - \sqrt{18} + \sqrt{72} + 2\sqrt{32}) \times 5\sqrt{3}$.

60. $(\sqrt{27} + 2\sqrt{75} - \sqrt{48} + 3\sqrt{12}) \times 3\sqrt{2}$.

61. $(\sqrt[3]{24} + 5\sqrt[3]{375} - \sqrt[3]{81} - 2\sqrt[3]{192}) \times 2\sqrt[3]{9}$.

62. $(3\sqrt{45} + \sqrt{80} + 2\sqrt{180} - 3\sqrt{245}) \times 4\sqrt{2}$.

63. $(3\sqrt[3]{32} + 5\sqrt[3]{500} - 4\sqrt[3]{256} + \sqrt[3]{108}) \times 2\sqrt[3]{6}$.

64. $(7\sqrt{150} + 2\sqrt{54} - 2\sqrt{96} + 3\sqrt{216}) \times 3\sqrt{3}$.

65. $(\sqrt{48} - 2\sqrt{45} + \sqrt{75} - \sqrt{80} - \sqrt{27}) \times 2\sqrt{5}$.

66. $(\sqrt{72} - \sqrt{63} + 4\sqrt{50} - \sqrt{28} - \sqrt{98}) \times 5\sqrt{2}$.

To the Teacher. — Teach 342.

Multiply:

1. $3+\sqrt{2}$ by $5-\sqrt{2}$.
2. $4-\sqrt{a}$ by $2+\sqrt{a}$.
3. $5+\sqrt{3}$ by $3+2\sqrt{3}$.
4. $2-\sqrt{a}$ by $5-3\sqrt{a}$.
5. $2+3\sqrt{2}$ by $4-5\sqrt{2}$.
6. $3-2\sqrt{a}$ by $2+4\sqrt{a}$.
7. $10+2\sqrt{5}$ by $3-3\sqrt{5}$.
8. $2a-3\sqrt{a}$ by $a+4\sqrt{a}$.
9. $\sqrt{3}-4\sqrt{6}$ by $\sqrt{3}+2\sqrt{6}$.
10. $\sqrt{a}+3\sqrt{x}$ by $\sqrt{a}+5\sqrt{x}$.
11. $2\sqrt{6}-3\sqrt{5}$ by $\sqrt{6}-2\sqrt{5}$.
12. $3\sqrt{a}+2\sqrt{x}$ by $\sqrt{a}-4\sqrt{x}$.
13. $5\sqrt{7}-4\sqrt{3}$ by $3\sqrt{7}+2\sqrt{3}$.
14. $4\sqrt{a}+3\sqrt{b}$ by $2\sqrt{a}+5\sqrt{b}$.
15. $2\sqrt[3]{4}-3\sqrt[3]{9}$ by $3\sqrt[3]{2}+4\sqrt[3]{3}$.
16. $a\sqrt{b}+x\sqrt{y}$ by $2\sqrt{b}+3\sqrt{y}$.
17. $3\sqrt[3]{4}+2\sqrt[3]{9}$ by $2\sqrt[3]{4}-3\sqrt[3]{6}$.
18. $2\sqrt{6}-3\sqrt{5}$ by $4\sqrt{3}-\sqrt{20}$.
19. $\sqrt{2}+\sqrt{3}-\sqrt{5}$ by $\sqrt{2}-\sqrt{3}+\sqrt{5}$.
20. $\sqrt{3}-\sqrt{5}+2\sqrt{7}$ by $3\sqrt{3}+2\sqrt{5}-\sqrt{7}$.
21. $5\sqrt{3}-3\sqrt{5}+2\sqrt{30}$ by $\sqrt{8}-\sqrt{5}+\sqrt{3}$.
22. $4\sqrt{7}-2\sqrt{3}+3\sqrt{5}$ by $3\sqrt{7}-5\sqrt{3}-\sqrt{5}$.
23. $\sqrt{8}-\frac{1}{4}\sqrt{32}+\sqrt{12}$ by $\sqrt{32}-4\sqrt{50}-\sqrt{2}$.

Multiply by inspection:

1. $\sqrt{5}+2$ by $\sqrt{5}+2$.
2. $\sqrt{a}-b$ by $\sqrt{a}-b$.
3. $\sqrt{3}+x$ by $\sqrt{3}-x$.
4. $a+\sqrt{7}$ by $a+\sqrt{7}$.
5. $5-\sqrt{a}$ by $5-\sqrt{a}$.
6. $\sqrt{6}+\sqrt{3}$ by $\sqrt{6}+\sqrt{3}$.
7. $\sqrt{a}-\sqrt{2}$ by $\sqrt{a}-\sqrt{2}$.
8. $\sqrt{a}-\sqrt{x}$ by $\sqrt{a}+\sqrt{x}$.
9. $\sqrt{a}+\sqrt{3}$ by $\sqrt{a}+\sqrt{3}$.
10. $\sqrt{a}-\sqrt{x}$ by $\sqrt{a}-\sqrt{x}$.
11. $2\sqrt{3}+\sqrt{5}$ by $2\sqrt{3}+\sqrt{5}$.
12. $3\sqrt{a}-\sqrt{2}$ by $3\sqrt{a}-\sqrt{2}$.
13. $a\sqrt{x}+\sqrt{y}$ by $a\sqrt{x}-\sqrt{y}$.
14. $a\sqrt{x}+\sqrt{b}$ by $a\sqrt{x}+\sqrt{b}$.
15. $a\sqrt{3}-\sqrt{x}$ by $a\sqrt{3}-\sqrt{x}$.
16. $2\sqrt{6}+3\sqrt{2}$ by $2\sqrt{6}+3\sqrt{2}$.
17. $a\sqrt{x}-b\sqrt{y}$ by $a\sqrt{x}-b\sqrt{y}$.
18. $a\sqrt{2}-3\sqrt{x}$ by $a\sqrt{2}+3\sqrt{x}$.
19. $3\sqrt{2}+a\sqrt{3}$ by $3\sqrt{2}+a\sqrt{3}$.
20. $2\sqrt{a}-3\sqrt{x}$ by $2\sqrt{a}-3\sqrt{x}$.
21. $a\sqrt{ax}-2\sqrt{a}$ by $a\sqrt{ax}-2\sqrt{a}$.
22. $3\sqrt{2\,a}+a\sqrt{2}$ by $3\sqrt{2\,a}+a\sqrt{2}$.
23. $a\sqrt{3\,a}+b\sqrt{5}$ by $a\sqrt{3\,a}-b\sqrt{5}$.

To the Teacher. — Teach 343.

Division of Radicals.

1. $3\sqrt{27} \div \sqrt{3}$.
2. $\sqrt{24} \div 2\sqrt{3}$.
3. $4\sqrt{84} \div \sqrt{7}$.
4. $\sqrt{72} \div 3\sqrt{8}$.
5. $6\sqrt{3} \div 2\sqrt[3]{3}$.
6. $\sqrt{72} \div 3\sqrt{2}$.
7. $2\sqrt{7} \div 4\sqrt[3]{7}$.
8. $\sqrt{54} \div 4\sqrt{2}$.
9. $3\sqrt[3]{80} \div \sqrt[3]{5}$.
10. $4\sqrt{3} \div 2\sqrt[3]{2}$.
11. $5\sqrt{8a} \div \sqrt{3}$.
12. $3\sqrt{8} \div 6\sqrt[3]{4}$.
13. $2\sqrt{9a} \div \sqrt{2}$.
14. $\sqrt{5x} \div 2\sqrt[3]{5}$.
15. $2\sqrt[5]{8} \div 3\sqrt[3]{2}$.
16. $\sqrt[3]{12} \div 2\sqrt{8}$.
17. $3\sqrt{8} \div \sqrt[4]{12}$.
18. $\sqrt[6]{500} \div \sqrt{5}$.
19. $4\sqrt{5} \div 8\sqrt[3]{5}$.
20. $\sqrt[6]{128} \div \sqrt[4]{4}$.
21. $x\sqrt{a} \div a\sqrt{x}$.
22. $\sqrt[4]{729} \div \sqrt[3]{9}$.
23. $\sqrt[3]{18} \div 2\sqrt{6}$.
24. $3\sqrt[3]{49} \div \sqrt{7}$.
25. $\sqrt[3]{12} \div 3\sqrt{6}$.
26. $2\sqrt[6]{72} \div \sqrt{2}$.
27. $8\sqrt{75} \div 2\sqrt{3}$.
28. $\sqrt[3]{162} \div 2\sqrt[3]{3}$.
29. $4\sqrt{90} \div 8\sqrt{5}$.
30. $\sqrt[4]{144} \div 3\sqrt{3}$.
31. $2\sqrt{8a} \div 3\sqrt{a}$.
32. $\sqrt[6]{160} \div 5\sqrt{2}$.
33. $6\sqrt{8a} \div 2\sqrt{b}$.
34. $\sqrt[6]{675} \div 2\sqrt[3]{5}$.
35. $4\sqrt{2a} \div 2\sqrt[3]{a}$.
36. $\sqrt[3]{4ax} \div \sqrt{2ax}$.
37. $3\sqrt[5]{2x} \div 6\sqrt[3]{x}$.
38. $\sqrt[6]{625} \div 2a\sqrt{5}$.
39. $2\sqrt{8a} \div 4\sqrt[3]{2}$.
40. $\sqrt[3]{3ax} \div 2\sqrt{xy^3}$.
41. $4\sqrt[5]{2x} \div a\sqrt[3]{x}$.
42. $\sqrt[5]{3bx} \div \sqrt[3]{ab^2x^3}$.
43. $\sqrt[4]{125} \div \sqrt[3]{5a}$.
44. $\sqrt{a^3b^3} \div \sqrt[3]{c^3x^3y^3}$.

Divide:

45. $\sqrt{12}-4+4\sqrt[3]{16}-3\sqrt{a}$ by $2\sqrt{2}$.

46. $4\sqrt[4]{32}-a\sqrt{3}-32+\sqrt[3]{256}$ by $2\sqrt{2}$.

47. $48-4\sqrt{a}-\sqrt[3]{216}+4\sqrt{75}$ by $\sqrt{24}$.

48. $2\sqrt[3]{25}+90-6\sqrt{x}-2\sqrt{27}$ by $\sqrt{45}$.

49. $3\sqrt[3]{32}+8\sqrt{12}+3\sqrt[3]{x}-16$ by $2\sqrt[3]{2}$.

50. $8\sqrt{27}+24-2\sqrt{a}-6\sqrt[3]{16}$ by $2\sqrt{3}$.

51. $27+2\sqrt{18}-6\sqrt{x}+3\sqrt[3]{81}$ by $3\sqrt{3}$.

52. $4a-5\sqrt{x}+3\sqrt{8a}-2\sqrt[3]{4a}$ by $\sqrt{2a}$.

To the Teacher. — Teach 344.

Involution of Radicals.

1. $(\sqrt[6]{2a})^3$.
2. $(2a\sqrt{2a})^3$.
3. $(2\sqrt{3a^3})^2$.
4. $(\sqrt[3]{ax})^4$.
5. $(3a\sqrt[4]{8x})^2$.
6. $(a\sqrt{ax^3})^5$.
7. $(\sqrt{3a})^5$.
8. $(ax\sqrt[3]{ax})^4$.
9. $(a\sqrt[3]{8x^3})^4$.
10. $(\sqrt[3]{5x})^2$.
11. $(2a\sqrt[4]{3a})^5$.
12. $(3\sqrt[4]{2a^3})^4$.
13. $(\sqrt[5]{2a})^3$.
14. $(ab\sqrt[12]{8a})^4$.
15. $(a\sqrt[3]{ax^2})^5$.
16. $(\sqrt[6]{8x})^3$.
17. $(3a\sqrt{2x})^3$.
18. $(2\sqrt[5]{2a^3})^5$.
19. $(\sqrt{ax})^6$.
20. $(ax\sqrt[4]{ax})^4$.
21. $(a\sqrt[9]{8a^4})^3$.
22. $(\sqrt[6]{8x})^2$.
23. $(ab\sqrt[4]{ab})^5$.
24. $(3\sqrt[3]{2a^2})^3$.
25. $(\sqrt{2x})^5$.
26. $(2a\sqrt[6]{8a})^3$
27. $(a\sqrt[9]{ax^5})^3$.
28. $(\sqrt[3]{7a})^3$.
29. $(ax\sqrt[3]{ax})^3$.
30. $(4\sqrt{3a^3})^2$.
31. $(\sqrt[3]{5x})^4$
32. $(2a\sqrt[3]{2a})^5$.
33. $(a\sqrt[3]{9a^2})^4$.

To the Teacher. — Teach 345.

Evolution of Radicals.

1. $(\sqrt{27})^{\frac{1}{3}}$.
2. $(2\sqrt[3]{a^4x^5})^{\frac{1}{2}}$.
3. $\sqrt[3]{\sqrt{8\,a^3x^6}}$.
4. $(\sqrt[3]{81})^{\frac{1}{4}}$.
5. $(a\sqrt{a^2x^6})^{\frac{1}{4}}$.
6. $\sqrt{\sqrt[3]{9\,a^2x^4}}$.
7. $(\sqrt[3]{45})^{\frac{1}{3}}$.
8. $(3\sqrt{a^3x^5})^{\frac{1}{4}}$.
9. $\sqrt[3]{\sqrt{4\,a^9x^3}}$.
10. $(\sqrt{64})^{\frac{1}{3}}$.
11. $(a\sqrt{a^3x^5})^{\frac{1}{5}}$.
12. $\sqrt{\sqrt[3]{5\,a^2x^2}}$.
13. $(\sqrt{32})^{\frac{1}{5}}$.
14. $(2\sqrt[3]{a^2b^2})^{\frac{1}{2}}$.
15. $\sqrt[5]{\sqrt{2\,a^3x^2}}$.
16. $(\sqrt[3]{125})^{\frac{1}{3}}$.
17. $(a\sqrt{a^3b^3})^{\frac{1}{4}}$.
18. $\sqrt[4]{\sqrt[3]{16\,a^4x^8}}$.
19. $(\sqrt{243})^{\frac{1}{5}}$.
20. $(3\sqrt[3]{a^2x^5})^{\frac{1}{2}}$.
21. $\sqrt[3]{\sqrt{27\,a^3x^6}}$.
22. $(\sqrt[3]{625})^{\frac{1}{4}}$.
23. $(a\sqrt[4]{a^2x^3})^{\frac{1}{3}}$.
24. $\sqrt[4]{\sqrt[3]{81\,a^8x^4}}$.

To the Teacher. — Teach 346 . . . . 349.

Rationalization.

Rationalize the denominators of the following:

1. $\frac{2}{\sqrt{3}}$.
2. $\frac{\sqrt{a}}{\sqrt{x}}$.
3. $\frac{\sqrt{a}}{a\sqrt{x}}$.
4. $\frac{2\sqrt{a}}{a\sqrt{x}}$.
5. $\frac{a}{\sqrt[3]{x}}$.
6. $\frac{\sqrt{a}}{\sqrt{2}}$.
7. $\frac{2}{2\sqrt[3]{9}}$.
8. $\frac{a\sqrt{2}}{2\sqrt{x}}$.
9. $\frac{3}{\sqrt[3]{4}}$.
10. $\frac{\sqrt{3}}{\sqrt{x}}$.
11. $\frac{a}{3\sqrt[4]{8}}$.
12. $\frac{4\sqrt{a}}{2\sqrt{x}}$.
13. $\frac{a}{\sqrt{7}}$.
14. $\frac{2}{\sqrt[3]{9}}$.
15. $\frac{\sqrt{5}}{a\sqrt{2}}$.
16. $\frac{a\sqrt{b}}{x\sqrt{y}}$.

17. $\frac{1}{\sqrt[3]{4}}$. 18. $\frac{3}{\sqrt[3]{2a}}$. 19. $\frac{1}{2\sqrt{a}}$. 20. $\frac{2\sqrt{x}}{3\sqrt{2a^3}}$.

21. $\frac{1}{\sqrt[4]{8}}$. 22. $\frac{1}{\sqrt[4]{2a}}$. 23. $\frac{a}{3\sqrt[3]{a}}$. 24. $\frac{2ax}{\sqrt[5]{16x^3}}$.

25. $\frac{1}{\sqrt[5]{8}}$. 26. $\frac{a}{\sqrt[5]{2a}}$. 27. $\frac{1}{a\sqrt[5]{4}}$. 28. $\frac{2a}{\sqrt[4]{27a^3}}$.

To the Teacher. — Teach 350.

Rationalize the denominators of the following:

1. $\frac{3}{4+\sqrt{3}}$. 2. $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$. 3. $\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}$.

4. $\frac{3+\sqrt{2}}{3-\sqrt{2}}$. 5. $\frac{\sqrt{a}-\sqrt{x}}{\sqrt{a}+\sqrt{x}}$. 6. $\frac{a+\sqrt{x+1}}{a-\sqrt{x+1}}$.

7. $\frac{x}{x-\sqrt{y}}$. 8. $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$. 9. $\frac{\sqrt{a+b}-c}{\sqrt{a+b}+c}$.

10. $\frac{a+\sqrt{x}}{a-\sqrt{x}}$. 11. $\frac{\sqrt{3}+\sqrt{7}}{\sqrt{5}-\sqrt{3}}$. 12. $\frac{a+\sqrt{a^2+2}}{a-\sqrt{a^2+2}}$.

13. $\frac{2}{2+\sqrt{2}}$. 14. $\frac{\sqrt{a}-\sqrt{x}}{\sqrt{x}-\sqrt{y}}$. 15. $\frac{\sqrt{x^2-9}-3}{\sqrt{x^2-9}+3}$.

16. $\frac{4-\sqrt{3}}{3+\sqrt{2}}$. 17. $\frac{\sqrt{3}-\sqrt{5}}{\sqrt{5}+\sqrt{3}}$. 18. $\frac{x+\sqrt{x^2+1}}{x-\sqrt{x^2+1}}$.

19. $\frac{a+\sqrt{x}}{x+\sqrt{y}}$. 20. $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{x}-\sqrt{y}}$. 21. $\frac{\sqrt{a^2+4}+2}{\sqrt{a^2+4}-2}$.

22. $\frac{3-\sqrt{6}}{2-\sqrt{2}}$. 23. $\frac{\sqrt{2}+2\sqrt{3}}{\sqrt{2}-2\sqrt{3}}$. 24. $\frac{\sqrt{x+3}-\sqrt{x}}{\sqrt{x+3}+\sqrt{x}}$.

25. $\frac{\sqrt{a}}{3-\sqrt{a}}$. 26. $\frac{\sqrt{a}-a\sqrt{b}}{\sqrt{a}+a\sqrt{b}}$. 27. $\frac{\sqrt{a+x}-\sqrt{a}}{\sqrt{a+x}+\sqrt{a}}$.

28. $\dfrac{\sqrt{a-x}-\sqrt{a+x}}{\sqrt{a-x}+\sqrt{a+x}}$.

29. $\dfrac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}}$.

30. $\dfrac{\sqrt{a-2}+\sqrt{a+2}}{\sqrt{a-2}-\sqrt{a+2}}$.

31. $\dfrac{\sqrt{a^2+x}-\sqrt{a^2-x}}{\sqrt{a^2+x}+\sqrt{a^2-x}}$.

To the Teacher. — Teach 351 and 352.

Square Root of Binomial Surds.

Find the square root of the following:

1. $5-2\sqrt{6}$.
2. $8+2\sqrt{12}$.
3. $15-4\sqrt{14}$.
4. $4+2\sqrt{3}$.
5. $16-4\sqrt{7}$.
6. $33-20\sqrt{2}$.
7. $6-4\sqrt{2}$.
8. $10+\sqrt{96}$.
9. $20+\sqrt{300}$.
10. $8+2\sqrt{7}$.
11. $18-8\sqrt{5}$.
12. $14-3\sqrt{20}$.
13. $7-4\sqrt{3}$.
14. $12+\sqrt{80}$.
15. $57-18\sqrt{2}$.
16. $9+4\sqrt{5}$.
17. $19-8\sqrt{3}$.
18. $47+\sqrt{360}$.
19. $9-2\sqrt{8}$.
20. $16-5\sqrt{7}$.
21. $15+2\sqrt{56}$.
22. $3+2\sqrt{2}$.
23. $10+\sqrt{64}$.
24. $67+7\sqrt{72}$.
25. $6-3\sqrt{3}$.
26. $50-8\sqrt{6}$.
27. $94-42\sqrt{5}$.
28. $7+2\sqrt{6}$.
29. $28+7\sqrt{7}$.
30. $28-5\sqrt{12}$.

To the Teacher. — Teach 353 . . . . 361.

IMAGINARY QUANTITIES.

Reduce the following to the form of $a\sqrt{-1}$.

1. $\sqrt{-4}$.
2. $\sqrt{-a^2}$.
3. $\sqrt{-4a}$.
4. $\sqrt{-16x^2}$.
5. $\sqrt{-9}$.
6. $\sqrt{-x^2}$.
7. $\sqrt{-3x}$.
8. $\sqrt{-20a^2}$.
9. $\sqrt{-8}$.
10. $\sqrt{-a^4}$.
11. $\sqrt{-8a}$.
12. $\sqrt{-27x^2}$.

Addition and Subtraction.

1. $\sqrt{-4}+\sqrt{-1}$.
2. $\sqrt{-12}+\sqrt{-27}$.
3. $\sqrt{-9}-\sqrt{-1}$.
4. $\sqrt{-75}-\sqrt{-48}$.
5. $\sqrt{-4}+\sqrt{-9}$.
6. $3\sqrt{-18}+\sqrt{-8}$.
7. $\sqrt{-a^2}-\sqrt{-1}$.
8. $2\sqrt{-32}-\sqrt{-8}$.
9. $\sqrt{-a^2}+\sqrt{-4}$.
10. $3\sqrt{-16}+\sqrt{-4}$.
11. $\sqrt{-x^2}-\sqrt{-9}$.
12. $5\sqrt{-72}-\sqrt{-2}$.
13. $\sqrt{-18}-\sqrt{-8}$.
14. $2\sqrt{-48}-\sqrt{-3}$.
15. $\sqrt{-32}+\sqrt{-8}$.
16. $3\sqrt{-a^3}+\sqrt{-4a^3}$.
17. $3\sqrt{-4}-\sqrt{-1}$.
18. $2\sqrt{-9x^3}-\sqrt{-x^3}$.
19. $2\sqrt{-9}+\sqrt{-1}$.
20. $\sqrt{-18a^2}+\sqrt{-8a^2}$.
21. $5\sqrt{-4}-\sqrt{-9}$.
22. $\sqrt{-32x^3}-\sqrt{-8x^3}$.

Multiplication.

1. $2\sqrt{-3}\times\sqrt{-4}$.
2. $2\sqrt{-4}\times-\sqrt{-3}\times4\sqrt{-3}$.
3. $3\sqrt{-6}\times2\sqrt{-3}\times\sqrt{-2}\times2\sqrt{-4}$.
4. $3\sqrt{-a}\times\sqrt{-x}$.
5. $3\sqrt{-5}\times-\sqrt{-2}\times2\sqrt{-3}$.
6. $2\sqrt{-a^2}\times2\sqrt{-9}\times\sqrt{-b^2}\times2\sqrt{-4}$.
7. $a\sqrt{-6}\times\sqrt{-3}$.
8. $3\sqrt{-2}\times-\sqrt{-6}\times2\sqrt{-3}$.
9. $3\sqrt{-a^3}\times a\sqrt{-6}\times\sqrt{-b^3}\times b\sqrt{-2}$.
10. $2\sqrt{-a^3}\times\sqrt{-b^3}$.
11. $2\sqrt{-a^3}\times-\sqrt{-b^3}\times a\sqrt{-c}$.
12. $\sqrt{-8}\times2\sqrt{-a^2}\times\sqrt{-4}\times-\sqrt{-b}\times3\sqrt{-c^3}$.
13. $a\sqrt{-x^3}\times\sqrt{-x}$.
14. $a\sqrt{-3}\times-\sqrt{-a^2}\times2\sqrt{-a^3}$.

15. $2\sqrt{-25} \times \sqrt{-36}$. 16. $\sqrt{-6} \times -\sqrt{-2} \times 2\sqrt{-4}$.

17. $\sqrt{-a^2} \times a\sqrt{-4} \times -\sqrt{-a^3} \times 2\sqrt{-a}$.

18. $3\sqrt{-16} \times \sqrt{-49}$. 19. $3\sqrt{-a^4} \times -\sqrt{-a^2} \times \sqrt{-x}$.

20. $\sqrt{-a^3} \times 2\sqrt{-8} \times -\sqrt{-a^3} \times 3\sqrt{-x}$.

21. $5\sqrt{-4a} \times \sqrt{-9x}$. 22. $a\sqrt{-a^3} \times -\sqrt{-b^3} \times \sqrt{-x^5}$.

23. $(a+2\sqrt{-x})(b-3\sqrt{-y})$.

24. $(\sqrt{5}-3\sqrt{-2})(\sqrt{5}+3\sqrt{-2})$.

25. $(\sqrt{-3}+\sqrt{-2})(\sqrt{-3}+\sqrt{-2})$.

26. $(\sqrt{-2}+2\sqrt{-3})(\sqrt{-4}-3\sqrt{-5})$.

27. $(a\sqrt{-x}-b\sqrt{-y})(a\sqrt{-x}+b\sqrt{-y})$.

28. $(2\sqrt{-3}+3\sqrt{-8})(3\sqrt{-6}-5\sqrt{-9})$.

Division.

Perform the following indicated divisions:

1. $\dfrac{2}{\sqrt{-1}}$. 2. $\dfrac{\sqrt{-x}}{\sqrt{-y}}$. 3. $\dfrac{2}{2-\sqrt{-3}}$.

4. $\dfrac{a}{\sqrt{-a}}$. 5. $\dfrac{\sqrt{-9}}{\sqrt{-3}}$. 6. $\dfrac{a}{a-\sqrt{-x}}$.

7. $\dfrac{x}{\sqrt{-4}}$. 8. $\dfrac{\sqrt{-8}}{\sqrt{-2}}$. 9. $\dfrac{2+\sqrt{-3}}{3-\sqrt{-2}}$.

10. $\dfrac{2}{\sqrt{-a^2}}$. 11. $\dfrac{\sqrt{-a^2}}{\sqrt{-a}}$. 12. $\dfrac{2\sqrt{-1}+x}{2\sqrt{-1}-x}$.

13. $\dfrac{a}{\sqrt{-a^2}}$. 14. $\dfrac{\sqrt{-x}}{\sqrt{-x^2}}$. 15. $\dfrac{\sqrt{-3}-\sqrt{-4}}{\sqrt{-6}+\sqrt{-3}}$.

To the Teacher. — Teach 362 and 363.

RADICAL EQUATIONS.

1. $\sqrt{x}+5=8.$
2. $\sqrt{x+7}+5=9.$
3. $\sqrt{x}-a=a.$
4. $\sqrt[3]{x+2}+3=5.$
5. $\sqrt{x+6}=3.$
6. $2\sqrt[3]{x+3}+3=6$
7. $\sqrt{x-a}=b.$
8. $\sqrt{x+7}+\sqrt{x}=7.$
9. $\sqrt[3]{3x+17}=5.$
10. $\sqrt{x+5}+\sqrt{x}=5.$
11. $\sqrt[3]{x+4}-1=2.$
12. $\sqrt{x+a}-\sqrt{x}=a.$
13. $\sqrt{x^2-7}+x=6.$
14. $\sqrt{x+3}+\sqrt{x}=3.$
15. $\dfrac{a-\sqrt{a^2-x}}{a+\sqrt{a^2-x}}=b.$
16. $\dfrac{\sqrt{x-2}}{\sqrt{x+3}}=\dfrac{\sqrt{x-3}}{\sqrt{x+5}}.$
17. $\dfrac{c}{x}+\dfrac{\sqrt{c^2+x^2}}{x}=\dfrac{1}{d}.$
18. $\dfrac{\sqrt{x}-1}{\sqrt{x}+2}=\dfrac{\sqrt{x}+1}{\sqrt{x}+6}.$
19. $\dfrac{\sqrt{x+7}+\sqrt{x}}{\sqrt{x+7}-\sqrt{x}}=7.$
20. $\dfrac{\sqrt{x}+2}{\sqrt{x}-4}=\dfrac{\sqrt{x}+6}{\sqrt{x}-3}.$
21. $\dfrac{\sqrt{a}+\sqrt{a-x}}{\sqrt{a}-\sqrt{a-x}}=\dfrac{1}{a}.$
22. $\dfrac{\sqrt{x}+13}{\sqrt{x}+10}=\dfrac{\sqrt{x}+7}{\sqrt{x}+5}.$
23. $\sqrt{x+5}=3\sqrt{x-3}.$
24. $\sqrt{x}-\sqrt{x-7}=\sqrt{7}.$
25. $\sqrt{x+a}=2\sqrt{x-a}.$
26. $\sqrt{x-a}+\sqrt{x}=\sqrt{a}.$
27. $\sqrt{x+9}-\sqrt{x-7}=2.$
28. $\sqrt{x}-\sqrt{5}=\sqrt{x-5}.$
29. $\sqrt{x+7}-2=\sqrt{x-5}.$
30. $\sqrt{x^2+\sqrt{x+1}}=x+1.$
31. $2\sqrt{x}-\sqrt{4x-9}=\sqrt{3}.$
32. $x+\sqrt{x^2-\sqrt{a^4-x}}=a.$
33. $3\sqrt{x}-\sqrt{a}=\sqrt{9x-c}.$
34. $\sqrt{x^2+2\sqrt{x+4}}=x+2.$

35. $\dfrac{4}{\sqrt{x-4}} = \sqrt{x} + \sqrt{x-4}.$

36. $\dfrac{2c}{\sqrt{x+c}} = \sqrt{x} + \sqrt{c+x}.$

37. $\dfrac{1}{2} - \dfrac{5}{x} = \sqrt{\dfrac{1}{4} + \dfrac{1}{x}\sqrt{25 - \dfrac{75}{x}}}.$

38. $\dfrac{2\sqrt{3x+6} + 2\sqrt{x-1}}{2\sqrt{3x+6} - 2\sqrt{x-1}} = 3.$

39. $\sqrt{c - \sqrt{x}} + \sqrt{\sqrt{x} + c} = \sqrt{x}.$

40. $\dfrac{b}{\sqrt{b-x}} - \dfrac{x}{\sqrt{c-x}} = \sqrt{c-x}.$

41. $\sqrt{x+1} + \sqrt{x-7} = \sqrt{8+4x}.$

42. $\dfrac{4}{\sqrt{7+3x}} = \sqrt{3x+7} - \sqrt{3x}.$

43. $\dfrac{\sqrt{x} + \sqrt{5}}{2} + 3\sqrt{5} = \dfrac{x-5}{\sqrt{x} - \sqrt{5}}.$

44. $\dfrac{a-x}{\sqrt{a} + \sqrt{x}} - 2\sqrt{a} = \dfrac{\sqrt{a} + \sqrt{x}}{2}.$

45. $\dfrac{55}{\sqrt{4x-11}} - \sqrt{4x} = \sqrt{4x-11}.$

46. $4\sqrt{x} - 2\sqrt{a} = \sqrt{a} + \sqrt{x} + 6\sqrt{a}.$

47. $\dfrac{(\sqrt{4x} - 5)(\sqrt{x} + 2)}{\sqrt{x} - 4} = 2\sqrt{x} + 10.$

48. $\sqrt{x+a} = \sqrt{4x + 3b + a} - \sqrt{x+b}.$

49. $\sqrt{x^2 + 2x + 10} + \sqrt{x^2 - 14x + 42} = 8.$

To the Teacher. — Teach 364 . . . . 366.

PURE QUADRATICS.

1. $x^2 - 5 = 3 - x^2$.
2. $x^2 - 9 = 7 - x^2$.
3. $x^2 + 4 = 2x^2 - 5$.
4. $x^2 - a = b - x^2$.
5. $\frac{x-10}{5} = \frac{5}{x+10}$.
6. $\frac{a}{\sqrt{x-a}} = \sqrt{a+x}$.
7. $x^2 - 14 = 18 - x^2$.
8. $3x^2 - 10 = x^2 - 9$.
9. $x^2 + 18 = 2x^2 + 6$.
10. $3x^2 - 7a = x^2 + a$.
11. $2x^2 + 5 = x^2 + 30$.
12. $4x^2 - 11 = x^2 - 9$.
13. $\frac{3x^2}{2} + 7 = 25 + x^2$.
14. $\frac{2a}{x} + \frac{x}{a} = \frac{6a^2 - x^2}{ax}$.
15. $(x+1)^2 = 2x + 9$.
16. $ax^2 - a = bx^2 - b$.
17. $\frac{x+a}{x-a} + \frac{x-a}{x+a} = 6$.
18. $\frac{4x^2+1}{3x^2-1} = \frac{2(4x^2+2)}{3(2x^2-1)}$.
19. $5x^2 - 7 = 3x^2 + 29$.
20. $x + \sqrt{x^2 + 2\sqrt{1-x}} = 1$.
21. $7x^2 + 5 = 4x^2 + 32$.
22. $\sqrt{x + \sqrt{x^2 - c^2}} = \sqrt{b+x}$.
23. $\frac{x+3}{x-3} + \frac{x-3}{x+3} = 3\frac{1}{8}$.
24. $\frac{\sqrt{2x^2+1} - \sqrt{2x^2-1}}{\sqrt{2x^2+1} + \sqrt{2x^2-1}} = \frac{1}{2}$.
25. $\frac{3}{2-x} + \frac{3}{2+x} = 4\frac{1}{2}$.
26. $\frac{2x+a}{2x-a} + \frac{2x-a}{2x+a} = \frac{2}{1-a}$.
27. $\frac{2(x+6)}{3} = \frac{6x+x^2}{9}$.
28. $\frac{24}{\sqrt{x^2-16}} = x + \sqrt{x^2-16}$.
29. $\frac{13x^2}{3} - 15 = \frac{13x^2}{4} + x^2$.
30. $x - \sqrt{x^2 - 2a} = \frac{3a}{\sqrt{x^2-2a}}$.
31. $\frac{2a}{x-a} + \frac{5x+2a}{3x} = -1$.
32. $\frac{40}{\sqrt{x^2+16}} - x = \sqrt{x^2+16}$.

33. $(x+3)^2-8=2(3x+13)$.

34. $\dfrac{4}{3x+1}-\dfrac{9x-7}{9x^2-1}=\dfrac{3x}{3x-1}$.

35. $\dfrac{6x^2+8}{16}-\dfrac{2x^2+6}{3x^2-2}=\dfrac{3x^2-4}{8}$.

36. $\dfrac{2x^2-6}{3}+\dfrac{x^2+9}{5}-\dfrac{x^2+20}{2}=3$.

37. $\dfrac{3x^2-3}{4}-\dfrac{5x^2+1}{9}-\dfrac{x^2-5}{5}=0$.

38. $\dfrac{2}{x+\sqrt{2-x^2}}+\dfrac{2}{x-\sqrt{2-x^2}}=\dfrac{a}{x}$.

39. $\dfrac{\sqrt{24+18x}}{\sqrt{6}}+\dfrac{\sqrt{24-18x}}{\sqrt{6}}=\sqrt{6}$.

40. $(x+1)(x^2+3)=(x-1)^3+x^2+5$.

41. $(x+4)(x+6)=2(x+3)(x+2)-4$.

42. $(4x+1)(x-3)+(2x-1)(x+6)=7$.

43. $\dfrac{1-x}{1-x+\sqrt{1+x^2}}=a-\dfrac{1+x}{1+x+\sqrt{1+x^2}}$.

44. $\dfrac{7x^2+7}{x^2-2}-\dfrac{2x^2+6}{x^2+3}-\dfrac{252}{(x^2-2)(x^2+3)}=5$.

PROBLEMS.

1. The square of a number exceeds the square of three-fourths of it by 175. Find the number.

2. The square of a number increased by the square of one-half of it equals 720. Find the number.

3. Two numbers are to each other as 4 to 7, and the sum of their squares is 585. Find the numbers.

4. The product of two numbers is 528, and the quotient of the greater divided by the less is $3\frac{3}{4}$. Find the numbers.

5. The smaller of two numbers is $\frac{1}{4}$ of the larger, and the difference of their squares is 540. Find the numbers.

6. If 156 be divided by a certain number, and the quotient added to the number itself, the sum will be fourteen times the number. Find the number.

7. A rectangular field of six acres is $1\frac{3}{4}$ times as long as it is wide. Find the dimensions of the field.

8. The sum of the squares of two consecutive numbers exceeds twice the smaller number by 33. Find the numbers.

9. The product of two numbers is 525, and the quotient of the less divided by the greater is $\frac{3}{7}$. Find the numbers.

10. A merchant bought a piece of silk for twenty-seven dollars. The number of yards in the piece was to the number of dollars paid per yard as sixteen to three. Find the number of yards in the piece and the price per yard.

11. Two numbers are to each other as 3 to 5, and the sum of their squares is 544. Find the numbers.

12. The smaller of two numbers is $\frac{2}{3}$ of the larger, and the difference of their squares is 400. Find the numbers.

13. The sum of the squares of two successive even numbers exceeds four times the smaller number by seventy-six. What are the numbers?

14. A man bought land for \$ 960, paying $\frac{2}{3}$ as many dollars per acre as there were acres in the piece. At what price per acre did he buy the land?

15. Three times the cube of a certain number exceeds 21 times the number by 27 times the number. Find the number.

16. The sum of the squares of two consecutive numbers exceeds twice the larger number by 71. Find the numbers.

17. The sides of two squares are to each other as 3 to 4, and the area of the larger square exceeds that of the smaller by 28 square yards. Find the side of each square.

18. Find three numbers that are to each other as 2, 3, and 5, and the sum of whose squares is 608.

19. There are sixteen square yards in a rectangular piece of paper, the length of which is nine times the width. Find the length of the paper.

20. A's age is to B's age as three to five, and the square of B's age exceeds the square of A's age by sixteen hundred. Find the age of each.

21. The dimensions of a rectangular field are to each other as 4 to 3, and the diagonal of the field is 100 rods. How many acres does the field contain?

22. Two numbers are to each other as 3 to 8, and the difference of their squares is 1375. Find the numbers.

23. Five equal squares of paper contain 304 square inches less than a square 32 inches on each side. Find the length of each of the five squares.

24. The sum of the squares of two successive odd numbers exceeds four times the larger number by forty-six. What are the numbers?

25. A man worked 12 times as many days as he received dollars a day and earned $147. How many days did he labor, and how much did he receive a day?

26. The dimensions of a rectangular field are to each other as 8 to 5, and the field contains twenty-five acres. Find the dimensions of the field.

27. The sum of two numbers is 24, and the less number divided by the greater is to the greater number divided by the less as 25 to 49. What are the numbers?

28. Two squares of paper contain 850 square inches, and the side of the larger square is as much greater than 20 inches as the side of the smaller square is less than 20 inches. Find the length of each of the squares.

To the Teacher. — Teach 367 . . . . 370.

AFFECTED QUADRATICS.

Solve the following equations by factoring:

1. $x^2 + x = 6$.
2. $x^2 + 7x = -12$.
3. $x^2 - x = 2$.
4. $x^2 - 8x = -15$.
5. $x^2 + x = 12$.
6. $x^2 + 9x = -14$.
7. $x^2 - 6x = 7$.
8. $x^2 - 9x = -18$.
9. $x^2 + x = 20$.
10. $x^2 + 8x = -12$.
11. $x^2 - 7x = 8$.
12. $x^2 - 7x = -10$.
13. $x^2 - x = 30$.
14. $x^2 + 8x = -16$.
15. $x^2 + 8x = 9$.
16. $x^4 - 7x^2 = -12$.
17. $4x^2 - 6x = 4$.
18. $x^4 - 10x^2 = -9$.
19. $6x^2 + 5x = 6$.
20. $x^2 - 13x = -12$.
21. $6x^2 - 8x = 8$.
22. $6x^2 + 11x = -4$.
23. $x^3 - 2x^2 = 8x$.
24. $x^4 - 13x^2 = -36$.
25. $x^3 + 7x^2 = 8x$.
26. $x^4 - 17x^2 = -16$.
27. $x^4 + x^3 = 12x^2$.
28. $4x^3 - 12x = -9$.
29. $x^4 - x^3 = 56x^2$.
30. $6x^3 + 17x = -5$.
31. $4x^3 + 31x = 8$.
32. $6x^3 - 49x = -8$.
33. $x^2 + ab - ax - bx = 0$.
34. $x^3 - 9x + 18 - 2x^2 = 0$.
35. $x(x^2 - 1) - 2(x + 1) = 0$.
36. $x(x^2 - 4) - 3(x - 2) = 0$.
37. $3x^3 + 8 - 2x^2 - 12x = 0$.
38. $2x^2 + 3a - 3x - 2ax = 0$.
39. $(x^2 - x - 2)(3x^2 - x - 2) = 0$.
40. $(x^2 + 5x + 6)(6x^2 - x - 5) = 0$.

To the Teacher. — Teach 371 and 372.

Form the equations whose roots are:

1. 4 and 2.
2. 3 and -5.
3. -6 and -1.
4. 5 and 7.
5. 5 and -2.
6. -3 and -6.
7. 3 and $2\frac{1}{2}$.
8. $2a$ and -3.
9. $-2\frac{1}{2}$ and -5.
10. 2 and $4\frac{1}{3}$.
11. $3a$ and -2.
12. $-1\frac{2}{3}$ and -3.
13. $1\frac{1}{2}$ and $2\frac{2}{3}$.
14. $5a^2$ and -1.
15. $-3\frac{1}{4}$ and -4.
16. $a+b$ and $a-b$.
17. $a+\sqrt{b}$ and $a-\sqrt{b}$.
18. $a+3$ and $a-3$.
19. $a+\sqrt{5}$ and $a-\sqrt{5}$.
20. $a+2b$ and $a-2b$.
21. $2\frac{1}{2}+\sqrt{2}$ and $2\frac{1}{2}-\sqrt{2}$.
22. $a+\frac{2}{3}b$ and $a-\frac{2}{3}b$.
23. $3+\frac{1}{2}\sqrt{3}$ and $3-\frac{1}{2}\sqrt{3}$.

To the Teacher. — Teach 373 and 374.

Complete the square by the first method, and solve:

1. $x^2-6x=3$.
2. $4x^2-4x+2=5$.
3. $x^2+8x=2$.
4. $2x^2-2x=x^2+8$.
5. $x^2-4x=4$.
6. $3x-12=2-2x^2$.
7. $x^2-2x=4$.
8. $3x^2-35=4-4x$.
9. $x^2+5=8x$.
10. $7x-32=7-2x^2$.
11. $x^2+1=4x$.
12. $3x^2-16=11x+4$.
13. $4x^2-4x=7$.
14. $3x^2+28=21x-8$.
15. $8x^2+x=30$.
16. $5x^2-48=7-14x$.
17. $x+21=2x^2$.
18. $25x-2x^2=21+4x^2$.
19. $4x^2-8x=5$.
20. $14x^2-3a^2=2ax-7x^2$.

To the Teacher. — Teach 375 . . . . 377.

Complete the square by the second method, and solve:

1. $9x^2 - 6x = 24$.
2. $3x^2 - 2x = 65$.
3. $3x^2 + 2x = 85$.
4. $2x^2 + 5x = 18$.
5. $2x^2 + 7x = 22$.
6. $5x^2 - 44x = 9$.
7. $4x^2 + 5x = 84$.
8. $2x^2 - 7x = 34$.
9. $5x^2 + 6x = 63$.
10. $3x^2 + 2x = 56$.
11. $9x^2 + 18x = 72$.
12. $x^2 - 13x = 14$.
13. $7x^2 + 5x = 150$.
14. $4x^2 + 6x = 10$.
15. $4x^2 - 6x = 108$.
16. $5x^2 - 6x = 41$.
17. $4x^2 - 7x = 102$.
18. $x^2 - 17x = 18$.
19. $7x^2 - 20x = 75$.
20. $5x^2 - 6x = 27$.
21. $9x^2 - 12x = 45$.
22. $4x^2 - 10x = 14$.

To the Teacher. — Teach 378.

Solve the following equations by formula:

1. $x^2 - 6x = 16$.
2. $x^2 - 7x = -10$.
3. $x^2 + 3x = 18$.
4. $6x^2 + 7x = -1$.
5. $x^2 - 5x = 84$.
6. $5x^2 - 7x = -2$.
7. $x^2 - 10x = 11$.
8. $x^2 - 7x = -12$.
9. $x^2 + 13x = 68$.
10. $4x^2 + 4x = -1$.
11. $2x^2 + 3x = 14$.
12. $x^2 - 6x = -11$.
13. $6x^2 - 7x = 10$.
14. $3x^2 - 8x = -4$.
15. $4x^2 - 9x = 28$.
16. $x^2 - 3b^2 = -2bx$.
17. $3x^2 - 5x = 12$.
18. $x^2 - 10a^2 = -3ax$.
19. $x^2 - 4ax = 12a^2$.
20. $x^2 - 14b^2 = -5bx$.
21. $x^2 + 3bx = 10b^2$.
22. $3x^2 - 4a^2 = -4ax$.

To the Teacher. — Teach 379 and 380.

Solve the following equations:

1. $x^4 - 3x^2 = 4.$
2. $x - 6 + \sqrt{x-6} = 12.$
3. $x^{\frac{1}{2}} - 2x^{\frac{1}{4}} = 3.$
4. $\sqrt{x+6} - \sqrt[4]{x+6} = 6.$
5. $x^3 - x^{\frac{3}{2}} = 56.$
6. $x + 3 + (x+3)^{\frac{1}{2}} = 30.$
7. $x^{\frac{3}{2}} + 2x^{\frac{1}{2}} = 8.$
8. $(x+5)^2 - x - 5 = 132.$
9. $x^{\frac{1}{2}} + x^{\frac{1}{4}} = 30.$
10. $x^2 - 6x - \sqrt{x^2 - 6x + 2} = 4.$
11. $x^6 + 2x^3 = 80.$
12. $x^2 + \sqrt{x^2 - 4x + 4} = 4x + 8.$
13. $x^3 - 5x^{\frac{3}{2}} = 24.$
14. $\sqrt{x+12} + \frac{1}{2}\sqrt[4]{x+12} = 5.$
15. $3x^{\frac{1}{2}} - 4x^{\frac{1}{4}} = 7.$
16. $(x+18)^2 - x - 18 = 600.$
17. $4x^{\frac{1}{3}} + x^{\frac{1}{6}} = 39.$
18. $(x^2+3)^2 - (x^2+3) = 42.$
19. $x^{\frac{1}{3}} + 7x^{\frac{1}{6}} = 44.$
20. $(x-2)^2 - 3(x-2) = 18.$
21. $x^n - 2ax^{\frac{n}{2}} = c^2.$
22. $(x^2-7)^2 - 5x^2 + 35 = 36.$
23. $ax^{2n} - bx^n = 2c.$
24. $(x^2+6)^2 - 7x^2 - 42 = 30.$
25. $x + 6 - 3(x+6)^{\frac{1}{2}} = 8 - (x+6)^{\frac{1}{2}}.$
26. $(x^2 - 3x + 4)^2 - x^2 + 3x - 4 = 56.$
27. $x^2 + 5 + 6(x^2 - 2x + 5) = 6 + 2x.$
28. $2\sqrt{x^2 - 6x} + 2 + 4x = x^2 - 2x - 1.$
29. $3x - 4 - \sqrt{4x^2 - 24x} = 9x - x^2 + 4.$
30. $5x - 2 - \sqrt{9x^2 - 18x} = 7x - x^2 + 2.$
31. $2x - 4x + 6 - 4\sqrt{2x - 7x + 6} = 3x.$
32. $2x - 2x - 6\sqrt{2x - 7x + 6} = 5x - 6.$
33. $3x - 30 + (4x^2 - 20x)^{\frac{1}{2}} = 8x - x^2 + 18.$
34. $x^2 - 2x - \sqrt{2x^2 - 3x + 29} = 4x - 3x^2 + 62.$
35. $2x^2 - 4x - \sqrt{3x^2 - 2x + 15} = 2x - 7x^2 + 57.$

MISCELLANEOUS EXERCISES.

1. $x^2 - 5x = 14$.
2. $2x^2 - 3x - 5 = 0$.
3. $x^3 + 8x^2 = 9x$.
4. $x^2 - 10x + 21 = 0$.
5. $5x^2 + x = 18$.
6. $x^2 - 2x + ax = 2a$.
7. $3x^2 - 96 = 2x$.
8. $3x^2 + 4x - 1 = 3$.
9. $x^4 + x^3 = 90x^2$.
10. $x^2 - (a - b)x = ab$.
11. $3x^2 + 2x = 56$.
12. $x^3 + 5x^2 - x = 5$.
13. $3x^2 + 3 = 10x$.
14. $(a - b)x^2 - b = cx$.
15. $x^3 + 5x = 6x^2$.
16. $x^3 - x^2 + 9 = 9x$.
17. $3x^2 + 9x = 30$.
18. $x^2 - cx + c = x$.
19. $x^2 - 91 = 6x$.
20. $4x^2 - x - 3 = 0$.
21. $x^4 + 4 = 5x^2$.
22. $3x^2 + x - 6 = 8$.
23. $2x^2 - x = 6$.
24. $x^3 + 7x^2 - 7 = x$.
25. $x^2 + 7x = 18$.
26. $x + \frac{2}{3}\sqrt{x+4} = 7$.
27. $x^4 - x^3 = 30x^2$.
28. $x(a^2 + 1) = ax^2 + a$.
29. $x^2 - 6x = 7$.
30. $x^3 + 5x^2 - 6x = 0$.
31. $3x^2 - x = 44$.
32. $ax^2 - bx - c = 0$.
33. $x^3 + x^2 = 30x$.
34. $\sqrt{x-5} + \sqrt{x} = 5$.
35. $cx^2 + 2ax = b$.
36. $2x^3 + 3x^2 - 2x = 3$.
37. $x - 5\sqrt{x} = 14$.
38. $x^2 - 3ax + 2a^2 = 0$.
39. $x^4 + 8 = 6x^2$.
40. $x - 3\sqrt{x-7} = 7$.
41. $3x + 7\sqrt{x} = 48$.
42. $a^2 + x^2 = 2ax + b$.
43. $10x^2 - 21 = x$.
44. $x^3 + 3x^2 - 4x = 12$.
45. $x^4 + 36x^2 = 13x^3$.
46. $6x^2 + 2bx - 3ax = ab$.
47. $x^3 - 42x = -x^2$.
48. $x(x^2 - 4) = 3(x + 2)$.

49. $abx^2 - acx - b^2x + bc = 0.$

50. $(x^2 - x - 6)(6x^2 - x - 7) = 0.$

51. $(x^2 - 4x + 3)(7x^2 - x - 6) = 0.$

52. $(x^2 + 3x + 2)(4x^2 + x - 5) = 0.$

53. $x^3 - x^2 = 20x.$

54. $x(x^2 - 9) = 4(x + 3).$

55. $\frac{x^2}{x+1} = \frac{a^2}{a+1}.$

56. $\frac{x}{4-x} + \frac{5-x}{x} = \frac{9}{2}.$

57. $x^4 + 7x^2 = -12x^2.$

58. $x^3 - x^2 - 56x = 0.$

59. $\frac{a}{b} - \frac{1}{x} = x - \frac{b}{a}.$

60. $\frac{2x^2}{x-6} - \frac{2-3x}{2} = \frac{5}{2}.$

61. $\frac{1}{x} + x = a + \frac{2}{x}.$

62. $\frac{4x + \sqrt{6x - x^2}}{4x - \sqrt{6x - x^2}} = 1\frac{1}{3}.$

63. $\frac{x+1}{\sqrt{x}} = \frac{a+1}{\sqrt{a}}.$

64. $\frac{4}{\sqrt{x-5}} + \sqrt{x-5} = 4.$

65. $x^4 - 3x^2 + 2 = 0.$

66. $x(x^2 - 9) = 4(x - 3).$

67. $\frac{2x(c-x)}{3c-2x} = \frac{c}{4}.$

68. $\frac{a-2y}{a+2y} + \frac{2a+y}{2a-y} = \frac{8}{3}.$

69. $\frac{2b}{x} + ax = c + \frac{b}{x}.$

70. $\frac{3x-7}{x-2} + \frac{1}{4} = \frac{3x-9}{x-3}.$

71. $\frac{x+2}{x-2} + \frac{x-1}{x-4} = \frac{7}{2}.$

72. $\frac{12x-6}{5x-7} = x - \frac{x^2-7}{x+4}.$

73. $\frac{16x}{x-1} = 12(x-1).$

74. $\frac{5\sqrt{x}-5}{2+\sqrt{x}} - \frac{18+4\sqrt{x}}{7+\sqrt{x}} = -1.$

75. $abx^2 + ab^2x - a^2bx - a^2b^2 = 0.$

76. $x^3 - 4x^2 + 12 = 3x.$ 77. $x^4 - 4x^2 + 3 = 0.$

78. $\frac{a+x}{x} = \frac{5}{2} - \frac{x}{x+a}.$ 79. $\frac{a+c}{2c+x} + \frac{2c+a}{3c-x} = 2.$

80. $\frac{5}{x+1} = \frac{1}{x-3} + \frac{1}{3}.$ 81. $\frac{3x+6}{x-2} - \frac{4x-9}{x} = \frac{7}{2}.$

82. $x^2 - \frac{a^2+b^2}{ab}x = -1.$ 83. $\frac{3x+2c}{4x-c} + \frac{2x-3c}{x+4c} = \frac{10}{7}.$

84. $\frac{x+1}{x-5} + \frac{x+2}{x-1} = -3.$ 85. $\frac{3a-2x}{a-x} - \frac{4a+5x}{a+x} = \frac{a+b}{a-b}.$

PROBLEMS.

1. The sum of two numbers is 13, and their product is 40. What are the numbers?

2. The sum of two numbers is 34, and the sum of their squares is 628. What are the numbers?

3. The sum of the squares of two successive even numbers is 340. What are the numbers?

4. The sum of the squares of three consecutive numbers is 194. What are the numbers?

5. Divide forty-two into two such parts that their product shall be two hundred and forty-five.

6. The sum of two numbers is 64, and their product is seven times as great. What are the numbers?

7. The difference between two numbers is 10, and their product is 375. What are the numbers?

8. The sum of the squares of two successive odd numbers exceeds their product by 103. Find the numbers.

9. The difference of the cubes of two successive odd numbers is 386. What are the numbers?

10. A rectangular field, which contains 18 acres, is 12 rods longer than it is wide. Find the dimensions.

11. The square of the sum of two consecutive numbers exceeds the sum of their squares by 312. Find the numbers.

12. Find two numbers whose difference is eight, and whose sum multiplied by the less is 384.

13. There are two square fields. The side of one is $3\frac{1}{2}$ rods longer than the side of the other, and the fields together contain $1041\frac{1}{4}$ square rods. How much does the area of the larger field exceed that of the smaller?

14. The product of two numbers is 112; and if one of them be divided by the other, the quotient will be 2 and the remainder 2. What are the numbers?

15. A rectangular park, 80 rods long and 60 rods wide, is surrounded by a street of uniform width, which contains 725 square rods. Find the width of the street.

16. A lady purchased 50 yards of silk in two pieces. Each piece cost as many dimes per yard as there were yards in the piece. If one piece cost $2\frac{1}{4}$ times as much as the other, how many yards of each did she buy?

17. A and B can do a piece of work in six days. It takes B nine days longer to do the whole work than it does A. In how many days can each do it?

18. A merchant bought silk for $\$40$, and the number of cents in the price exceeded the number of yards by 30. At what price per yard did he buy it?

19. The sum of two numbers is a, and their product is b. What are the numbers?

20. The hypotenuse of a certain right triangle is 3 yards greater than the base and 2 feet greater than the perpendicular. Find the length of the perpendicular.

21. A merchant bought lace for \$180. He kept 20 yards and sold the remainder for what it all cost, gaining \$.75 a yard. How many yards did he buy?

22. The length of a rectangular field exceeds its breadth by 4 rods. If the length and breadth of the field were each increased six rods, the field would contain 3 acres. Find the dimensions of the field.

23. The denominator of a fraction exceeds the numerator by 1. If three be added to both terms, the fraction will be increased $\frac{1}{18}$. What is the fraction?

24. An agent sold a number of books for \$56, and gained a per cent equal to the number of dollars he paid for them. How much did he pay for them?

25. A man bought a farm for \$2700. He sold it at \$35 an acre, making a profit equal to the cost of 15 acres. How many acres were there in the farm?

26. A rectangular mirror, 18 inches by 12 inches, is to be surrounded by a velvet frame of uniform width, whose area is equal to that of the glass. Find the width of the frame.

27. A certain number is expressed by two digits. The units' digit exceeds the square of half the tens' digit by 5, and if 45 be added to the number, the digits will be interchanged. What is the number?

28. A merchant sold silk for \$90. If he had sold 5 yards more for the same money, he would have received 25 cents less per yard. How many yards did he sell?

29. A man can row 4 miles up a river and back in 6 hours. At what rate can he row in still water, if the rate of the current is half a mile an hour?

30. The perimeter of a rectangular field is 66 rods, and its area is 270 square rods. Find the dimensions.

31. A lady bought some velvet for $39 and silk for $45, paying 25 cents a yard more for the velvet than for the silk. If she bought 3 yards more of silk than velvet, how many yards of each did she buy?

32. The larger of two pipes will fill a cistern in 2 hours 30 minutes less than the other, and both will fill it in 3 hours. How long will it take each to fill it?

33. A man bought a piece of land for $1020. He reserved twenty acres and sold the remainder at an advance of $1.50 an acre, receiving for it $20 less than it all cost. At what price per acre did he sell it?

34. What is the price of oranges per dozen, when 5 less for 50 cents increases the price 10 cents a dozen?

35. A rectangle, which is 6 feet longer and 4 feet wider than a given square, contains double the area of the square. What is the length of the rectangle?

36. The denominator of a fraction exceeds twice the numerator by 2. If 5 be added to the numerator and subtracted from the denominator, the result will be the reciprocal of the given fraction. Find the fraction.

37. A merchant bought silk for $45. He was obliged to sell it at $.65 a yard, thus losing as much as 8 yards cost. At what price per yard did he buy it?

38. A grocer sold tea for $45, and the number of pounds sold exceeded the number of cents in the price by 15. How many pounds did he sell?

39. The number of square inches in the surface of a cube of wood exceeds the number of inches in the sum of its edges by 378. Find the volume of the cube.

40. If 13,725 soldiers are arranged in two solid squares with 15 more men on a side of one square than on the other, how many men are there in each square?

41. A rectangular garden, 50 yards by 40 yards, has a walk of uniform width around it. The area of the walk equals $\frac{1}{2}$ the area of the garden. Find the width of the walk.

42. A and B set out from the same point and travel along the sides of a right angle. At the end of 9 days they are 45 miles apart. Find the rate at which each travels, if A travels a mile a day more than B.

43. A farmer sold some sheep for \$150. If he had sold 10 less for the same money, he would have received \$.50 more per head. How many did he sell?

44. A merchant sold a quantity of goods for \$21, and lost a per cent equal to the number of dollars he paid for the goods. How much did he lose?

45. A crew can row 3 miles down a river and back in 2 hours 30 minutes. Their rate of rowing in still water exceeds three times the rate of the current by 1 mile. At what rate can they row in still water?

46. The length of a rectangle is 10 inches less than twice its width, and the length of its diagonal is 25 inches. Find the dimensions of the rectangle.

47. A speculator bought apples for \$300. He sold all but ten barrels of them at a profit of 25 cents a barrel, receiving for them \$45 more than they all cost. How many barrels did he sell?

TO THE TEACHER. — Teach 389 . . . . 393.

SIMULTANEOUS QUADRATIC EQUATIONS.

1. $\begin{cases} x^2+y^2=41 \\ x+3y=19 \end{cases}$

2. $\begin{cases} 4x^2-xy=90 \\ 2x+3y=16 \end{cases}$

3. $\begin{cases} x^2-y^2=40 \\ 2x+y=17 \end{cases}$

4. $\begin{cases} y^2+2x^2=11 \\ 3x+2y=9 \end{cases}$

5. $\begin{cases} x^2+2y^2=17 \\ 4x+y=14 \end{cases}$

6. $\begin{cases} 3xy-2y^2=10 \\ 5x-y=13 \end{cases}$

7. $\begin{cases} x+4y=14 \\ 4x-2y+y^2=11 \end{cases}$

8. $\begin{cases} 7x^2-3xy=22 \\ 5x+4y=14 \end{cases}$

9. $\begin{cases} 2x-3y=1 \\ 2x^2+xy-5y^2=20 \end{cases}$

10. $\begin{cases} 3x^2-2xy=15 \\ 2x+3y=12 \end{cases}$

11. $\begin{cases} 3x+2y=22 \\ 5x^2-3xy+y^2=45 \end{cases}$

12. $\begin{cases} x+3y=2xy \\ 2x-y=5 \end{cases}$

To the Teacher. — Teach 394.

1. $\begin{cases} x^2-xy=35 \\ xy+y^2=18 \end{cases}$

2. $\begin{cases} 3x^2+xy=18 \\ 4y^2+3xy=54 \end{cases}$

3. $\begin{cases} x^2-xy=70 \\ xy-y^2=21 \end{cases}$

4. $\begin{cases} 5x^2-3xy=56 \\ 5y^2+xy=28 \end{cases}$

5. $\begin{cases} x^2-xy=6 \\ x^2+y^2=61 \end{cases}$

6. $\begin{cases} 2x^2+3xy=26 \\ 3y^2+2xy=39 \end{cases}$

7. $\begin{cases} 3xy+x^2=10 \\ 5xy-2x^2=2 \end{cases}$

8. $\begin{cases} x^2+3xy=28 \\ xy+4y^2=8 \end{cases}$

9. $\begin{cases} 2x^2-3xy=0 \\ y^2+5xy=34 \end{cases}$

10. $\begin{cases} 5x^2+3xy=26 \\ 3y^2+2xy=7 \end{cases}$

Note. — These equations may also be solved by letting $x=vy$.

To the Teacher. — Teach 395 and 396.

1. $\begin{cases} x+y=5 \\ xy=-14 \end{cases}$

2. $\begin{cases} x^2+y^2=49-2xy \\ x^2-2xy=36-y^2 \end{cases}$

3. $\begin{cases} x^2-y^2=40 \\ x-y=4 \end{cases}$

4. $\begin{cases} x^2+xy+y^2=61 \\ x+y=9 \end{cases}$

5. $\begin{cases} x^2+y^2=90 \\ xy=27 \end{cases}$

6. $\begin{cases} x^2+y^2=1+2xy \\ x^2+y^2=37-xy \end{cases}$

7. $\begin{cases} x-y=3 \\ 2xy=+36 \end{cases}$

8. $\begin{cases} x^2-xy+y^2=76 \\ x+y=14 \end{cases}$

9. $\begin{cases} x^2-y^2=9 \\ x+y=9 \end{cases}$

10. $\begin{cases} x^2+xy+y^2=67 \\ x^2-xy+y^2=39 \end{cases}$

11. $\begin{cases} x^2+y^2=65 \\ x+y=11 \end{cases}$

12. $\begin{cases} x^2-xy+y^2=19 \\ x-y=3 \end{cases}$

13. $\begin{cases} x^3+y^3=35 \\ x+y=5 \end{cases}$

14. $\begin{cases} x^2-xy+y^2=21 \\ x^3+y^3=126 \end{cases}$

MISCELLANEOUS EXAMPLES.

1. $\begin{cases} x^2+y^2=13 \\ x+3y=9 \end{cases}$

2. $\begin{cases} x^4-y^4=369 \\ x^2+y^2=41 \end{cases}$

3. $\begin{cases} x^2+y^2=29 \\ x-y=3 \end{cases}$

4. $\begin{cases} x^2+xy=77 \\ xy-y^2=12 \end{cases}$

5. $\begin{cases} x^2+3y^2=28 \\ xy+2y^2=16 \end{cases}$

6. $\begin{cases} x^2-y^2=65 \\ 2x-y=14 \end{cases}$

7. $\begin{cases} x^3+y^3=3473 \\ x+y=23 \end{cases}$

8. $\begin{cases} x^2y+xy^2=30 \\ x+y=5 \end{cases}$

9. $\begin{cases} 4x^2 - 3xy = 91 \\ 2x - 3y = -1 \end{cases}$

10. $\begin{cases} 3x^2 + 2xy + y^2 = 33 \\ x^2 + xy + y^2 = 19 \end{cases}$

11. $\begin{cases} x^2 - 2y^2 = 8 \\ 3y^2 - xy = 4 \end{cases}$

12. $\begin{cases} x^2 + xy + y^2 = 39 \\ x - y = 3 \end{cases}$

13. $\begin{cases} 4x^2 - 4y^2 = 15 \\ 2x - 2y = 3 \end{cases}$

14. $\begin{cases} x^2 + 3xy - 2y^2 = 52 \\ 2x + 3y = 36 \end{cases}$

15. $\begin{cases} xy + y^2 = 40 \\ y - 3x = -4 \end{cases}$

16. $\begin{cases} 2x^2 + 4xy + 3y^2 = 33 \\ 4x^2 + 5xy = 51 \end{cases}$

17. $\begin{cases} 4x^2 - 2xy = 12 \\ 2y^2 + 3xy = 8 \end{cases}$

18. $\begin{cases} x^2 + xy + y^2 = 61 \\ x^2 - y^2 = 61 \end{cases}$

19. $\begin{cases} x^3 - y^3 = 37 \\ x - y = 1 \end{cases}$

20. $\begin{cases} x^4 + x^2y^2 + y^4 = 91 \\ x^2 + xy + y^2 = 13 \end{cases}$

21. $\begin{cases} 2x - 5y = 0 \\ x^2 - 3y^2 = 13 \end{cases}$

22. $\begin{cases} 2x^2 - xy + y^2 = 29 \\ x^2 + 2xy - 2y^2 = 22 \end{cases}$

23. $\begin{cases} x^2 - 2xy = 45 \\ 2xy + y^2 = 40 \end{cases}$

24. $\begin{cases} x^2 - xy + y^2 = 73 \\ x^3 + y^3 = 1241 \end{cases}$

25. $\begin{cases} x + 2y = 27 \\ xy = 85 \end{cases}$

26. $\begin{cases} x^2 + y^2 - x - y = 18 \\ xy + x + y = 19 \end{cases}$

27. $\begin{cases} 4x - 5y = 29 \\ x^2 - 5y^2 = 76 \end{cases}$

28. $\begin{cases} x^2 + xy + 2y^2 = 44 \\ 2x^2 - xy + y^2 = 16 \end{cases}$

29. $\begin{cases} x^2 - 3xy = 0 \\ 5x^2 + 3y^2 = 48 \end{cases}$

30. $\begin{cases} x^2 + xy + y^2 = 84 \\ x + \sqrt{xy} + y = 14 \end{cases}$

31. $\begin{cases} 2x - 3y = 9 \\ xy = 6 \end{cases}$

32. $\begin{cases} x^2 + y^2 + x + y = 18 \\ xy = 6 \end{cases}$

33. $\begin{cases} 3x - 2y = 23 \\ x^2 - 2xy = 45 \end{cases}$

34. $\begin{cases} 2x - y = 35 \\ xy = 57 \end{cases}$

35. $\begin{cases} 3x^2 - y^2 = 23 \\ xy + y^2 = 45 \end{cases}$

36. $\begin{cases} \sqrt{x} + \sqrt{y} = 7 \\ x + y = 25 \end{cases}$

37. $\begin{cases} x^2 - xy = 80 \\ x - y = 16 \end{cases}$

38. $\begin{cases} x^2y + xy^2 = 30 \\ x^3 + y^3 = 35 \end{cases}$

39. $\begin{cases} x^2 - y^2 = 16 \\ 2x + y = 13 \end{cases}$

40. $\begin{cases} x^2 - 2y^2 = 31 \\ x^2y^2 = 441 \end{cases}$

41. $\begin{cases} 4x^2 + 5xy = 14 \\ 7xy + 9y^2 = 50 \end{cases}$

42. $\begin{cases} xy = 2a^2 + 2ab \\ x - y = a - b \end{cases}$

43. $\begin{cases} 4x^2 - 9y^2 = 19 \\ 2x + 3y = 19 \end{cases}$

44. $\begin{cases} x^4 + x^2y^2 + y^4 = 133 \\ x^2 - xy + y^2 = 7 \end{cases}$

45. $\begin{cases} x^2 + 4y = 76 \\ 3x - y = 21 \end{cases}$

46. $\begin{cases} x^2 + xy + y^2 = 84 \\ x - \sqrt{xy} + y = 6 \end{cases}$

47. $\begin{cases} x^4 - y^4 = 240 \\ x^2 - y^2 = 12 \end{cases}$

48. $\begin{cases} 2x^2 + 3xy + y^2 = 70 \\ 6x^2 + xy - y^2 = 50 \end{cases}$

49. $\begin{cases} x - y = 5 \\ \sqrt{xy} = 6 \end{cases}$

50. $\begin{cases} x^2 + y^2 - x - y = 50 \\ xy = 30 \end{cases}$

51. $\begin{cases} 3x + 4y = 2xy \\ 2x - y = 5 \end{cases}$

52. $\begin{cases} x^2 - x - y + y^2 = 78 \\ xy + x + y = 39 \end{cases}$

53. $\begin{cases} xy^2 - x^2y = -70 \\ x^3 - y^3 = 218 \end{cases}$

54. $\begin{cases} 4x^2 - 4y^2 = 7x + 7y \\ 2x^2 - 2y^2 = 5x - 5y \end{cases}$

55. $\begin{cases} x^2 + y^2 = 5x^2y^2 \\ x + y = xy \end{cases}$

56. $\begin{cases} x^2 + 2xy + 3x + y = 73 \\ y^2 + x + 3y = 44 \end{cases}$

PROBLEMS.

1. The sum of two numbers is a, and the sum of their squares is b. What are the numbers?

2. Find two numbers the difference of whose squares is 33, and the product of their squares 784.

3. The combined area of two square fields is $8\frac{1}{8}$ acres, and the sum of their perimeters is 200 rods. What is the area of each field?

4. The sum of the squares of two numbers exceeds twice their product by 4, and the difference of their squares is 11 less than their product. Find the numbers.

5. The difference of two numbers is 28, and half their product is equal to the cube of the smaller number. What are the numbers?

6. The area of the ceiling of a hall is 700 square feet, and its length is six feet less than four times the width. Find the dimensions.

7. If the sum of two numbers is multiplied by their product, the result is 210; and the sum of the cubes of the numbers is 370. Find the numbers.

8. If the dimensions of a rectangle were each increased 1 foot, the area would be 99 square feet; if they were each diminished 1 foot, the area would be 63 square feet. What are the dimensions?

9. A number is expressed by two figures the sum of which is 14, and the sum of the squares of the digits exceeds the number by 11. Find the number.

10. The combined area of two adjoining square fields is 900 square rods, and it requires 150 rods of fence to inclose them. If they are so situated as to require the least amount of fence, what is the dimension of each?

11. The area of a rectangle is 192 square inches, and its diagonal is 20 inches. Find the dimensions.

12. A rectangular field contains 270 square rods. If it were two rods longer and one rod wider, it would contain 50 square rods more. Find the dimensions of the field.

13. A farmer bought 12 sheep and 4 calves for $ 60. At the same prices, he could buy 3 more sheep for $ 24 than calves for $ 30. Find the price of each.

14. The perimeter of a rectangular piece of ground is 200 rods, and its area is fifteen acres. Find the dimensions of the field.

15. The hypotenuse of a right triangle is thirty feet, and its area is two hundred sixteen square feet. Find the length of the other two sides.

16. The sum of the cubes of two numbers is 468, and the sum of their squares exceeds their product by 39. What are the numbers?

17. A merchant bought two kinds of silk, paying $ 6 for each piece, and buying 8 yards more of one kind than the other. The difference in price was 50 cents a yard. How many yards of each kind did he buy?

18. A rectangular piece of paper contains 1350 square inches; but if the dimensions were each 5 inches less, it would contain 1000 square inches. Find the dimensions.

19. If the sum of two numbers is added to their product, the result is 31; and the sum of their squares exceeds their sum by 48. What are the numbers?

20. A man bought sheep for $ 136. He kept 22 of them, and sold the remainder at a profit of $ 1 a head, receiving for them $ 2 more than they all cost. At what price per head did he buy them?

RATIO.

Simplify the following ratios:

1. $3\frac{1}{3} : 2\frac{1}{4}$.
2. 9 in. : 2 yd.
3. \$1.75 : \$2.
4. $8\frac{3}{4} : 1\frac{5}{8}$.
5. 4 rd. : 6 ft.
6. \$16 : \$.80.
7. $2\frac{5}{6} : 6\frac{1}{3}$.
8. 6 oz. : 3 lb.
9. \$4.20 : \$9.
10. $a^2b : ab^2$.
11. $4\sqrt{3} : 2\sqrt{2}$.
12. $12\sqrt{54} : \sqrt{2}$
13. $xy^4 : x^2y$.
14. $2\sqrt[3]{3} : 3\sqrt{2}$.
15. $14\sqrt[3]{9} : 2\sqrt{3}$.
16. $x^2 - y^2 : x - y$.
17. $x^2 - 8x + 15 : x^2 - 5x$.
18. $x + y : x^2 - y^2$.
19. $x^2 + 7x + 12 : 3x + x^2$.

20. Arrange the ratios $3 : 4$, $5 : 6$, $\sqrt{25} : \sqrt{64}$, $2 : 3$, and $5\sqrt{40} : \sqrt{2}$ in descending order of magnitude.

21. What is the ratio compounded of $4 : 5$, $15 : 16$, and $12 : 6$? Of $14 : 18$, $15 : 7$, and $12 : 25$?

22. What is the ratio compounded of $a^3x : by^2$ and $b^2y^2 : a^2x$? Of $4a^2b : 3ab^2$, $3ax^2 : 2by^2$, and $b^2y^2 : 3ax^2$?

23. What is the ratio compounded of $32 : 15$ and the duplicate ratio of $5 : 4$?

24. What is the ratio compounded of $a^4x^4 : 2b^2y^2$ and the triplicate ratio of $2by : ax$?

25. What is the ratio compounded of $4 : 3$ and the subduplicate ratio of $3 : 8$?

26. What is the value of x when the ratio of $17 - x : 18 + x$ is two-thirds?

27. By what number must the terms of the ratio, $13 : 17$, be equally diminished to make it equal to the ratio of $5 : 8$?

PROPORTION.

Find the value of x in the following proportions:

1. $3\frac{1}{3} : .02 :: x : .012$.
2. $\frac{4}{5} : x = 4 : 4\frac{1}{2}$.
3. $15 : 5 :: \$48.75 : x$.
4. $\frac{5}{7} : 2\frac{1}{7} = x : 5$.

5. $\$8.20 : \$3.28 :: x : 12$.
6. $x : 12 = \frac{2}{3} : \frac{1}{3}$.
7. 4 yd. : 5 ft. :: $\$2.04 : x$.
8. $\frac{5}{7} : x = 4\frac{1}{3} : 3\frac{1}{2}$.
9. $5 : x :: \$12.05 : \$21.69$.
10. $4\frac{1}{2} : x = 1\frac{2}{3} : 1\frac{5}{6}$.
11. $\$8.75 : x ::$ 4 rd. : 11 yd.
12. $x : 5 = .7 : 1.4$.
13. $x : 30 :: \$14.28 : \$35.70$.
14. $9\frac{1}{2} : 19 = x : 1\frac{1}{4}$.
15. $x-a : x-b :: x-c : x-d$.
16. $x+7 : x-2 :: 2x-2 : x-3$.
17. $ax+m : bx+n :: ax+c : bx+d$.
18. $4x+a : 4x+c :: 3x-d : 3x-b$.
19. $(x+7)(x-6) : (x-5)(x-6) :: (x+5)(x-3) : (x-2)(x-6)$.

Find the mean proportional between

20. 16 and 4.
21. a^4 and a^2.
22. $12a^2x^2$ and $3a^2$.
23. 16 and 9.
24. a^3 and a^2.
25. $27a^2x^3$ and $3a^3$.
26. 25 and 2.
27. a^6 and x^4.
28. $18a^4x^3$ and $6x^3$.

Find the third proportional to

29. 12 and 6.
30. a^4 and a^3.
31. $48a^2$ and $8a^3x^2$.
32. 15 and $\sqrt{3}$.
33. a^5 and a^3.
34. $24x^3$ and $6a^2x^2$.
35. 14 and $\sqrt{7}$.
36. a^2 and a^4.
37. $30a^4$ and $5a^2x^3$.

Find the fourth proportional to

38. 8, 6, and 4.
39. $2a^2$, $3a^2b$, and $4b^2$.
40. 7, 5, and 6.
41. $3a^3$, $2ab^2$, and $5a^2$.
42. 4, 9, and 7.
43. $5a^4$, $3a^2b$, and $6a^2$.
44. 3, 7, and 9.
45. $4x^2$, $5b^2x$, and $8x^2$.
46. 9, 4, and 7.
47. $7a^4$, $3a^2x$, and $8a^3$.

48. If $a : b :: c : d$, show that $ac : bd = c^2 : d^2$.

49. The sum of two numbers is 15, and the ratio of their squares is $2\frac{1}{4}$. Find the numbers.

50. The ratio of two numbers is $\frac{1}{3}$, and the sum of their squares is 468. Find the numbers.

51. If $a : b :: c : d$, show that $ab : cd = a^2 : c^2$.

52. The sides of a triangle are as $2 : 3 : 4$, and the perimeter is 135 feet. Find the sides.

53. The area of a rectangular field is 4 acres, and the ratio of the dimensions is $1\frac{2}{3}$. Find the dimensions.

54. Divide \$64 between two men so that their shares will be to each other as 3 to 5.

55. There are two numbers which are to each other as 5 to 3; and if 5 be added to each number, the ratio of the resulting numbers will be $1\frac{1}{2}$. Find the numbers.

56. The ratio of the areas of two squares, whose sides differ by 20 rods, is $2\frac{1}{4}$. Find the side of each.

57. Eight years ago A's age was to B's as 3 to 2, but the ratio of their ages now is $1\frac{2}{3}$. Find their ages now.

VARIATION.

1. If $x \propto y$, and $x = 5$ when $y = 7$, find x when $y = 21$.

2. If x varies inversely as y, and $x = 4$ when $y = 9$, find x when $y = 13$.

3. If x varies jointly as y and z, and $x = 48$ when $y = 4$ and $z = 3$, find x when $y = 5$ and $z = 6$.

4. If $x \propto y$ and $y \propto z$, show that $xz \propto y^2$.

5. If x varies directly as y and inversely as z, and $x = 6$ when $y = 3$ and $z = 9$, find x when $y = 4$ and $z = 8$.

6. If $x \propto y$, and $x = 3$ when $y = 11$, find y when $x = 5$.

7. If x varies inversely as y, and $x = 11$ when $y = 7$, find y when $x = 154$.

8. If $3x+5$ varies as $5y-3$, and $x=7$ when $y=11$, find x when $y=9$.

9. If x varies directly as y and inversely as z, and $x=\frac{1}{2}$ when $y=\frac{1}{3}$ and $z=2\frac{1}{4}$, find y when $x=3$ and $z=7$.

10. If x^2 varies as y^3, and $x=3$ when $y=2$, find the equation between x and y.

11. If $x \propto y^3$ and $y^3 \propto z$, show that $xz \propto y^5$.

12. If x^2 varies as y^3, and $x=3$ when $y=6$, what is the value of y when $x=7$?

13. If x varies inversely as y and y varies inversely as z, show that x varies as z.

14. If x varies jointly as y and z, and $x=\frac{1}{2}$ when $y=\frac{1}{3}$ and $z=\frac{1}{5}$, find y when $x=60$ and $z=6$.

15. If x varies as y, and $x=4$ when $y=2$, find the value of x in terms of y.

16. If x varies inversely as y, and $x=2$ when $y=3$, find the value of y in terms of x.

17. If $3x+2a$ varies as $y-3a$, and $x=6a$ when $y=13a$, find y when $x=4$.

18. If x varies directly as y and inversely as z, show that y varies directly as the product of x and z.

19. If $3x+7y$ varies as $3x+13y$, and $x=5$ when $y=3$, find the equation between x and y.

20. If x varies inversely as z^2-1, and $x=10$ when $z=5$, find z when $x=16$.

21. If x varies as 1 divided by y, plus 1 divided by z, and $x=8$ when $y=1$ and $z=3$, show that $xyz=6(y+z)$.

22. If $x \propto ay+z$, and $x=3$ when $y=1$ and $z=2$; and $x=5$ when $y=2$ and $z=3$, find the value of a.

23. If x varies directly as y, and z varies inversely as y, and $x+z=5$ when $y=3$; and $x+z=-11$ when $y=-5$, find the value of x and z in terms of y.

24. The area of a circle whose diameter is 10 is 785.4. Find the area of a circle whose diameter is 30.

25. If a body falls 48 feet in two seconds, how far will it fall in 10 seconds?

26. How many circles 5 inches in diameter are equivalent to one circle 20 inches in diameter?

27. The area of a rectangle is x; its base, y; its altitude, z. Find the area when its base is $4y$ and its altitude $\frac{5}{8}z$.

28. If the velocity of a falling body at the end of 3 seconds is 144 feet, what is its velocity at the end of 9 seconds?

29. How many spheres 2 inches in diameter are equivalent to one sphere 8 inches in diameter?

30. The volume of a sphere whose radius is 1 is 4.188. Find the volume of a sphere whose radius is 5.

ARITHMETICAL PROGRESSION.

1. Find the 15th term of the series 5, 8, 11, ...
2. Find the 17th term of the series 19, 15, 11, ...
3. Find the 25th term of the series $1\frac{1}{2}$, 2, $2\frac{1}{2}$, ...
4. Find the 18th term of the series -37, -32, -27, ...
5. Find the nth term of the series 5, 16, 27, ...

Find a when

6. $n=9$, $d=4$, $l=75$.
7. $n=16$, $d=\frac{1}{3}$, $l=23$.
8. $n=8$, $d=-5$, $l=-9$.
9. $n=27$, $d=\frac{3}{5}$, $l=-5$.

Find d when

10. $a=5$, $n=15$, $l=103$. 11. $a=72$, $n=13$, $l=12$.
12. $a=9$, $n=21$, $l=-1$. 13. $a=45$, $n=19$, $l=-45$.

Find n when

14. $a=3$, $d=2$, $l=39$. 15. $a=4\frac{1}{2}$, $d=\frac{1}{2}$, $l=15$.
16. $a=47$, $d=-3$, $l=2$. 17. $a=37$, $d=-4$, $l=-7$.

1. Insert 7 arithmetical means between 7 and 47.
2. Insert 15 arithmetical means between 55 and 7.
3. Insert 9 arithmetical means between 3 and -9.
4. Insert 7 arithmetical means between 3 and 5.

1. Find the sum of 27 terms of the series 5, 9, 13, ...
2. Find the sum of 13 terms of the series 11, 8, 5, ...
3. Find the sum of 11 terms of the series 3, $4\frac{1}{2}$, 6, ...
4. Find the sum of 23 terms of -17, -13, -9, ...

Find n and d if

5. $a=7$, $l=35$, $s=84$. 6. $a=16$, $l=-12$, $s=30$.
7. $a=4\frac{1}{2}$, $l=23\frac{1}{2}$, $s=126$. 8. $a=-3$, $l=-9$, $s=-78$.

Find a and d if

9. $n=14$, $l=13$, $s=126$. 10. $n=12$, $l=62$, $s=414$.
11. $n=11$, $l=19$, $s=126\frac{1}{2}$. 12. $n=13$, $l=-21$, $s=-39$

Find l and d if

13. $a=5$, $n=17$, $s=901$. 14. $a=7$, $n=13$, $s=143$.
15. $a=33$, $n=11$, $s=-22$. 16. $a=-11$, $n=15$, $s=150$.

1. Find three numbers in arithmetical progression, whose sum is 21 and the sum of whose squares is 179.

2. The sum of four numbers in arithmetical progression is 38, and their product is 6160. Find the numbers.

3. The product of five numbers in arithmetical progression is 945, and their sum is 25. Find the numbers.

1. Insert 7 arithmetical means between 5 and 61.

2. Insert 5 arithmetical means between $\frac{1}{8}$ and $\frac{8}{4}$.

3. If a wheelman rides 3 miles the first day of June, 5 miles the second day, increasing the distance daily at the same rate, how many miles will he ride during the month?

4. A debt can be discharged in a year by paying \$3 the first week, \$5 the second, \$7 the third, and so on. Find the last payment and the amount of the debt.

5. Find four numbers in arithmetical progression, the sum of whose squares is 84, and whose product is 105.

6. Five heirs inherited an estate of \$17,000. Their shares formed an arithmetical progression, the common difference of which was \$600. How much did each receive?

7. A man paid \$2100 in 7 installments. The first payment was \$150, and the last \$450, all the payments being in arithmetical progression. Find each payment.

8. The sum of five numbers in arithmetical progression is 30, and the sum of their squares 220. Find the numbers.

9. Find the sum of the numbers $1, 2, 3, \cdots$, to 800.

10. If the fourth term of an A. P. is 24, and the 16th term is 108, what is the common difference?

11. If the fifth term of an A. P. is 5, and the twentieth term is -40, what is the common difference?

12. The first term of an A. P. is 5, and its fourth term is 26. What is the thirteenth term?

13. The first term of an A. P. is 53, and its twentieth term is -4. What is the eleventh term?

14. Find the sum of the series 1, $2\frac{3}{4}$, $4\frac{1}{2}$, $\cdots$, to 13 terms.

15. Find the sum of -9, -4, 1, $\cdots$, to 16 terms.

16. Find the sum of all the numbers between 100 and 300 that are divisible by 2.

17. A body falls $16\frac{1}{12}$ feet the first second, and in each succeeding second $32\frac{1}{8}$ feet more than in the next preceding one. How far will it fall in half a minute?

18. The product of five numbers in arithmetical progression is 10,395, and their sum is 35. Find the numbers.

19. The sum of the squares of the extremes of four numbers in arithmetical progression is 185, and the sum of the square of the means is 149. Find the numbers.

20. If a man travels eight miles the first day of his journey, and increases his rate two miles a day, how far will he travel in two full weeks?

21. If the first term of an arithmetical progression is 150, and the number of terms 26, what must the common difference be that the sum of the series may be 2925?

22. If 50 apples are placed in a line three yards apart, and the first three yards from a basket, how far must a boy travel, starting from the basket, to gather them up singly, and return with each to the basket?

23. A man agrees to work 300 days at the following wages: 5 cents the first day, 10 cents the second day, 15 cents the third day, increasing at the same rate each day. How much will he receive?

24. A man saves \$360 the first year, \$320 the second, and so on. Another man saves \$220 a year. In how many years will the two have saved equal amounts?

25. If a body falling to the earth descends a feet the first second, $3a$ feet the second, $5a$ feet the third, and so on, how far will it fall during the nth second?

GEOMETRICAL PROGRESSION.

1. Find the sum of 10 terms of the series 3, 9, 27, ...
2. Find the sum of 9 terms of the series 16, 8, 4, ...
3. Find the sum of 11 terms of the series 2, 6, 18, ...

Find the sum of the following infinite series:

4. 9, 6, 4, ...
5. 1, $\frac{1}{3}$, $\frac{1}{9}$, ...
6. $\frac{1}{2}$, $-\frac{1}{4}$, $\frac{1}{8}$, ...
7. 8, 4, 2, ...
8. 1, $\frac{1}{2}$, $\frac{1}{4}$, ...
9. $\frac{7}{1}$, -1, $\frac{3}{7}$, ...

1. Insert 3 geometrical means between 5 and 405.
2. Insert 4 geometrical means between 9375 and 3.
3. Insert 7 geometrical means between 7 and 1792.

1. Find the 7th term of the series 3, 9, 27, ...
2. Find the 9th term of the series 16, 8, 4, ...
3. Find the nth term of the series 1, 4, 16, ...
4. Find the 7th term of the series -25, 5, -1, ...
5. Find a, if $l = 128$, $r = 2$, and $n = 7$.
6. Find a, if $l = 1458$, $n = 6$, and $r = 3$.
7. Find r, if $a = 7$, $n = 8$, and $l = 896$.
8. Find r, if $a = -3$, $n = 8$, and $l = 384$.

9. Find n, if $a = 3$, $r = 2$, and $l = 192$.

10. Find n, if $a = 3$, $r = 3$, and $l = 243$.

Find the value of the following repeating decimals:

1. $.2\dot{3}\dot{7}$. 2. $7.27\dot{1}0\dot{7}$. 3. $.67\dot{3}$.

4. $.2\dot{3}\dot{8}$. 5. $3.4\dot{1}74\dot{3}$. 6. $.0\dot{5}\dot{7}$.

1. The sum of a geometrical series containing 8 terms is 1785, and the ratio is 2. Find the first term.

2. Insert 5 geometrical means between 5 and 3645.

3. The first term of a G. P. is 3, and the third term is 4. Find the seventh term.

4. Insert one geometrical mean between 12 and 972.

5. Insert three geometrical means between 3 and 1875.

6. The fourth term of a G. P. is .016, and the seventh term is .000128. Find the first term.

7. The last term of a geometrical progression is 1701, the ratio 3, and the sum 2548. Find the first term.

8. The first term of a G. P. is 8, the number of terms 7, and the last term 5832. What is the ratio?

Find the value of the following repeating decimals:

9. $.2\dot{7}$. 10. $.4\dot{1}\dot{7}$. 11. $.\dot{1}0\dot{4}$. 12. $.3\dot{2}7\dot{3}$.

13. Find the sum of 8 terms of the series 6, 24, 96, ...

14. The last term of a G. P. is 2673, the number of terms 6, and the ratio 3. What is the first term?

15. The fifth term of a geometrical progression is 48, and the eighth term is -384. Find the first term.

16. Find the sum of 6 terms of the series $-\frac{2}{3}, \frac{1}{2}, -\frac{3}{8}, \ldots$.

17. The first term of a geometrical progression is 3, the last term 12,288, and the sum 16,383. Find the ratio.

Find the sum of the following infinite series:

18. $75, 15, 3, \cdots$. 19. $2, \frac{4}{3}, \frac{8}{9}, \cdots$. 20. $\frac{3}{4}, -\frac{1}{2}, \frac{2}{3}, \cdots$.

21. $98, 14, 2, \cdots$. 22. $5, 2, \frac{4}{5}, \cdots$. 23. $\frac{5}{6}, -\frac{1}{2}, \frac{3}{10}, \cdots$.

24. What is the distance passed through by a ball that falls 50 feet and each time rebounds half the distance?

25. The first term of a geometrical progression is 7, the ratio 3, and the sum 2548. What is the last term?

26. The second term of a geometrical progression exceeds the first by 15, and the fourth term exceeds the third by 540. Find the four terms of the series.

NOTE. — Let x = the first term, and y = the ratio.

INEQUALITIES.

Simplify the following inequalities and find the limits of the unknown quantities:

1. $5x - 4 > 2x + 8$. 2. $7x - 4 < 4x + 5$.

3. $3x - a > 7a - x$. 4. $a - 4x < x - 4a$.

5. $\frac{4x}{5} + 3 < \frac{3x}{2} - 4$. 6. $\frac{3x}{2} - 4 < \frac{2x}{3} + 6$.

7. $\frac{ax}{5} + bx - ab > \frac{a^2}{5}$. 8. $\frac{2x}{3} - 8 - \frac{x}{2} < \frac{5x}{6}$

9. If a and b are unequal positive numbers, is $(a-b)^2$ positive or negative? How does it compare with 0?

10. If a and b are unequal positive numbers, prove that $a^2 + b^2 > 2ab$.

11. Show that for any value of x except 2, $x^2 + 4 > 4x$.

12. If x and y are unequal positive numbers, prove that $x^2y + xy^2 > 2x^2y^2$.

13. Show that for any value of x except $\frac{3}{4}$, $24x < 16x^2 + 9$.

14. If a and b are unequal positive numbers, is $a^2b + ab$ greater or less than $a^3 + b^3$?

15. Simplify $(4x-1)^2 - 38 > (2x-3)(8x-5)$.

16. Simplify $(5x+2)^2 - 42 < (3x-4)^2 + (4x-3)^2$.

17. If a and b are unequal positive numbers, is $2b(a+b)$ greater or less than $a^2 + 3b^2$?

18. Simplify $(2x-3)(8x+2) > (4x-5)(4x+6) - 48$.

19. A farmer has a number of lambs such that 3 times the number, minus 14, is greater than twice the number, plus 33; and twice the number, plus 26, is greater than 4 times the number, minus 72. How many lambs has he?

20. $\begin{cases} 5x - 5 > 3x + 3 \\ 3x - 7 < 2x + 2 \end{cases}$

21. $\begin{cases} 4x + 3 > 5y + 7 \\ 3x - 2 = 8y - 5 \end{cases}$

22. $\begin{cases} 2x + y = 3y + 4 \\ 3x - 7 < 6 + 5y \end{cases}$

23. $\begin{cases} 7x - 3 > 3y - 8 \\ 4x + 5 = 3 + 2y \end{cases}$

24. $\begin{cases} x + 5y = 13 - y \\ x + 2y > 7 - 4x \end{cases}$

25. $\begin{cases} 4x + y = 38 - y \\ x - 22 > 8 - 5y \end{cases}$

26. $\begin{cases} 3x - 2 < \frac{x}{2} + 5\frac{1}{2} \\ \frac{4x-2}{3} > \frac{2-4x}{5} \end{cases}$

27. $\begin{cases} \frac{5x}{6} + \frac{y}{5} = 14 - \frac{y}{5} \\ \frac{3x}{4} - \frac{y}{5} > \frac{y}{5} + 4 \end{cases}$

28. $\begin{cases} \frac{cx}{3} + dx > cd + \frac{c^2}{3} \\ \frac{dx}{8} + cd < cx + \frac{d^2}{8} \end{cases}$

29. $\begin{cases} \frac{x}{2a-b} > \frac{2a-b}{x} \\ ax - bx < \frac{a^2-b^2}{x} \end{cases}$

30. If the letters represent unequal positive numbers, is $xy + xz + yz$ greater or less than $x^2 + y^2 + z^2$?

31. Simplify $8ax - 6bc > 12ac - 4bx$, and find the limit of x, if $2a + b$ is negative.

32. If $\frac{1}{2}x$ and y are unequal positive numbers, is $(x+2y)(x-2y)$ greater or less than $4y(x-y)$?

33. If a and b are unequal positive numbers, is $\frac{a}{b} + \frac{b}{a}$ greater or less than 2?

34. If $\frac{3}{4}x$ and y are unequal positive numbers, is $x(9x - 4y)$ greater or less than $4y(2x - y)$?

35. If $\frac{1}{2}x$ and y are unequal positive numbers, show that $\frac{2x}{y} + \frac{2y}{x} > 2$.

36. If the letters are unequal positive numbers, show that $abc > (a + b - c)(a - c + b)(b + c - a)$.

INDETERMINATE EQUATIONS.

Assuming that n in the following equations is zero or integral, determine which of them are capable of solution in positive integers, and how many solutions each has.

1. $\begin{cases} x = 2n + 1 \\ y = 5 - 7n \end{cases}$

2. $\begin{cases} x = 2n - 7 \\ y = 3n + 5 \end{cases}$

3. $\begin{cases} x = 2n - 1 \\ y = 2 - 3n \end{cases}$

4. $\begin{cases} x = 3n - 1 \\ y = 2n - 8 \end{cases}$

5. $\begin{cases} x = 3n - 7 \\ y = 1 - 5n \end{cases}$

6. $\begin{cases} x = 5n + 2 \\ y = 4 - 3n \end{cases}$

7. $\begin{cases} x = 1 - 5n \\ y = 2 - 4n \end{cases}$ 8. $\begin{cases} x = 3n + 9 \\ y = 7 - 2n \end{cases}$ 9. $\begin{cases} x = 2n - 7 \\ y = 8 - 3n \end{cases}$

10. $\begin{cases} x = 5n - 1 \\ y = 7 - 2n \end{cases}$ 11. $\begin{cases} x = 3n - 1 \\ y = 2 - 2n \end{cases}$ 12. $\begin{cases} x = 2n + 7 \\ y = 3 - 8n \end{cases}$

13. $\begin{cases} x = 1 - 3n \\ y = 5 - 2n \end{cases}$ 14. $\begin{cases} x = 2n - 2 \\ y = 8 - 3n \end{cases}$ 15. $\begin{cases} x = 3n - 8 \\ y = 1 - 2n \end{cases}$

Solve the following in positive integers:

1. $3x + 7y = 58.$
2. $5x - 2y = 21.$
3. $4x + 5y = 34.$
4. $7y - 4x = 17.$
5. $5x + 9y = 37.$
6. $5x - 3y = 24.$
7. $4x + 7y = 65.$
8. $x - 7y = -50.$
9. $\begin{cases} x - 3z = -24. \\ 5x + 2y = 33. \end{cases}$
10. $\begin{cases} 8y - z = -21. \\ 7x - 5y = 57. \end{cases}$
11. $5x - 8y = -23.$
12. $3x - 8y = -21.$

13. A man bought cattle at \$34 a head and sheep at \$4 a head, paying \$210 for all. How many of each did he buy?

14. Find the least number which, being divided by 4 and 7, gives remainders 3 and 5, respectively.

15. In how many ways can the sum of \$4.40 be paid with quarters and dimes?

16. A farmer bought sheep and calves for \$117, paying \$3 a head for the sheep and \$7 a head for the calves. Find the least number and the greatest number of sheep he could have bought.

17. The sum of two fractions is $3\frac{5}{12}$, and their denominators are 3 and 4. Find the fractions.

18. Divide 230 into two parts, one of which is a multiple of 5, and the other a different multiple of 11.

19. A man paid a bill of $5.70 with half-dollars, quarters, and dimes, giving twenty-five coins in payment. How many of each did he give?

20. What is the least number which, being divided by 3, 4, and 5, gives remainders 2, 3, and 3, respectively?

21. A merchant sold two pieces of silk at 90 cents and 60 cents a yard, respectively, receiving $24 for it. Find the least number and the greatest number of yards he could have sold.

22. A dealer sold 100 books for $100, selling some at $2, some at $1, and the rest at 50 cents apiece. Find the least number and the greatest number of books he could have sold at the highest price.

23. A miller has oats worth 30 cents, corn worth 45 cents, and rye worth 75 cents a bushel. How many different mixtures of 200 bushels, worth 54 cents a bushel, can he make, and what is the least number and the greatest number of bushels of corn he can use?

EXPANSION OF FRACTIONS.

Expand to five terms in ascending powers of x:

1. $\frac{1+2x}{1-3x}$.
2. $\frac{1+x-x^2}{1-x+x^2}$.
3. $\frac{3}{1+2x-3x^2}$.
4. $\frac{1}{1-2x}$.
5. $\frac{4-x}{2+x+x^2}$.
6. $\frac{1+x-x^2}{1-3x+2x^2}$.
7. $\frac{1-2x}{2+3x}$.
8. $\frac{x(1-x)}{1+x-x^2}$.
9. $\frac{2-2x-3x^2}{3+2x-2x^2}$.
10. $\frac{2}{3-2x}$.
11. $\frac{3}{3-x+x^2}$.
12. $\frac{x+1}{x^2+3x^3-x^4}$.

Resolve into partial fractions:

1. $\frac{30x}{9x^2-4}$.
2. $\frac{2x-1}{(x+3)^2}$.
3. $\frac{x-9}{2x^2-3x}$.
4. $\frac{3-x}{x^3-x}$.
5. $\frac{4x}{(x-4)^2}$.
6. $\frac{12x-31}{x^2-5x+6}$.
7. $\frac{-24}{x^3+8}$.
8. $\frac{2x^3}{(x-5)^3}$.
9. $\frac{2x^2-29}{2x^2+x-6}$.
10. $\frac{x^3}{x^2-1}$.
11. $\frac{3x^3}{(x-2)^4}$.
12. $\frac{18x-25}{2-3x+x^2}$.
13. $\frac{5x-2}{x^4-x}$.
14. $\frac{4x+1}{x(x+1)^3}$.
15. $\frac{4x^2+10x}{4x^3-x-3}$.
16. $\frac{6x}{x^2-9}$.
17. $\frac{9x^2-1}{(3x-2)^3}$.
18. $\frac{6x}{x^2-8-2x}$.
19. $\frac{x^5-2}{x^4-1}$.
20. $\frac{3-x^3}{x^3(x+1)^2}$.
21. $\frac{6x^2+5}{6x^2+x-5}$.
22. $\frac{x^3+4}{x^3-4x}$.
23. $\frac{3x-1}{x^3(x^3+1)}$.
24. $\frac{2x^3}{x^4-3x^3+1}$.
25. $\frac{x^3+8}{x^4-2x^3}$.
26. $\frac{x+48}{15(x-12)}$.
27. $\frac{4x^3-2x}{4x^3-4x+1}$.
28. $\frac{4x^2+9}{8x^3-27}$.
29. $\frac{3x-4}{14(x-20)}$.
30. $\frac{a^2-ax+2a}{a^2-2ax+x^2}$.
31. $\frac{2x^2-7a^2}{x^2-3ax-4a^2}$.
32. $\frac{27x^3-5x+3}{(1+2x)(3x-2)^2}$.
33. $\frac{12x^3-19}{(3x+2)(x^2-5)}$.
34. $\frac{7x^3-10x+10}{(2x-1)(x^2-2x+3)}$.
35. $\frac{3x^2-x-4}{x(x-1)(x-2)^2}$.
36. $\frac{36x^3+23x+14}{(3x-1)(6x^2+7x-3)}$.
37. $\frac{8x^3-20x^2+24}{(x^2-3x)(x^2-4)}$.
38. $\frac{52(x^2+1)}{(3-2x)(6+5x-6x^2)}$.

BINOMIAL THEOREM.

1. $(2a^{\frac{2}{3}} - x^{\frac{1}{2}})^4$.
2. $(\sqrt{x^3} - 3\sqrt[3]{y})^6$.
3. $(2a^2 + \frac{1}{2}\sqrt{x})^5$.
4. $(a^3 + 2x^{-2})^5$.
5. $(3x^{-\frac{2}{3}} - \sqrt{y})^4$.
6. $(\frac{3}{4}a^3 - a\sqrt{b})^4$.
7. $\left(\frac{3a}{b} - \frac{2b}{a}\right)^4$.
8. $\left(\frac{\sqrt{a}}{\sqrt{b}} - \frac{2\sqrt{b}}{\sqrt[3]{a^2}}\right)^4$.
9. $\left(3x^{\frac{3}{2}} + \frac{1}{3x^{\frac{3}{2}}}\right)^4$.
10. $\left(\frac{a^3}{b} - \frac{\sqrt{b}}{3a}\right)^5$.
11. $\left(\frac{\sqrt{x}}{\sqrt[3]{y^2}} + \frac{1}{3}\sqrt{\frac{y}{x}}\right)^4$.
12. $\left(\frac{a^{-\frac{1}{4}}}{3} - \sqrt[4]{x^3}\right)^4$.
13. $(a^{-\frac{1}{2}} + \frac{1}{2}b)^5$.
14. $(x^{2n} - y^{2n})^5$.
15. $(a^{2n} + 2a^{-2n})^5$.
16. $(a - b + c)^3$.
17. $(x - y - z - u)^3$.
18. $(2a - x - y)^3$.
19. $(a - 2b - x + y)^3$.
20. $(x + 3y + 2z)^4$.
21. $(a + 3b - 2x - y)^3$.

22. Show that the exponent of a in the mth term of the expansion is $n - m + 1$.

23. Write an expression to represent the rth term of the expansion of $(a + x)^n$.

Find the following:

24. 6th term of $(a + x)^{15}$.
25. 5th term of $(x^2 + y^2)^{10}$.
26. 9th term of $(x - y)^{14}$.
27. 7th term of $(a^3 - a^2)^{11}$.
28. 8th term of $(x + 1)^{12}$.
29. 4th term of $(x^n - y^2)^{14}$.
30. 6th term of $(1 - x)^{13}$.
31. 7th term of $(a^{-2} + 2)^{16}$.
32. 5th term of $(a + 3)^{11}$.
33. 4th term of $(x^{-\frac{2}{3}} - \frac{2}{3})^{12}$.
34. 7th term of $(2 - x)^{10}$.
35. 5th term of $(a^{-\frac{1}{2}} + a^{\frac{1}{2}})^9$.
36. 6th term of $\left(\frac{a}{2} - x\right)^{12}$.
37. 4th term of $\left(x^{\frac{1}{2}} + \frac{y^3}{3}\right)^8$.

Find the middle term of the following:

38. $\left(\frac{\sqrt[4]{x^3}}{2} - y^{-\frac{2}{5}}\right)^{10}$.
39. $\left(2a^{\frac{3}{2}} + \frac{b^2}{2}\right)^{12}$.
40. $\left(\frac{2a}{b^2} - \frac{b^2}{2a}\right)^{14}$.

Find the fourth term from the last.

41. $(2\sqrt{a}-\sqrt{-b})^8$. 42. $(\sqrt{-x}+\sqrt{-y})^9$.

Expand each of the following to five terms:

1. $(a+x)^{\frac{1}{2}}$. 2. $(a-b)^{-2}$. 3. $(1+2x)^{\frac{3}{2}}$.
4. $(x-y)^{\frac{2}{3}}$. 5. $(a+x)^{-\frac{1}{2}}$. 6. $(2a-1)^{\frac{1}{2}}$.
7. $(x+2)^{\frac{4}{3}}$. 8. $(2-x)^{-\frac{3}{2}}$. 9. $(2+3a)^{\frac{1}{4}}$.
10. $(x-1)^{\frac{2}{3}}$. 11. $(1+x)^{-3}$. 12. $(2x-3)^{\frac{2}{5}}$.
13. $\sqrt{a-x}$. 14. $\sqrt{2x+1}$. 15. $\sqrt[4]{2a-3x}$.
16. $\dfrac{1}{(a-x)^{\frac{1}{2}}}$. 17. $\dfrac{1}{(x+y)^{-\frac{1}{2}}}$. 18. $\dfrac{1}{(3a-2b)^3}$.
19. $\dfrac{1}{\sqrt[3]{(a+b)^2}}$. 20. $\dfrac{1}{\sqrt{(x^{\frac{1}{2}}-y)^{-\frac{2}{3}}}}$. 21. $\dfrac{1}{\sqrt[5]{(x^{-\frac{1}{2}}+yz)^{-3}}}$.
22. $\sqrt{[(a+x)^{-3}]}$. 23. $\sqrt[3]{[(x^{-\frac{2}{3}}-2y)^{-2}]}$. 24. $\sqrt{[(a^{\frac{1}{2}}+2x)^3]}$.

Find the following:

25. 5th term of $(1-x)^{\frac{2}{3}}$. 26. 7th term of $(2a-3x)^{-\frac{1}{4}}$.
27. 6th term of $(a+2x)^{-3}$. 28. 4th term of $(\frac{2}{3}a-2b^{-1})^{-\frac{5}{2}}$.
29. 3d term of $(1-2x)^{-\frac{1}{2}}$. 30. 8th term of $(\sqrt{(x^2+2y)^{-7}}$.

Find the approximate value of each of the following:

31. $\sqrt{10}$. 32. $\sqrt[3]{28}$. 33. $\sqrt[4]{18}$. 34. $\sqrt[3]{127}$.
35. $\sqrt[3]{10}$. 36. $\sqrt{27}$. 37. $\sqrt[5]{35}$. 38. $\sqrt[4]{259}$.

LOGARITHMS.

Find the logarithms of the following numbers:

1. 4218. 2. 416.3. 3. .2463. 4. 425.42.
5. 3720. 6. 40.25. 7. .0378. 8. 31.604.
9. 5004. 10. 3.006. 11. .0097. 12. 7.0005.

N	0	1	2	3	4	5	6	7	8	9
10	00000	00432	00860	01284	01703	02119	02531	02928	03342	03743
11	04139	04532	04922	05308	05690	06070	06446	06819	07188	07555
12	07918	08279	08636	08991	09342	09691	10037	10380	10721	11059
13	11394	11727	12057	12385	12710	13033	13354	13672	13988	14301
14	14613	14922	15229	15534	15836	16137	16435	16732	17026	17319
15	17609	17898	18184	18469	18752	19033	19312	19590	19866	20140
16	20412	20683	20952	21219	21484	21748	22011	22272	22531	22789
17	23045	23300	23553	23805	24055	24304	24551	24797	25042	25285
18	25527	25768	26007	26245	26482	26717	26951	27184	27416	27646
19	27875	28103	28330	28556	28780	29003	29226	29447	29667	29885
20	30103	30320	30535	30750	30963	31175	31387	31597	31806	32015
21	32222	32428	32634	32838	33041	33244	33445	33646	33846	34044
22	34242	34439	34635	34830	35025	35218	35411	35603	35793	35984
23	36173	36361	36549	36736	36922	37107	37291	37475	37658	37840
24	38021	38202	38382	38561	38739	38917	39094	39270	39445	39620
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330
26	41497	41664	41830	41996	42160	42325	42488	42651	42813	42975
27	43136	43297	43457	43616	43775	43933	44091	44248	44404	44560
28	44716	44871	45025	45179	45332	45484	45637	45788	45939	46090
29	46240	46389	46538	46687	46835	46982	47129	47276	47422	47567
30	47712	47857	48001	48144	48287	48430	48572	48714	48855	48996
31	49136	49276	49415	49554	49693	49831	49969	50106	50243	50379
32	50515	50651	50786	50920	51055	51188	51322	51455	51587	51720
33	51851	51983	52114	52244	52375	52504	52634	52763	52892	53020
34	53148	53275	53403	53529	53656	53782	53908	54033	54158	54283
35	54407	54531	54654	54777	54900	55023	55145	55267	55388	55509
36	55630	55751	55871	55991	56110	56229	56348	56467	56585	56703
37	56820	56937	57054	57171	57287	57403	57519	57634	57749	57864
38	57978	58092	58206	58320	58433	58546	58659	58771	58883	58995
39	59106	59218	59329	59439	59550	59660	59770	59879	59988	60097
40	60206	60314	60423	60531	60638	60746	60853	60959	61066	61172
41	61278	61384	61490	61595	61700	61805	61909	62014	62118	62221
42	62325	62428	62531	62634	62737	62839	62941	63043	63144	63246
43	63347	63448	63548	63649	63749	63849	63949	64048	64147	64246
44	64345	64444	64542	64640	64738	64836	64933	65031	65128	65225
45	65321	65418	65514	65610	65706	65801	65896	65992	66087	66181
46	66276	66370	66464	66558	66652	66745	66839	66932	67025	67117
47	67210	67302	67394	67486	67578	67669	67761	67852	67943	68034
48	68124	68215	68305	68395	68485	68574	68664	68753	68842	68931
49	69020	69108	69197	69285	69373	69461	69548	69636	69723	69810
50	69897	69984	70070	70157	70243	70329	70415	70501	70586	70672
51	70757	70842	70927	71012	71096	71181	71265	71349	71433	71517
52	71600	71684	71767	71850	71933	72016	72099	72181	72263	72346
53	72428	72509	72591	72673	72754	72835	72916	72997	73078	73159
54	73239	73320	73400	73480	73560	73640	73719	73799	73878	73957

N	0	1	2	3	4	5	6	7	8	9
55	74036	74115	74194	74273	74351	74429	74507	74586	74663	74741
56	74819	74896	74974	75051	75128	75205	75282	75358	75435	75511
57	75587	75664	75740	75815	75891	75967	76042	76118	76193	76268
58	76343	76418	76492	76567	76641	76716	76790	76864	76938	77012
59	77085	77159	77232	77305	77379	77452	77525	77597	77670	77743
60	77815	77887	77960	78032	78104	78176	78247	78319	78390	78462
61	78533	78604	78675	78746	78817	78888	78958	79029	79099	79169
62	79239	79309	79379	79449	79518	79588	79657	79727	79796	79865
63	79934	80003	80072	80140	80209	80277	80346	80414	80482	80550
64	80618	80686	80754	80821	80889	80956	81023	81090	81158	81224
65	81291	81358	81425	81491	81558	81624	81690	81757	81823	81889
66	81954	82020	82086	82151	82217	82282	82347	82413	82478	82543
67	82607	82672	82737	82802	82866	82930	82995	83059	83123	83187
68	83251	83315	83378	83442	83506	83569	83632	83696	83759	83822
69	83885	83948	84011	84073	84136	84198	84261	84323	84386	84448
70	84510	84572	84634	84696	84757	84819	84880	84942	85003	85065
71	85126	85187	85248	85309	85370	85431	85491	85552	85612	85673
72	85733	85794	85854	85914	85974	86034	86094	86153	86213	86273
73	86332	86392	86451	86510	86570	86629	86688	86747	86806	86864
74	86923	86982	87040	87099	87157	87216	87274	87332	87390	87448
75	87506	87564	87622	87679	87737	87795	87852	87910	87967	88024
76	88081	88138	88195	88252	88309	88366	88423	88480	88536	88593
77	88649	88705	88762	88818	88874	88930	88986	89042	89098	89154
78	89209	89265	89321	89376	89432	89487	89542	89597	89653	89708
79	89763	89818	89873	89927	89982	90037	90091	90146	90200	90255
80	90309	90363	90417	90472	90526	90580	90634	90687	90741	90795
81	90849	90902	90956	91009	91062	91116	91169	91222	91275	91328
82	91381	91434	91487	91540	91593	91645	91698	91751	91803	91855
83	91908	91960	92012	92065	92117	92169	92221	92273	92324	92376
84	92428	92480	92531	92583	92634	92686	92737	92788	92840	92891
85	92942	92993	93044	93095	93146	93197	93247	93298	93349	93399
86	93450	93500	93551	93601	93651	93702	93752	93802	93852	93902
87	93952	94002	94052	94101	94151	94201	94250	94300	94349	94399
88	94448	94498	94547	94596	94645	94694	94743	94792	94841	94890
89	94939	94988	95036	95085	95134	95182	95231	95279	95328	95376
90	95424	95472	95521	95569	95617	95665	95713	95761	95809	95856
91	95904	95952	95999	96047	96095	96142	96190	96237	96284	96332
92	96379	96426	96473	96520	96567	96614	96661	96708	96755	96802
93	96848	96895	96942	96988	97035	97081	97128	97174	97220	97267
94	97313	97359	97405	97451	97497	97543	97589	97635	97681	97727
95	97772	97818	97864	97909	97955	98000	98046	98091	98137	98182
96	98227	98272	98318	98363	98408	98453	98498	98543	98588	98632
97	98677	98722	98767	98811	98856	98900	98945	98989	99034	99078
98	99123	99167	99211	99255	99300	99344	99388	99432	99476	99520
99	99564	99607	99651	99695	99739	99782	99826	99870	99913	99957

13. 6200.	14. 800.3.	15. .0008.	16. 3204.6.
17. 7403.	18. 470.0.	19. .7004.	20. 570.02.
21. 9000.	22. 350.2.	23. .0516.	24. 85.407.
25. 8007.	26. 500.0.	27. .0048.	28. 6000.4.
29. 200065.	30. 724358.1.	31. 8000004.32.	
32. 400174.	33. 6454.072.	34. 3247.00006.	

Given $\log 2 = .30103$; $\log 3 = .47712$; $\log 5 = .69897$; $\log 7 = .84510$; $\log 13 = 1.11394$; find the logarithms of the following without referring to the table:

1. 98.	2. $\frac{8}{3}$.	3. $12^{\frac{1}{7}}$.	4. $\frac{10}{33}$.	5. 2^4.
6. 42.	7. $\frac{3}{4}$.	8. $28^{\frac{1}{5}}$.	9. $\frac{14}{65}$.	10. $4^{\frac{3}{5}}$.
11. 70.	12. $\frac{2}{7}$.	13. $24^{\frac{3}{7}}$.	14. $\frac{15}{91}$.	15. 6^7.
16. 48.	17. $\frac{3}{5}$.	18. $40^{\frac{2}{3}}$.	19. $\frac{33}{35}$.	20. $5^{\frac{4}{3}}$.
21. 75.	22. $\frac{3}{5}$.	23. $20^{\frac{1}{3}}$.	24. $3\frac{1}{2}$.	25. 4^8.
26. 80.	27. $\frac{8}{7}$.	28. $25^{\frac{1}{4}}$.	29. $4\frac{2}{7}$.	30. $6^{\frac{2}{3}}$.
31. $\sqrt{18}$.	32. $\frac{5}{3}$.	33. $52^{\frac{1}{2}}$.	34. $7\frac{1}{2}$.	35. 5^5.
36. $\sqrt[3]{63}$.	37. $\frac{6}{7}$.	38. $56^{\frac{2}{3}}$.	39. $1\frac{2}{3}$.	40. $8^{\frac{3}{4}}$.
41. $\sqrt[5]{27}$.	42. $\frac{5}{8}$.	43. $\sqrt[3]{14^2}$.	44. $4\frac{2}{3}$.	45. $\sqrt{7}$.
46. $\sqrt[7]{45}$.	47. $\frac{8}{3}$.	48. $\sqrt{21^3}$.	49. $5\frac{1}{3}$.	50. $\sqrt[4]{8}$.
51. $\sqrt[5]{15^4}$.	52. $\sqrt[3]{9}$.	53. $\sqrt[4]{26^3}$.	54. $5\frac{4}{7}$.	55. $\sqrt[5]{6}$.
56. $3\frac{3^{\frac{1}{2}}}{7^{\frac{1}{2}}}$	57. $\frac{4^7}{5^5}$	58. $\frac{5\sqrt{3^3}}{3\sqrt[3]{7^2}}$	59. $2\sqrt{\frac{8}{15}}$	60. $\frac{6\sqrt{2}}{5\sqrt{7}}$
61. 1625.	62. 17199.	63. 27783.	64. 81900.	
65. 4.2.	66. .26.	67. .105.	68. .0015.	69. .00009.
70. 3.6.	71. .64.	72. 32.4.	73. $(3.9)^5$.	74. $(12.6)^{\frac{3}{2}}$.

Find the antilogs of the following logarithms:

1. 2.18469.	2. 4.41372.	3. $\bar{1}$.29226.	4. 0.36507.
5. $\bar{1}$.52894.	6. 0.37291.	7. $\bar{5}$.71245.	8. 2.42975.
9. 0.49554.	10. 5.91347.	11. 3.57403.	12. $\bar{4}$.84763.
13. 3.25672.	14. $\bar{2}$.80277.	15. 0.16347.	16. $\bar{2}$.94349.

Find the following products by the use of logarithms:

1. 758 × .0054 × 9870.	2. 46.7 × .008 × 940.4 × .0078.
3. 480 × .0807 × 6587.	4. 8.94 × .072 × 80.70 × .0945.
5. 900 × .7964 × 9670.	6. 75.2 × .854 × 6.143 × .8762.
7. 648 × .0009 × 8900.	8. 3.87 × .009 × 978.4 × .0008.
9. 590 × .0870 × 0683.	10. 27.8 × .097 × 46.80 × .0007.
11. 895 × .9780 × 6548.	12. 5.39 × .840 × 7.940 × .0058.

Find the following quotients by the use of logarithms:

1. 793 ÷ .008.	2. 4.78 ÷ 78.6.	3. 7640 ÷ 23.8.
4. 658 ÷ .075.	5. 58.4 ÷ 6.47.	6. 8594 ÷ 4.26.
7. 462 ÷ .954.	8. 3.96 ÷ 97.2.	9. 6070 ÷ .075.
10. 879 ÷ .007.	11. .008 ÷ 7.40.	12. 4932 ÷ .005.

Find the following powers by the use of logarithms:

1. 75^5.	2. 496^3.	3. 7.42^4.	4. 46.21^3.
5. 95^4.	6. 783^2.	7. 97.5^5.	8. 657.4^2.
9. 84^6.	10. 697^4.	11. 4.87^6.	12. 7.250^5.

Find the following roots by the use of logarithms:

1. $95^{\frac{1}{2}}$.	2. $492^{\frac{1}{3}}$.	3. $245^{\frac{1}{4}}$.	4. $739^{\frac{1}{4}}$.
5. $68^{\frac{1}{3}}$.	6. $378^{\frac{1}{3}}$.	7. $932^{\frac{1}{3}}$.	8. $463^{\frac{1}{7}}$.
9. $\sqrt[5]{82}$.	10. $\sqrt[4]{241^{\frac{1}{2}}}$.	11. $\sqrt{742^{\frac{1}{3}}}$.	12. $\sqrt[3]{573^{\frac{1}{4}}}$.

PERMUTATIONS AND COMBINATIONS.

1. How many numbers can be expressed by the 9 digits taken 5 at a time?

2. In how many ways can the letters a, c, e, i, m, n, be arranged taken 3 at a time? Taken 5 at a time?

3. In how many ways could a party of 8 persons be arranged around a table?

4. How many different selections of 5 pieces each can be made from 11 different pieces of money?

5. From a company of 15 persons, how many different parties of 7 each can be formed?

6. From 5 consonants and 4 vowels, how many words containing 4 consonants and 2 vowels can be formed?

nP of $5_4 = 5 \cdot 4 \cdot 3 \cdot 2 = 120$ $\quad$ nP of $4_2 = 4 \cdot 3 = 12$

$$120 \times 12 = 1440$$

7. How many different pickets of 6 men and an officer can be formed from a squad of 18 men and 3 officers?

8. How many permutations can be made out of the letters of the word *promotion* taken all together?

9. In a school of 26 boys and 18 girls, how many classes of 8 boys and 6 girls can be formed?

10. In how many different orders of succession can 12 pupils leave the class when dismissed?

11. How many different groups of 8 persons can be formed from 12 men and half as many women?

12. In how many ways could the 7 prismatic colors be arranged taken all together?

13. How many divisors has a number which is the product of 6 odd numbers greater than 1?

These exercises have been taken mainly from the entrance examinations of the various colleges.

1. The first digit of a number is 3 times the second; and if the number, increased by 3, be divided by the difference of its digits, the quotient is 16. Find the number.

2. Divide

$$\sqrt[3]{x^4} - 4xy + 4y\sqrt[3]{x^2} + 4y^2 \text{ by } \sqrt[3]{x^2} + 2\sqrt{xy} + 2y.$$

3. Factor $x^4 + x^2 + 1$. 4. $10x^2 + 9x - 40$.

5. Solve $2\sqrt{x} + 3\sqrt{y} = 3\sqrt{x} + 2\sqrt{y} = 10$.

6. $\dfrac{2x-1}{x+2} = \dfrac{b^2}{a^2} \cdot \dfrac{x+2}{2x+1}$. 7. $\dfrac{a+x}{b+x} + \dfrac{a-x}{b-x} = n$.

8. Simplify $2^{\frac{1}{2}} \cdot 3^{-\frac{1}{6}} \cdot 4^{\frac{1}{3}} \cdot 9^{\frac{1}{3}}$.

9. If a varies as b, and $a = 11$ when $b = 7$, what is the value of a when $b = 10$?

10. $\begin{cases} x^2 - xy = 2 \\ 2x^2 + y^2 = 9 \end{cases}$ 11. $\begin{cases} 4(x+y) = 3xy \\ x + x^2 + y + y^2 = 26 \end{cases}$

12. Factor $a^2 + x^2 - (y^2 + z^2) - 2(yz - ax)$.

13. $\dfrac{x^3 - 4x}{x-2} + \dfrac{x^2 - 1}{x+1} = 39$. 14. $\dfrac{2^{n+1}}{2^{n(n-1)}} \div \dfrac{4^{n+1}}{(2^{n-1})^{n+1}}$.

Expand to four terms in ascending powers of x:

15. $\dfrac{2+x}{3-x^2}$. 16. $\dfrac{2}{3x^2 - 4x^3}$. 17. $\dfrac{1-3x}{1+2x^2}$.

18. Solve $\sqrt{(a+x)(x+b)} + \sqrt{(a-x)(x-b)} = 2\sqrt{ax}$.

19. $4x^3 - 4x^2 + 9 = 9x$. 20. $\sqrt[3]{ab^{-1}c^{-2}} \times (a^{-1}b^{-2}c^{-4})^{-\frac{1}{6}}$

21. If $a : b = c : d$, prove that

$$a^2 + b^2 : a^2 - b^2 = c^2 + d^2 : c^2 - d^2.$$

22. $\left(\dfrac{2\sqrt{a}}{\sqrt[3]{b^2}} - \dfrac{\sqrt{b}}{\sqrt{a}}\right)^4 \cdot \left(\dfrac{2x}{\sqrt{y}} - \dfrac{y}{2\sqrt{x}}\right)^5 \cdot \left(\sqrt{\dfrac{x}{3}} - \sqrt{\dfrac{3}{2}y}\right)^6$.

23. Divide \$3150 among A, B, C, and D, so that B and C shall have half as much as A and D, that B's money shall be $\frac{7}{8}$ of C's, and D's money $\frac{2}{5}$ of A's.

24. Simplify $\dfrac{1}{1-\sqrt{1-x^2}}-\dfrac{1}{1+\sqrt{1-x^2}}+\dfrac{2}{\sqrt{1-x^2}}$.

25. The sums of three numbers, taken two by two, are 20, 29, and 27. What are the numbers?

26. Simplify $x-(2x-y-[3x-2y-(4x-3y)])$.

27. Show that $(x+1)(x+2)(x+3)(x+4)+1$ is a perfect square.

28. Find the highest common factor of $x^4+9x^3+29x^2-39x+18$, $4x^3-27x^2+58x-39$, and $x^3-8x^2+19x-12$.

29. If $a:b::c:d$, prove that

$$ab+cd:b^2+d^2::a:b.$$

30. A takes 3 hours longer than B to walk 30 miles; but if he doubles his pace, he takes two hours less time than B. Find their rates of walking.

31. Find the sum of 10 terms of $-\frac{1}{2}, -\frac{1}{6}, \frac{1}{6}, \cdots$.

32. $\dfrac{[(a^m)^{\frac{1}{r}}(a^q)^{\frac{1}{n}}]^{nr}}{[\sqrt[q]{b^n}(\sqrt[m]{b})^r]^{mq}}\div\left[\left(\dfrac{a}{b}\right)^q\right]^r$.

33. $\dfrac{\sqrt[p]{(xyz)^m}}{(x^{\frac{3}{p}}y^{\frac{4}{p}}z^{\frac{5}{p}})^m}\cdot\dfrac{}{(x^{\frac{3}{p}}y^{\frac{4}{p}}z^{\frac{5}{p}})^{-m}}$.

34. By what number must 7, 10, 19, 31, be diminished so that the remainders may form a proportion?

35. $\begin{cases} x^2+y^2:x^2-y^2::29:21 \\ x+y=7 \end{cases}$

36. $\begin{cases} x+\sqrt{xy}=a \\ y+\sqrt{xy}=n \end{cases}$

37. $\dfrac{x-2}{x^2-x}\cdot\dfrac{x^3+x^2y}{xy-y^2}\div\dfrac{xy^2+y^3}{x^3-x^2y}\cdot\dfrac{x^{\frac{1}{2}}y^{\frac{1}{2}}}{z^{-\frac{1}{2}}}\div\dfrac{y^{-1}z^2}{x^{\frac{1}{2}}}\cdot\dfrac{z^3}{\sqrt{xy^{-\frac{1}{2}}}}$.

38. The sum of two numbers is 200, and their difference is equal to two-thirds of the less. What are the numbers?

39. How many terms must there be in the series $1+3+5+7+\cdots$ to amount to 1,234,321?

40. $\begin{cases} x^2+xy=8x+2 \\ xy-y^2=8y-1 \end{cases}$ **41.** $\begin{cases} x^3+y^3=b \\ x+y=a \end{cases}$

42. Factor $a^2+abx+ac+b^2y+aby+bc$.

43. $\dfrac{\sqrt{x}-8}{\sqrt{x}-6}=\dfrac{\sqrt{x}-4}{\sqrt{x}+2}$. **44.** $\dfrac{x}{\sqrt{(a^2+x^2)}}=\dfrac{x-c}{\sqrt{b^2+(c-x)^2}}$.

45. Insert four geometric means between 160 and 5.

46. $\dfrac{\dfrac{1}{a-x}-\dfrac{1}{a-y}+\dfrac{x}{(a-x)^2}-\dfrac{y}{(a-y)^2}}{\dfrac{1}{(a-y)(a-x)^2}-\dfrac{1}{(a-x)(a-y)^2}}$. **47.** $\begin{cases} \dfrac{a}{x}+\dfrac{b}{y}=c \\ \dfrac{b}{x}-\dfrac{a}{y}=n \end{cases}$

48. Simplify $\{(a^{\frac{4}{15}}b^{\frac{3}{5}})^{-\frac{1}{2}}\times(a^{-\frac{2}{10}}b^{-\frac{2}{5}})^{\frac{1}{2}}\}^{-120}$.

49. If x varies inversely as y, and $x=4$ when $y=2$, what is the value of y when $x=\frac{4}{3}$?

50. Factor $a^4+a^2b^2+b^4$. **51.** $5x^4-15x^3-90x^2$.

52. $(2x^3-y^{-\frac{1}{2}})^4$. **53.** $(2a^{\frac{1}{2}}-\frac{1}{2}x)^5$. **54.** $(1-\frac{2}{3}x^{-1})^6$.

55. Solve $(a^2-b^2)(x^2+1)=2(a^2+b^2)x$.

56. If $a:b::c:d$, prove that

$$2a+3b:2a-3b::2c+3d:2c-3d.$$

57. $\begin{cases} \dfrac{x-y}{2}+\dfrac{3y}{8}=\dfrac{7}{8} \\ \dfrac{3x+y}{5}-\dfrac{4x}{15}=\dfrac{5x+y}{10} \end{cases}$ **58.** $3=\dfrac{3}{4-\dfrac{3}{4-\dfrac{3}{4-x}}}$.

59. Find the middle term of $(\frac{1}{2}x-2y)^6$.

60. A and B together own 175 shares of railway stock. They agree to divide. A takes 85 shares; B takes 90 shares and pays A \$ 500. Find the value of a share.

61. Two numbers are as 3 : 4; and if 7 be subtracted from each, the remainders are as 2 : 3. Find the numbers.

Expand to four terms:

62. $(1-2x^2)^{\frac{3}{2}}$. 63. $(a^2-x)^{-\frac{1}{2}}$. 64. $(a^2n-x^{\frac{1}{2}})^{-\frac{2}{3}}$.

65. Form the quadratic equation whose roots are 0 and $\frac{1}{a}$.

66. $\frac{1}{10}\sqrt{72}+\sqrt{12}+8\frac{1}{\sqrt{32}}$. 67. $\frac{3\sqrt{2}}{3\sqrt{2}-\sqrt{6}}$.

68. Simplify $\frac{1-a^2}{(1-ax)^2-(a+x)^2}\div\frac{1}{2}\left(\frac{1}{1-x}+\frac{1}{1+x}\right)$.

Expand to five terms in ascending powers of x:

69. $\frac{1-4x+2x^2}{2-3x^2-x^3}$. 70. $\frac{1-x}{1-3x-2x^2}$.

71. $(\sqrt{a}-\sqrt{b})(\sqrt[4]{a}+\sqrt[4]{b}\sqrt{-1})(\sqrt[4]{a}-\sqrt[4]{b}\sqrt{-1})$.

72. How many different numbers of three figures each can be expressed by the digits 1, 5, 6, 8, 9?

Factor the following:

73. $x^2-3\sqrt{3}x-12$. 74. $a^2x^3-b^2xy^2-a^2cx^2+b^2cy^2$.

75. $\begin{cases} ax+\frac{n}{y}=1 \\ nx+\frac{a}{y}=1 \end{cases}$ 76. $\begin{cases} x+\frac{1}{y}=3 \\ y+\frac{1}{x}=\frac{12}{5} \end{cases}$ 77. $\begin{cases} \frac{1}{x}+\frac{1}{y}=11 \\ \frac{1}{xy}=18 \end{cases}$

78. Given l, r, s, derive a rule for finding the number of terms of a geometric series.

79. $3x^2-x+2=0$. 80. $ax^2+bx+c=0$.

81. Prove that if $\frac{a}{b}=\frac{c}{d}$, $\frac{a^2+b^2}{c^2+d^2}=\frac{(a+b)^2}{(c+d)^2}$; also prove that $(a^2+c^2)(b^2+d^2)=(ab+cd)^2$.

82. If $3x-5$ varies as $4y-7$, and $x=20$ when $y=-1$, what is the value of y when $x=23\frac{1}{3}$?

83. Show that a quadratic equation can not have more than two roots. Find the values of a for which the equation $a^2x^2+(a^2+a)x+1=0$ will have its roots equal.

84. Find the lowest common multiple of $4x^4-11x^3-3x^2$, $2x^3-5x^2-3x$, and $6x^5-x^4-2x^3$.

85. Form the equation whose roots are $3\pm2\sqrt{-1}$.

Rationalize the denominators of:

86. $\dfrac{\frac{1}{2}\sqrt{2}+\sqrt{\frac{3}{4}}}{\sqrt{\frac{1}{2}}-\frac{1}{2}\sqrt{3}}$.

87. $\dfrac{\sqrt{-2}-3\sqrt{-6}}{\sqrt{-2}+2\sqrt{-5}}$.

88. Define a^m and prove that $(a^m)^n=a^{mn}$.

89. $4x^{-\frac{1}{2}}-5x^{-\frac{1}{4}}+1=0$.

90. $x^4-25x^2=-144$.

91. Divide 81 into two such parts that one shall be a multiple of 8 and the other a different multiple of 5.

92. Factor $x^2-6\sqrt{2}x+18$.

93. $x^5-16x^3+x^2-16$.

94. Find how many terms of the progression

$$16+24+32+40+\cdots \text{ equal } 1840.$$

95. $\begin{cases}\dfrac{x}{y}-\dfrac{y}{x}=\dfrac{8}{3}\\ x-y=4\end{cases}$

96. $\begin{cases}\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{6}\\ x-y=5\end{cases}$

97. How many permutations of seven letters each can be formed from the letters of the word *figures*?

Expand to five terms:

98. $\dfrac{1}{(a^{-1}-3y^{-2})^4}$.

99. $\dfrac{1}{\sqrt[3]{(3a^{-1}+3x^{\frac{1}{2}})}}$.

100. A general can form all his men into a solid square, or a hollow square four deep. The number of men in the front of the solid square is sixteen less than the number in the front of the hollow square. How many men has he?

101. State and demonstrate the relation which exists between the roots of an affected quadratic and the coefficient of the first power of the unknown quantity.

102. If $\sqrt{p+q} : \sqrt{p-q} :: m : n$, find $p : q$.

103. $9\frac{1}{8}x^2 - 90\frac{1}{8}x + 195 = 0$. 104. $2\sqrt{3} + 15\sqrt{27} - \frac{1}{3}\sqrt{\frac{1}{3}}$.

105. State and illustrate how to form a quadratic equation which shall have given roots.

106. Resolve into partial fractions $\frac{8 - 3x - x^2}{x(x+2)^2}$.

107. If $2x + y = m$, $x + 2y = n$, and $(x+y)^2 = 2(x-y)^2$, prove that $17(m^2 + n^2) = 38mn$.

108. Find the middle term of $(3a - \frac{1}{3}x)^8$.

Solve in positive integers:

109. $8x + 65y = 81$. 110. $31x + 8y = 225$.

111. The third term of an A. P. is four times the first term, and the sixth term is 17. Find the series.

112. Simplify $(x^m)^{\frac{1}{m}} - (x^{1+\frac{1}{m}})^{\frac{m}{m+1}} + m\sqrt{x^{2m}}$.

113. Find the sum of $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \cdots$ to 7 terms.

Extract the square root of:

114. $57 - 12\sqrt{15}$. 115. $41 - 24\sqrt{2}$. 116. $16 + 5\sqrt{7}$.

117. Divide 20 into three parts, such that the products of the three pairs may be in the ratios $6 : 10 : 15$.

118. Express as one fraction $x^{-\frac{1}{2}}y + x^{\frac{3}{2}}y^{-1} - x^{\frac{1}{2}}y^{-\frac{3}{2}}$.

119. $\frac{1}{\sqrt{16+2\sqrt{63}}} + \frac{1}{\sqrt{16-2\sqrt{63}}}$. 120. $\left(x^{\sqrt{2}} - \frac{x^{-\sqrt{2}}}{2}\right)^2$.

121. Solve $x + 2 = \sqrt{[4 + x\sqrt{(8-x)}]}$.

122. Derive the formula for the nth term and the sum of n terms of an arithmetical progression.

123. Prove that the sum of the roots of an affected quadratic is equal to the coefficient of x with its sign changed, and the product of the roots is equal to the independent term.

124. $\begin{cases} \frac{x}{a}+\frac{y}{b}=1 \\ \frac{a}{x}+\frac{b}{y}=1 \end{cases}$ **125.** $\begin{cases} x^2+\frac{x^4}{y^2}+y^2=84 \\ x+\frac{x^2}{y}+y=14 \end{cases}$

126. Solve $1+\sqrt[3]{8x^3-3x^3}=2x$.

Find the fourth term of:

127. $(2x-5y)^{12}$. $(\frac{1}{2}a-\frac{2}{3}x)^{11}$. $(a^{-\frac{1}{2}}-2a^2)^{12}$.

128. If $x+1=\frac{y}{y-1}$, show by proportion that $\frac{x+2}{x}=2y-1$, and demonstrate a principle of proportion used.

129. Factor $x^2+5\sqrt{3}x+18$. **130.** $8(x+y)^3-(2x-y)^3$.

131. Simplify $\sqrt{a}(\sqrt{a}-\sqrt{x})^{-1}+(\sqrt{a}-\sqrt{x})(\sqrt{x})^{-1}$.

132. Separate into partial fractions $\frac{2x^2+x+1}{x^3+3x^2-x-3}$.

133. From a squad of 10 soldiers, how many different pickets of six men can be taken?

134. $\frac{\sqrt[11]{\{(\sqrt[3]{a^4b^2c})^5(\sqrt{a^2b^2c})^4\}^3}}{a^{\frac{5}{11}}}$. **135.** $\sqrt{\frac{x}{a^3}}\cdot\sqrt[3]{\frac{a}{y}}\cdot\sqrt[6]{\frac{y^4}{a^{-1}x}}$.

136. If z varies as $px+y$, and $z=3$ when $x=1$ and $y=2$, and $z=5$ when $x=2$ and $y=3$, find the value of p.

137. Prove $\frac{15}{\sqrt{10}+\sqrt{20}-\sqrt{5}-\sqrt{40}}=(1+\sqrt{2})\sqrt{5}$.

138. Show that $2a^2(a+3b)$, $(a+b)^3$, $2b^2(b+3a)$ are in arithmetical progression.

139. A man sold a acres more than the mth part of his estate, and there remained b acres less than the nth part. Find the number of acres in the original estate.

140. Prove the value of the rth term in $(a+x)^n$.

Write from inspection the roots of:

141. $z^2-(a+b)z+c=0$. 142. $x^4+4x^2+1=0$.

143. Find the sum of a given geometric series, and explain how the sum of an infinite series may have a finite value. Prove that $.3333\cdots$ to infinity $=\frac{1}{3}$.

144. Why is $(a^p)^{\frac{1}{q}}=(a^{\frac{1}{q}})^p$? $\sqrt[3]{(16a+24)}+\sqrt[3]{(54a+81)}$.

145. Find the highest common factor of a^3-3a^2-6a+8, a^3+4a^2+a-b, and $a^3+a^2-14a-24$.

Find the fifth term of

146. $\left(\frac{3a^2}{\sqrt[3]{b}}-\frac{\sqrt[5]{a}}{3b^2}\right)^7$. $\left(\frac{2a}{\sqrt{x}}-\frac{\sqrt{x}}{2\sqrt{a}}\right)^9$. $\left(\sqrt{\frac{a}{2}}-\sqrt{\frac{2}{3}}x\right)^{10}$.

147. If a days : b days = \$ c : \$ d, and e men : f men = \$ g : \$ h, show that ae days' labor : bf days' labor = \$ cg : \$ dh.

148. Factor $x^2+6\sqrt{y}x+9y$. 149. $ax^3+x+a+1$.

150. Solve $2(x+x^{-1})-x-x^{-1}=10$.

151. Resolve into partial fractions $\frac{8x^3-36x^2-2}{4x^2-8x-5}$.

152. Find the middle term of $(2x-\frac{1}{2}y)^{10}$.

153. $\left\{\frac{\sqrt{x+1}}{\sqrt{x-1}}-\frac{\sqrt{x-1}}{\sqrt{x+1}}\right\}\frac{\sqrt{x^3-1}}{\sqrt{x^2+x+1}}$. $\sqrt{\frac{3}{x}}+\sqrt{\frac{x}{3}}=2$.

154. How many different words of 3 consonants and 2 vowels can be formed from 7 consonants and 4 vowels.

155. Assume that y varies directly as x, and z varies inversely as x. If $y+z=7$ when $x=2$, and $y+z=-13$ when $x=-3$, find the value of y and z in terms of x.

156. Solve $2(x+a)[x-(x^2-a^2)^{\frac{1}{2}}]=a^2$.

157. Prove that if ax^2+bx+c is divisible by $x-n$, then will $an^2+bn+c=0$.

158. Solve $10\,x^{\frac{1}{12}}=3(x^{\frac{1}{3}}+x^{\frac{1}{4}})$. **159.** $3\,x^{\frac{1}{2}}-4\,x^{\frac{1}{4}}=7$.

160. Form the quadratic equation whose roots exceed by h the roots of $x^2+ax+b=0$.

161. Simplify $\dfrac{a^{-\frac{m}{n}}b^{\frac{1}{m}}+a^{\frac{1}{n}}b^{-\frac{n}{m}}}{ab}$. **162.** $\dfrac{\sqrt{18}}{\sqrt{3}+\sqrt{2}}+\dfrac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$.

163. $\sqrt{(xy)^{\frac{1}{2}}(x^{\frac{1}{2}}+4\,y^{\frac{1}{12}})+\dfrac{4\,y^{\frac{1}{2}}}{x^{\frac{1}{2}}}+\dfrac{1}{x}+2\,y^{\frac{1}{4}}(1+2\,y^{\frac{5}{12}})}$.

164. Solve $ax^2+bx+c=0$, and show what conditions will make both roots, (*a*) real and unequal, (*b*) real and equal, (*c*) imaginary.

165. What must be the value of x in order that the fourth term of the expansion of $(x-a)^5$ may be equal to the fourth term of the expansion of $(a+x)^{2n}$?

166. The number of permutations of n things taken r at a time : the number taken $r-1$ at a time as 10 : 1, and the corresponding combinations are as 5 : 3. Find n and r.

167. Derive the formula for the nth term and the sum of n terms of a geometrical progression. In what case and how is the formula for the sum modified?

Unless otherwise directed, solve each of the following equations for each letter it contains:

169. $v = \frac{s}{t}$. 170. $v = gt$. 171. $s = \frac{1}{2}gt^2$.

172. Eliminate t between 170 and 171. 173. $v = u + gt$.

174. $s = ut + \frac{1}{2}gt^2$. 175. Eliminate t between 173 and 174.

176. $v = u - gt$. 177. $s = ut - \frac{1}{2}gt^2$.

178. Eliminate t between 176 and 177.

179. Suppose $v = 0$ in 176, find s in terms of g and t from 176 and 177.

180. $s = \frac{g}{2}(2t - 1)$. 181. $Ft = mv$.

182. Eliminate v between 170 and 181. 183. $K = Fs$.

184. Find K in terms of m, v, and s from 170, 171, 181, 183.

185. $F = \frac{kgg'}{t^2}$. 186. $d = \frac{m_1d_1 + m_2d_2}{m_1 + m_2}$. 187. $t = \pi\sqrt{\frac{l}{g}}$.

188. $f = \frac{v^2}{r}$. 189. $\frac{v}{v'} = \frac{tp'}{t'p}$. 190. $\frac{a}{g} = \frac{h}{l}$.

191. Eliminate g between 171 and 190. 192. $\frac{a}{g} = \frac{h - l}{h + l}$

193. Eliminate g between 171 and 192. 194. $k = \frac{h}{l}$.

195. Eliminate h and l between 192 and 194.

196. Find a special case of 186 when $m_1 = m_2$.

197. $D = \frac{M}{V}$. 198. $v = \sqrt{\frac{e}{d}}$. 199. $n = \frac{1}{2l}\sqrt{\frac{t}{m}}$.

200. $\frac{1}{q} + \frac{1}{p} = \frac{1}{f}$. 201. $\frac{1}{q} - \frac{1}{p} = \frac{1}{f}$. 202. $F = \frac{9C}{5} + 32$.

203. $a = \frac{l - l_0}{l_0t}$. 204. $I = \frac{E}{R}$. 205. $I = \frac{E}{R + r}$.

206. $I = \frac{nE}{R + nr}$. 207. $I = \frac{E}{R + \frac{r}{n}}$. 208. $I = \frac{mE}{R + \frac{mr}{n}}$.

209. $\frac{a}{b} = \frac{c}{d}$. 210. $\frac{a + b}{a - b} = \frac{c + d}{c - d}$. 211. $\frac{a}{b} = \frac{b}{c}$.

212. $c^2 = a^2 + b^2$. 213. $a^2 = b^2 + c^2 - 2cp$. 214 $h_c^2 = b^2 - p^2$.

215. Eliminate p between 213 and 214. Solve for h_c.

216. $a^2 + b^2 = \left(\frac{c}{2}\right)^2 + m_c^2$. 217. $ab = t^2 - pq$. 218. $\frac{a}{b} = \frac{p}{q}$.

219. $p + q = c$. Eliminate p and q between 217, 218, 219.

220. $ab = Rh_c$. Eliminate h_c between 220 and the result of 215 and solve for R.

221. $\frac{a}{x} = \frac{x}{a - x}$. 222. $C = 2\pi R$. 223. $A = \pi R^2$.

GRAPHS.

574. We locate places on the earth with reference to two imaginary lines perpendicular to each other.

These two lines are the equator and the meridian which passes through Greenwich (London).

A place is located by giving its latitude, north or south, and its longitude, east or west.

This location indicates the *distances* and *directions* of the place from these two lines of reference.

So we may locate any point in the plane of two straight lines intersecting at right angles, provided we know its distances and directions from those two lines.

Let XX' and YY' on the next page be two straight lines intersecting at right angles at some point, as O, which point is called the *origin*.

This *horizontal* line of reference is called the ***x*-axis**, and the *vertical* line is called the ***y*-axis**.

575. The **Abscissa**, or **x-distance**, of a point is its perpendicular distance from the y-axis.

The abscissa of each lettered point in this figure is the line drawn from that point to the y-axis.

Abscissas measured to the right of the y-axis are *positive;* those measured to the left of it are *negative.*

We say the abscissa of F is -7, because it is 7 units of measure to the left of the y-axis.

Using a side of the square as the unit of measure, give the abscissa of each lettered point.

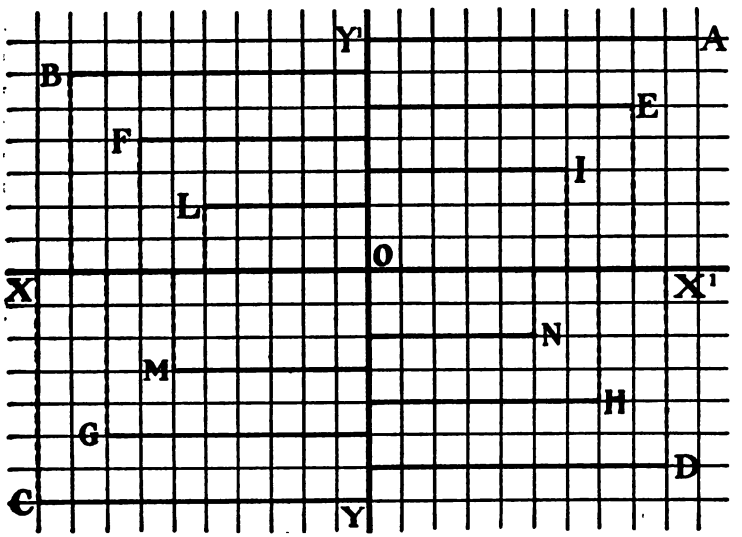
576. The **Ordinate**, or **y-distance**, of a point is its perpendicular distance from the x-axis.

The ordinate of each lettered point in this figure is the dotted line drawn from that point to the x-axis.

Ordinates measured above the x-axis are *positive*, and those measured below it are *negative*.

We say the ordinate of H is -4, because it is 4 units of measure below the x-axis.

The abscissa of any point on YY' is 0, and the ordinate of any point on XX' is 0.



Using a side of the square as the unit of measure, give the ordinate of each lettered point.

577. The **Coördinates** of any given point are the abscissa and the ordinate of that point.

The coördinates of a point determine the location of that point with respect to the lines of reference.

A point is designated by its coördinates, written (m, n), m denoting the abscissa and n the ordinate.

It is important to remember that the coördinates of a point represent units of distance.

The point $(4, 2)$ is 4 units of measure to the *right* of the y-axis and 2 units *above* the x-axis.

The point $(5, -2)$ is 5 units of measure to the *right* of the y-axis and 2 units *below* the x-axis.

The point $(-2, 5)$ is 2 units of measure to the *left* of the y-axis and 5 units *above* the x-axis.

The point $(-3, -3)$ is 3 units of measure to the *left* of the y-axis and 3 units *below* the x-axis.

The axes of coördinates divide the whole space in the plane into four quadrants, named as below.

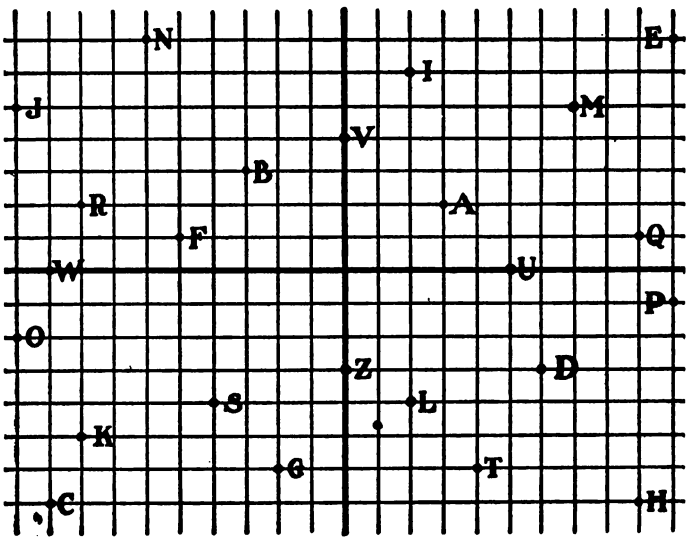
Quadrant 2	Quadrant 1
Quadrant 3	Quadrant 4

The signs of the coördinates of the points in the four quadrants are as follows:

Quadrant 1 $(+, +)$	Quadrant 3 $(-, -)$
Quadrant 2 $(-, +)$	Quadrant 4 $(+, -)$

Since the coördinates of any point are measured in like units, the side of a square is generally taken as the unit of measurement, and *squared paper* is used for determining the coördinates of a point, and for locating a point when the coördinates of it are known.

1. Write the coördinates of all the lettered points in the figure below. Thus, A is (3, 2).



578. **Plotting a point** is the process of indicating it on a plane as determined by its coördinates.

2. On squared paper plot the points whose coördinates are: (5, 3), (−3, 4), (2, −5), (−6, −4), (7, 0).

3. Plot the points whose coördinates are: (−5, 3), (0, 5), (4, 4), (3, −6), (−3, −7), (−4, 6), (5, −4).

Plot the following pairs of points, and join the points in each pair by a straight line:

4. (3, −5) and (4, 7).
5. (1, 4) and (5, 1).
6. (−6, 0) and (0, −8).
7. (4, 0) and (7, 0).
8. (3, 6) and (−3, −6).
9. (0, 3) and (0, 8).
10. (3, 3) and (−3, −3).
11. (0, 0) and (6, 9).

12. What may be noted about the straight line in Ex. 10? In Ex. 7? In Ex. 9? In Ex. 11? In Ex. 8?

13. Plot the points (4, 8), (4, 3), (7, 3), (7, 8), and join these points in succession by straight lines. Find the area of the figure formed by the four lines.

14. Plot (2, 5) and (− 6, 15) and join them by a straight line. Through what point does the line pass?

15. Plot the pairs of points, (− 8, 3) and (− 3, 2), (6, − 2) and (− 6, 7), and join the points in each pair by a straight line. Give the coördinates of the intersecting points.

16. Draw about the point (0, 5) a circle whose radius is 5, and name 12 points on this circle.

GRAPHS OF EQUATIONS.

579. We have learned that an indeterminate equation is an equation containing two or more general numbers, and that such equations are satisfied by an indefinite number of sets of values for the unknown numbers.

We may give y any value in the equation $x - y = 3$, and find a value for x that will satisfy the equation.

Assigning values to y and determining the corresponding values of x, we find the following solutions:

$x = 3$	$x = 4$	$x = 5$	$x = 6$	$x = 7$
$y = 0$	$y = 1$	$y = 2$	$y = 3$	$y = 4$
$x = 2$	$x = 1$	$x = 0$	$x = -1$	$x = -2$
$y = -1$	$y = -2$	$y = -3$	$y = -4$	$y = -5$

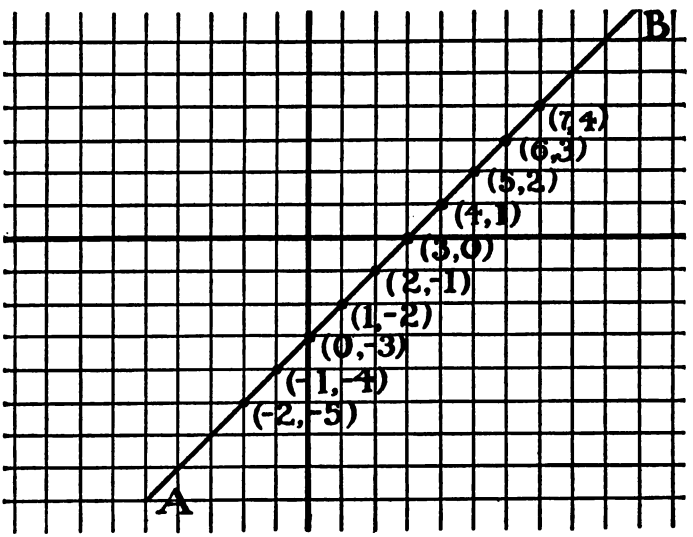
Each value of x with its corresponding value of y is called a *root* of the indeterminate equation.

Any root of such an equation may be written (m, n), m denoting the value of x, and n the corresponding value of y.

If we consider the values of x and y to denote units of distance from two lines of reference, each root of such an equation represents the coördinates of a point.

Plotting the points denoted by these roots, we find that they all lie on the straight line AB.

580. The **Graph of an Equation** is the line upon which are found all the points indicated by its roots.



It is proved in geometry that the graph of every simple equation is a straight line, and for this reason, simple equations are also called *linear equations.*

Since two points on a straight line determine the direction of it, to plot the graph of a linear equation, we need to find only two roots of the equation, plot the points denoted by those roots, and then draw a straight line through the two points.

The two points most easily located are those in which the graph intersects the coördinate axes.

For all points on XX', $y = 0$; hence, to find where the graph cuts XX', put $y = 0$ and find the value of x.

For all points on YY', $x = 0$; hence, to find where the graph cuts YY', put $x = 0$ and find the value of y.

The value of y which makes x equal to 0 is called the *intercept on the y-axis*, and the value of x which makes y equal to 0 is called the *intercept on the x-axis*.

To plot the graph of $x - 2y = 8$. If $x = 0$, $y = -4$, $(0, -4)$. If $y = 0$, $x = 8$, $(8, 0)$. Locate $(0, -4)$ and $(8, 0)$ and draw a straight line through them.

Plot the graphs of the following equations:

1. $x + 2y = 6$.	2. $y = 2x + 8$.	3. $2x + 3y = 12$.
4. $x - 3y = 9$.	5. $y = 3x - 9$.	6. $5x - 3y = 15$.
7. $x + 2y = 8$.	8. $y + 2x = 6$.	9. $3x + 6y = 18$.

Plot the graphs of the following equations in which one or both values are fractional:

1. $x + 2y = 7$.	2. $y = 3x - 7$.	3. $4x - 3y = 10$.
4. $x - 4y = 9$.	5. $y = 4x + 6$.	6. $2x + 3y = 11$.
7. $x + 3y = 8$.	8. $y = 2x - 9$.	9. $3x - 4y = 15$.

581. If an equation in two unknown numbers has a root (0, 0), its graph passes through the origin.

An equation can have such a root only when it has no absolute term; for if we assign 0 as the value of one unknown number, the value of the other cannot be 0, if there is an absolute term.

If an equation in two unknown numbers has no absolute term, its graph passes through the origin.

As the origin is one point on the graph, we have only to locate one other point to plot the graph. Find a root by letting x equal the coefficient of y.

Plot the graphs of the following equations:

1. $x = 2y$. 2. $x = 3y$. 3. $2x = -3y$.

4. $2x = -y$. 5. $y = 4x$. 6. $4y = -3x$.

582. Since x and y must have the same value in $ax = ay$, the coördinates of any point in the graph are equal.

This means that the graph passes through the origin and is equally distant from the coördinate axes at all points.

583. The equation $x = 4$ means that the abscissa of every point in the graph is equal to 4.

The graph of $x = 4$ is a straight line parallel to the y-axis and 4 units to the right of it.

The graph of $x = -2$ is a straight line parallel to the y-axis and 2 units to the left of it.

584. The equation $y = 3$ means that the ordinate of every point in the graph is equal to 3.

The graph of $y = 3$ is a straight line parallel to the x-axis and 3 units above it.

The graph of $y = -5$ is a straight line parallel to the x-axis and 5 units below it.

It follows from the above that the graph of $x = 0$ is the axis YY', and the graph of $y = 0$ is the axis XX'.

Describe the graphs of the following equations:

1. $x = -3$. 2. $x - 6 = 0$. 3. $x + 4 = 0$.

4. $y = -4$. 5. $y - 1 = 0$. 6. $y + 6 = 0$.

585. The usual arrangement of a linear equation for the purpose of plotting the graph is $y = mx + n$.

Other forms of simple equations in x and y may be reduced to this form by transposition and division.

By using this form we can find the equation whose graph passes through any two points except the origin.

Let $y = mx + n$ represent the equation two points in the graph of which are $(7, 5)$ and $(-4, 3)$. We have

$$5 = 7m + n \quad \text{and} \quad 3 = -4m + n.$$

Solving these equations for m and n, substituting the values in $y = mx + n$, and clearing of fractions,

$$11y = 2x + 41.$$

When one point in the graph of an equation is $(0, 0)$ the equation has no absolute term; therefore,

Let $y = mx$ represent the equation two points in the graph of which are $(3, 4)$ and $(0, 0)$. We have

$$4 = 3m.$$

Solving this equation for m, substituting the value of m in $y = mx$, and clearing of fractions, we have

$$3y = 4x.$$

In general, if two points in the graph of an equation are (a, b) and $(0, 0)$, the equation is $ay = bx$.

Form the equations which pass through the following pairs of points, and plot their graphs:

1. $(-4, 3)$ and $(5, 3)$.
2. $(7, 3)$ and $(2, 4)$.
3. $(-7, 8)$ and $(0, 0)$.
4. $(4, 4)$ and $(6, 6)$.
5. $(-4, 1)$ and $(3, -7)$.
6. $(2, 5)$ and $(4, 8)$.
7. $(-2, 7)$ and $(-2, -6)$.
8. $(5, 5)$ and $(0, 0)$.

9. Select from the following points those which fall on the graph of $y + 2x = 7$. Check by plotting.

$(2, 3)$ $(4, 1)$ $(5, -2)$ $(4, -1)$ $(3, 1)$ $(6, 0)$ $(0, 7)$

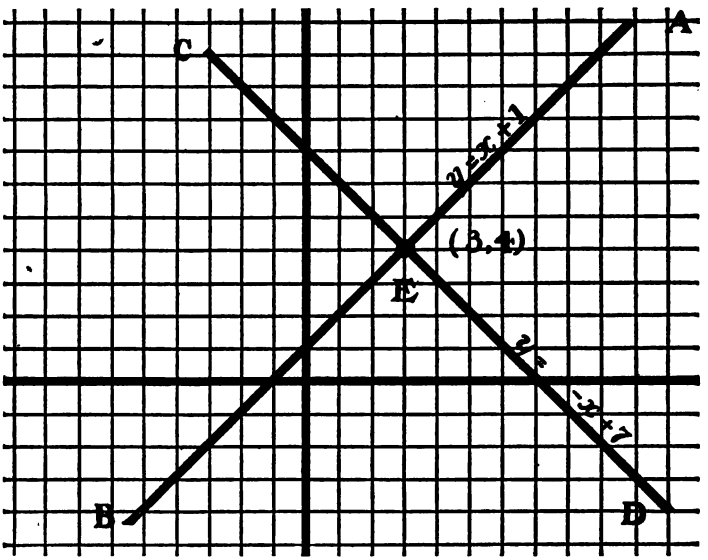
10. Form the equation whose graph is parallel to the graph of $2x - 6y = 12$, and passes through $(5, 0)$.

GRAPHS OF TWO OR MORE EQUATIONS.

586. We shall now consider the relations of the graphs of a system of linear equations in two unknown numbers.

Plotting the graphs of $y = x + 1$ and $y = -x + 7$, we find them to be AB and CD, which intersect at E.

The point $E(3, 4)$ lies on both graphs, and its coördinates indicate the common root of the equations.



Hence, $x = 3$ and $y = 4$, as may be verified by the usual method of solving simultaneous equations.

It follows that simultaneous linear equations may be solved by plotting their graphs, and taking for the root the coördinates of the point of intersection.

The graphs of any two simultaneous linear equations intersect. But since two straight lines can intersect in only one point, such equations have only one common root.

Solve the following equations by plotting their graphs and check by solving by elimination:

1. $\begin{cases} x-y=2 \\ x+y=6 \end{cases}$ 2. $\begin{cases} x+2y=4 \\ x=-4 \end{cases}$ 3. $\begin{cases} 2x-y=1 \\ x+y=5 \end{cases}$

4. $\begin{cases} x+y=8 \\ x-y=0 \end{cases}$ 5. $\begin{cases} y+2x=6 \\ y=-6 \end{cases}$ 6. $\begin{cases} x-2y=10 \\ x+y=-2 \end{cases}$

The graphical solution of simultaneous equations is little used, as it is difficult in most cases to determine exactly the coördinates of the point of intersection.

587. The graphs of two inconsistent equations do not intersect, that is, they are parallel.

$$2x+3y=12.$$
$$6x+9y=18.$$

The two points located on the coördinate axes are (0, 4) and (6, 0) for the first equation and (0, 2) and (3, 0) for the second, hence the graphs are parallel.

588. The graphs of any number of equivalent equations are identical.

This is evident, for by definition of equivalent equations each equation has all the roots of the others.

Determine from their graphs whether the following systems are independent, equivalent, or inconsistent.

1. $\begin{cases} 2x-y=20 \\ 4x+y=16 \end{cases}$ 2. $\begin{cases} x+y=7 \\ y=5-x \end{cases}$ 3. $\begin{cases} 2x=3y+10 \\ 9y=6x-30 \end{cases}$

4. $\begin{cases} 3y=2x-7 \\ 2x=7+3y \end{cases}$ 5. $\begin{cases} x=y-6 \\ y=8-x \end{cases}$ 6. $\begin{cases} 3x-4y=16 \\ 6x+4y=20 \end{cases}$

7. $\begin{cases} 2x+y=12 \\ 3x-y=0 \end{cases}$ 8. $\begin{cases} x=5+y \\ y=4+x \end{cases}$ 9. $\begin{cases} 2x=12+4y \\ 6y=3x-18 \end{cases}$

GRAPHICAL SOLUTION OF QUADRATICS.

589. The graphical solution of any quadratic equation in x is obtained from the graphs of a system, to which the given equation is equivalent.

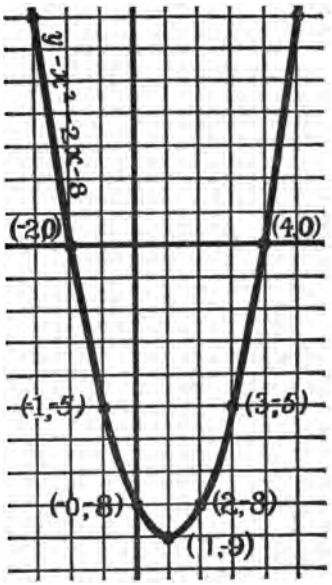
The quadratic equation, $x^2-2x-8=0$, is equivalent to the two equations,

$$\begin{cases} y=0 & (1) \\ y=x^2-2x-8 & (2) \end{cases}$$

for, if in (2) we substitute the value of y in (1), we get the equation, $x^2-2x-8=0$.

Therefore, all solutions of the system are solutions of the given equation, and the abscissas of the intersection of the graphs are the solutions of the given equation.

We know that the graph of (1) is the x-axis. We must plot the graph of (2) and determine the abscissas of the points where the graph cuts the x-axis.



When $x=$ 0, 1, 2, 3, 4, 5, −1, −2, −3,
$y=$ −8 −9 −8 −5 0 7 −5 0 7

Plotting these points and drawing a smooth curve through them, we have the graph of $y=x^2-2x-8$.

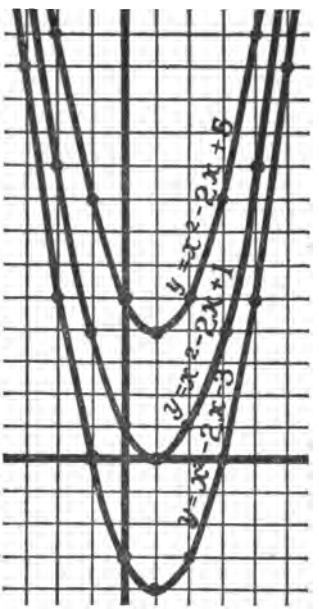
The curve obtained by plotting the graph of any quadratic equation of the form $y=x^2+ax+b$ is a **Parabola.**

We observe that the parabola intersects the x-axis at the points 4 and -2, which are the solutions of the system and also of the equation $x^2-2x-8=0$.

To solve graphically any quadratic in x, plot the graph of $y = x^2 + ax + b$, and determine the abscissas of the points where the graph cuts the x-axis.

Solve graphically the following equations:

1. $x^2 - 3x - 10 = 0$.
2. $x^2 - 8x + 12 = 0$.
3. $x^2 + 2x - 15 = 0$.
4. $x^2 + 9x + 14 = 0$.



590. Plotting the graphs of the equations,

$$y = x^2 - 2x - 3,$$
$$y = x^2 - 2x + 1,$$
$$y = x^2 - 2x + 5,$$

we obtain the three parabolas in this figure.

We observe that one parabola cuts the x-axis at the points 3 and -1, which are the solutions of $x^2 - 2x - 3 = 0$.

Another parabola is tangent to the x-axis at the point $(1, 0)$.

This means that the solutions of $x^2 - 2x + 1$ are both $+1$.

The third parabola, being above the x-axis, means that the solutions of $x^2 - 2x + 5 = 0$ are imaginary.

These are the only positions the parabola can have with reference to the x-axis. Hence,

If the roots of any quadratic equation in one unknown number, as $x^2 + ax + b = 0$, are:

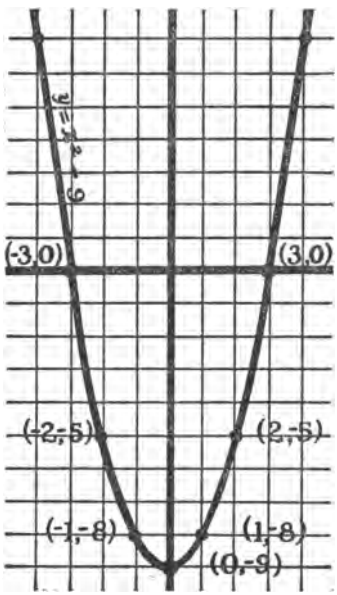
(1) *real and unequal, the graph of* $y = x^2 + ax + b$ *intersects the x-axis at two points.*

(2) *real and equal, the graph of* $y = x^2 + ax + b$ *is tangent to the x-axis.*

(3) *imaginary, the graph of* $y = x^2 + ax + b$ *is wholly above the x-axis.*

591. If the equation is a pure quadratic, $x^2 - 9 = 0$, the two equations to which it is equivalent are,

$$\begin{cases} y = 0 & (1) \\ y = x^2 - 9 & (2) \end{cases}$$



The graph of $y = x^2 - 9$ is the parabola in this figure.

The parabola of $y = x^2 - 9$ cuts the x-axis at the points 3 and -3, which are the solutions of the system, and also of the given equation.

Since the real roots of a pure quadratic are equal in absolute value and of opposite sign, the parabola obtained in solving graphically such an equation cuts the x-axis at points equally distant from the y-axis.

592. The lowest point of the parabola of any quadratic expression in x may be determined as follows:

$$x^2 - 2x - 8 = (x - 1)^2 - 9.$$

This expression has its negative value of greatest absolute value when $x = 1$, being then equal to -9.

Hence, the coördinates of the lowest point of the parabola of $y = x^2 - 2x - 8$ are $(1, -9)$.

$$x^2 + 2x + 8 = (x + 1)^2 + 7.$$

For no value of x is this expression negative. Hence, the lowest point of the curve is above the x-axis.

It has its least positive value when $x = 0$, and the coördinates of the lowest point of the curve are (0, 8).

$$2x^2 - 8x - 1 = x^2 - 4x - \tfrac{1}{2} = (x-2)^2 - \tfrac{9}{2}.$$

This expression has its negative value of greatest absolute value when $x = 2$, being then equal to $-\frac{9}{2}$.

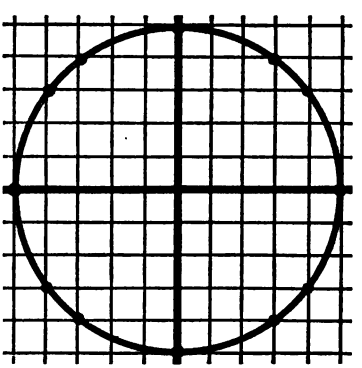
Hence, the coördinates of the lowest point of the parabola of $y = 2x^2 - 8x - 1$ are $(2, -\frac{9}{2})$.

$$x^2 + 8x + 16 = (x+4)^2.$$

For no value of x is this expression negative, but its value for $x = -4$ is 0. Hence, the coördinates of the lowest point of the parabola are $(-4, 0)$.

Solve the following equations graphically, determining fractional roots approximately:

1. $x^2 + 6x = -9$.
2. $x^2 - 16 = 0$.
3. $2x^2 + x = 15$.
4. $x^2 - 3x = -8$.
5. $x^2 + 10 = 0$.
6. $3x^2 - x = 30$.
7. $x^2 - 6x = -9$.
8. $x^2 - 25 = 0$.
9. $4x^2 + x = 18$.



593. We shall now consider the graphical solution of several quadratic equations in two unknown numbers.

1. $x^2 + y^2 = 25$.

As any value of x greater than 5 makes y imaginary, we substitute for x values between 5 and -5 only.

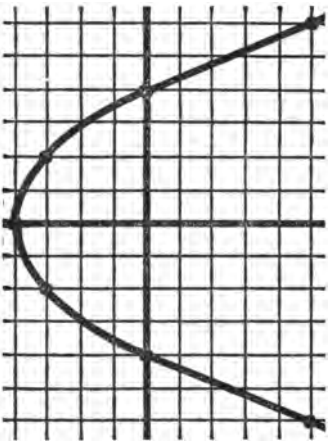
When $x = 0, \quad 3, -3, \quad 4, -4, 5, -5,$
$y = \pm 5, \pm 4, \pm 4, \pm 3, \pm 3, 0, \quad 0.$

Plotting these points and drawing a smooth curve through them, the graph appears to be a circle. It is a circle whose radius is 5, as may be proved by geometry.

The graph of $x^2+y^2=a^2$ is a circle whose radius is a and whose center is at the origin.

2. $y^2=4x+16$.

For any value of x less than -4, y^2 is negative and y imaginary; therefore no part of the graph lies to the left of $x=-4$.



Beginning with $x=-4$, we substitute values for x and find corresponding values of y, as follows:

When

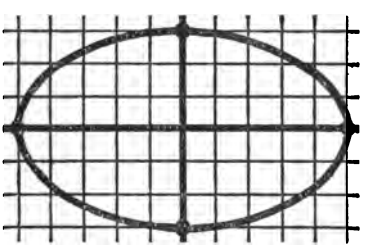
$x=-4,\ -3,\ \ 0,\ \ 5,$

$y=\ \ 0,\ \pm 2,\ \pm 4,\ \pm 6.$

The graph is a parabola and extends indefinitely to the right of the y-axis.

In the equation, $y^2=4x$, when $x=0$, $y=0$, and the parabola is tangent to the y-axis at the origin.

The graph of any equation in the form of $y^2=ax+c$ or $y^2=ax$ is a parabola.



3. $9x^2+25y^2=225$.

When

$x=0,\ \ \pm 1,\ \ \pm 4,\ \ \pm 5,$

$y=\pm 3,\ \pm 2.9,\ \pm 1.8,\ \ 0.$

For any value of x numerically greater than 5, y is imaginary, and no point of the graph can be more than 5 units to the right or to the left of the origin.

For any value of y numerically greater than 3, x is imaginary, and no point of the graph can be more than 3 units above or below the origin.

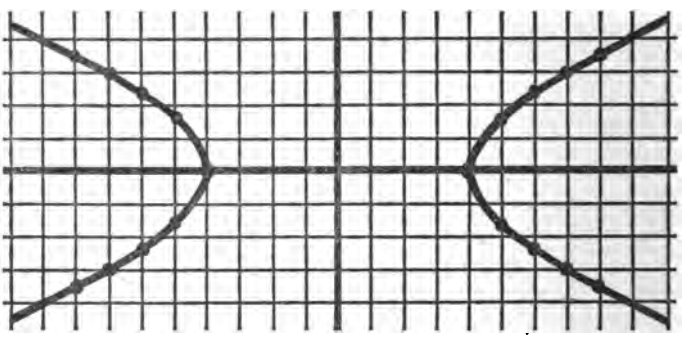
In plotting points for the graph, we therefore substitute for x only values between -5 and $+5$.

Plotting these points and drawing a smooth curve through them, we have the graph, which is an **Ellipse.**

The graph of any equation in the form of $a^2x^2 + b^2y^2 = a^2b^2$ *or* $ax^2 + by^2 = c$ *is an ellipse.*

4. $4x^2 - 16y^2 = 64$.

For any value of x less than 4, y is imaginary, and no part of the graph lies between $x = +4$ and $x = -4$.



In plotting points for the graph, we therefore substitute for x only values numerically greater than 4.

When $x =$	± 4,	± 5,	± 6,	± 7,	± 8,
$y =$	0,	± 1.5,	± 2.2,	± 2.8,	± 3.4.

We observe that half of the plotted points are on one side of the y-axis and half on the other side.

The two branches of the graph, which extend indefinitely from the origin, constitute an **hyperbola.**

The graph of $xy = a$ is an hyperbola the branches of which are wholly in diagonally opposite quadrants.

The graph of any equation in the form of $a^2x^2 - b^2y^2 = a^2b^2$, $ax^2 - by^2 = c$, *or* $xy = a$ *is an hyperbola.*

Construct the graphs of:

1. $4x^2 + 9y^2 = 36$.
2. $x^2 + y^2 = 36$.
3. $y^2 = 9x$.
4. $4x^2 - 9y^2 = 36$.
5. $x^2 - y^2 = 12$.
6. $xy = 8$.
7. $4x^2 - y^2 = -4$.

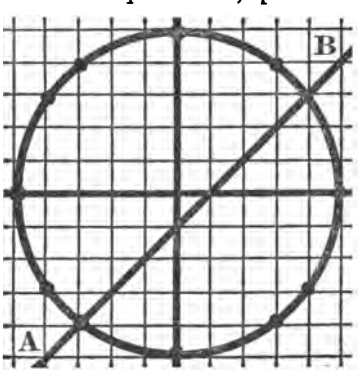
Several types of equations have been studied and their graphs determined, as follows:

$ax + by = c$	**Straight line.**
$x^2 + y^2 = a$	**Circle.**
$y^2 = ax + b$	**Parabola.**
$y^2 = ax$	**Parabola.**
$a^2x^2 + b^2y^2 = a^2b^2$	**Ellipse.**
$ax^2 + by^2 = c$	**Ellipse.**
$a^2x^2 - b^2y^2 = a^2b^2$	**Hyperbola.**
$xy = a$	**Hyperbola.**

594. We shall now consider the graphical solutions of a system of simultaneous quadratic equations.

To solve graphically simultaneous quadratics, plot the graph of each equation and determine the coördinates of the points where the graphs intersect or touch.

1. $\begin{cases} x^2 + y^2 = 25 & (1) \\ x - y = 1 & (2) \end{cases}$



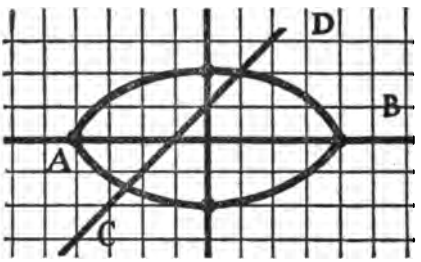
The graph of (1) is the circle whose center is at the origin and whose radius is 5. The graph of (2) is the straight line AB.

The straight line intersects the circle at the two points (4, 3) and (−3, −4), and the solutions are,

$$x = 4 \text{ and } 3, \quad y = -3 \text{ and } -4.$$

Interpret other relations that the graphs of two equations of this form might have.

2. $\begin{cases} 4x^2 + 16y^2 = 64 & (1) \\ x - y = -1 & (2) \end{cases}$



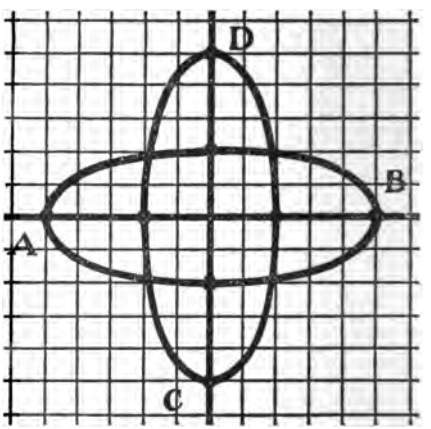
The graph of (1) is the ellipse *AB*, and the graph of (2) is the straight line *CD*.

These graphs intersect in two points, and the system has two solutions.

Interpret other relations that the graphs of two equations of this form might have.

3. $\begin{cases} 4x^2 + 25y^2 = 100 & (1) \\ 25x^2 + 4y^2 = 100 & (2) \end{cases}$

The graph of (1) is the ellipse *AB*, and the graph of (2) is the ellipse *CD*.



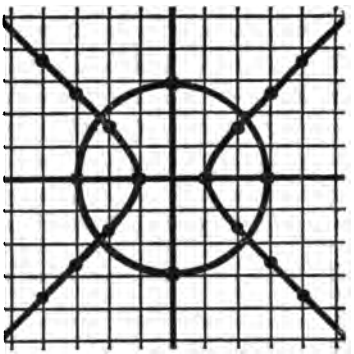
These ellipses intersect in four points, and the system has four solutions.

Interpret other relations that the graphs of two equations of this form might have.

Construct the graphs of the following systems and interpret their meaning:

1. $\begin{cases} x^2 + y^2 = 64 \\ 3x + 4y = 12 \end{cases}$ 2. $\begin{cases} x^2 + y^2 = 49 \\ y^2 = 4x \end{cases}$ 3. $\begin{cases} 9x^2 + 25y^2 = 225 \\ x^2 + y^2 = 4 \end{cases}$

4. $\begin{cases} 4x^2 - 4y^2 = 4 & (1) \\ x^2 + y^2 = 9 & (2) \end{cases}$



The graph of (1) is the hyperbola, and the graph of (2) is the circle.

These graphs intersect in four points, and the system has four solutions.

Interpret other relations that the graphs of two equations in this form might have.

It has been shown that an equation in two unknown numbers is represented by some kind of a line, straight or curved, and that the real roots of a system of such equations are the coördinates of the points where their graphs intersect or are tangent. Also, that when their graphs have no point in common, the roots are imaginary.

Since the coördinates of the points of intersection or tangency are often fractional or surd, they cannot be determined with any degree of accuracy. For this reason, graphical solutions are made use of more to determine the *nature* of the roots than to find the roots.

Plot enough points to determine the graphs of these systems, and interpret their meaning:

1. $\begin{cases} y^2 = 4x + 16 \\ x^2 + y^2 = 36 \end{cases}$

2. $\begin{cases} x - y = 4 \\ xy = -2 \end{cases}$

3. $\begin{cases} x^2 - 16y^2 = 16 \\ x^2 + y^2 = 9 \end{cases}$

4. $\begin{cases} 2y^2 - 3x = 5 \\ x^2 + y^2 = 5 \end{cases}$

5. $\begin{cases} x^2 + 25y^2 = 25 \\ 4x^2 - 3y^2 = 36 \end{cases}$

6. $\begin{cases} 4x^2 - 9y^2 = 36 \\ y^2 = 6x \end{cases}$

FACTORING.

595. Factor Theorem. — *If a rational integral polynomial containing x becomes zero when a is substituted for x, the polynomial is exactly divisible by $x - a$.*

Let D represent any rational integral polynomial containing x, which becomes zero when a is substituted for x.

Suppose that D is divided by $x - a$ until the remainder does not contain x. Let Q denote the quotient and R the remainder. Since the dividend is the product of the divisor and quotient, plus the remainder,

$$D = (x - a)\,Q + R.$$

By supposition, D becomes zero when a is substituted for x, and $x - a$ also becomes zero when a is substituted for x. Hence,

$$0 = 0 \cdot Q + R \text{ and } R = 0;$$

that is, the remainder is zero, and the division is exact.

Similarly, we may prove that if such a polynomial becomes 0 when $-a$ is substituted for x, it is exactly divisible by $x + a$.

$$x^3 + 3x^2 - 13x - 15.$$

Substituting 3 for x in this expression, the expression becomes 0, and it is therefore exactly divisible by $x - 3$. We also find that it becomes 0 when -1 and -5 are substituted for x. It is therefore exactly divisible by $x + 1$ and $x + 5$.

We shall now make use of the factor theorem to prove certain important truths in relation to the exact divisibility of one binomial by another.

596. *The difference of the same powers of two numbers is exactly divisible by the difference of the numbers.*

Let x represent any number, y any smaller number, and n any positive integer. Then $x^n - y^n$ will represent the difference of the same powers of any two numbers, and $x - y$ their difference.

On substituting y for x in $x^n - y^n$, the expression becomes $y^n - y^n$, or 0. Hence, $x^n - y^n$ is exactly divisible by $x - y$.

597. *The difference of the same even powers of two numbers is exactly divisible by the sum of the numbers.*

On substituting $-y$ for x in $x^n - y^n$, the expression becomes $y^n - y^n$, or 0. Hence, $x^n - y^n$ is exactly divisible by $x + y$.

598. *The sum of the same odd powers of two numbers is exactly divisible by the sum of the numbers.*

On substituting $-y$ for x in $x^n + y^n$, the expression becomes $-y^n + y^n$, or 0. Hence, $x^n + y^n$ is exactly divisible by $x + y$.

When we divide $x^5 - y^5$ by $x - y$, the quotient is

$$x^4 + x^3y + x^2y^2 + xy^3 + y^4.$$

When we divide $x^6 - y^6$ by $x + y$, the quotient is

$$x^5 - x^4y + x^3y^2 - x^2y^3 + xy^4 - y^5.$$

When we divide $x^7 + y^7$ by $x + y$, the quotient is

$$x^6 - x^5y + x^4y^2 - x^3y^3 + x^2y^4 - xy^5 + y^6.$$

599. We observe the following laws of signs and exponents when one binomial is divided by another.

We find x in every term of the quotient except the last, and y in every term except the first.

When the divisor is the difference of two numbers, all the terms of the quotient are positive.

When the divisor is the sum of two numbers, the terms of the quotient are alternately positive and negative.

The exponent of x in the first term of the quotient is the exponent of x in the dividend diminished by the exponent of x in the divisor, and decreases in each succeeding term by the exponent of x in the divisor.

The exponent of y in the second term of the quotient is the exponent of y in the divisor, and increases in each succeeding term by the exponent of y in the divisor.

FACTORING BY THE FACTOR THEOREM.

600. We have learned that if any rational integral polynomial containing x reduces to 0 when a is substituted for x, the polynomial is exactly divisible by $x - a$.

$$x^3 + 5x^2 - 2x - 24.$$

The second term of any binomial factor of this polynomial must be a factor of 24; hence, the *only numbers* that need be substituted for x in this trial are the *factors* of the *last term*.

By trial we find that this polynomial reduces to 0 when 2 is substituted for x. Hence,

$$x^3 + 5x^2 - 2x - 24$$

is divisible by $x - 2$. Dividing it by $x - 2$, we obtain $x^2 + 7x + 12$, and factoring this quotient, we have

$$x^3 + 5x^2 - 2x - 24 = (x - 2)(x + 3)(x + 4).$$

If such a polynomial reduces to 0 when $-a$ is substituted for x, the polynomial is divisible by $x + a$.

$$x^3 + 7x^2 + 14x + 8.$$

It is evident that the substitution of a positive number for x will not reduce this polynomial to 0.

By trial we find that this polynomial reduces to 0 when -1 is substituted for x. Hence,

$$x^3 + 7x^2 + 14x + 8$$

is divisible by $x + 1$. Dividing it by $x + 1$, we obtain $x^2 + 6x + 8$, and factoring this quotient, we have

$$x^3 + 7x^2 + 14x + 8 = (x + 1)(x + 2)(x + 4).$$

1. $a^3 - 14a^2 + 35a - 22$.
2. $x^3 - 12x^2 + 39x - 28$.
3. $a^3 - 11a^2 + 31a - 21$.
4. $x^4 - 15x^2 - 10x + 24$.
5. $a^3 - 16a^2 + 71a - 56$.
6. $x^4 - 25x^2 + 60x - 36$.
7. $a^3 + 10a^2 - 17a - 66$.
8. $x^3 + 10x^2 + 33x + 36$.
9. $a^3 - 13a^2 + 47a - 35$.
10. $x^4 - 35x^2 + 90x - 56$.
11. $a^3 - 14a^2 + 35a - 22$.
12. $x^3 - 10x^2 + 31x - 30$.
13. $a^3 - 12a^2 + 41a - 42$.
14. $x^4 - 23x^2 - 18x + 40$.
15. $a^3 + 15a^2 + 48a + 44$.
16. $x^4 - 33x^2 - 28x + 60$.

601. Polynomial squares of more than three terms can be resolved into two like polynomial factors.

By Article 271, such polynomials consist of the squares of three or more numbers together with twice the product of each number by each one that follows it.

$$a^2 + b^2 + c^2 - 2ab + 2ac - 2bc.$$

Since this polynomial consists of the squares of the numbers a, b, and c, and twice the product of each number by each one that follows it, the polynomial may be the square of a trinomial.

Since $2ab$ is negative, a and b must have unlike signs. If a is positive, b is negative; if a is negative, b is positive.

Since $2ac$ is positive, a and c must have like signs. If a is positive, c is positive; if a is negative, c is negative.

We find by trial that the polynomial is the square of $a - b + c$ or $-a + b - c$, and the factors are

$$(a - b + c)(a - b + c) \text{ or } (-a + b - c)(-a + b - c).$$

In factoring such polynomial squares, it is customary to give only those factors whose first terms are positive.

The square roots of the squares are the terms of each of the factors, if the polynomial is a square.

After we find what the terms of the factors must be, we consider the first term of them positive, and then ascertain from terms that are products whether the other terms have like or unlike signs. Knowing the terms and their signs, we know the factors.

$$x^2 - 6x + y^2 - 6y + 9 + 2xy.$$

This polynomial consists of the squares of x, y, and 3 and twice the product of each number by each one that follows it.

Since $2xy$ is positive, x and y must have like signs. Since $6x$ is negative, x and 3 must have unlike signs. We find by trial that the polynomial is the square of $x + y - 3$, and

$$x^2 - 6x + y^2 - 6y + 9 + 2xy = (x + y - 3)(x + y - 3).$$

1. $a^2 + b^2 - 2ab - 2a + 2b + 1.$
2. $x^2 + y^2 - 8x + 8y - 2xy + 16.$
3. $x^2 - 2xz + y^2 - 2yz + z^2 + 2xy.$
4. $4a^2 + 20a - 8ab + 4b^2 - 20b + 25.$
5. $9a^2 - 12b + 4b^2 + 18a - 12ab + 9.$
6. $a^2 - 6ac + 4b^2 - 12bc + 9c^2 + 4ab.$
7. $4x^2 + 24yz + 16z^2 - 12xy + 9y^2 - 16xz.$
8. $16a^2 - 40ay + 20xy - 16ax + 4x^2 + 25y^2$